Mass Scales of the String Unification

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Although mass scales associated with string tension, compactification of extra dimension, and Newtonian gravity are *a priori* independent, these scales must be approximately equal to each other in any unified string theory with realistic couplings. Consequently, the compactification cannot be adequately described in terms of ten-dimensional field theory.

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Superstrings[1] offer an exciting possibility of constructing a consistent theory of *all* particle interactions. The naive low-energy limit of superstring theories is a ten-dimensional $N=1$ supergravity coupled to an SO(32)[2,3] or $E_8 \otimes E_8$[3] gauge supermultiplet. However, superstrings also allow for vacua with six dimensions forming a compact manifold Q, and the low-energy physics being described by a four-dimensional field theory; in particular, when Q is a Calabi-Yau manifold, low-energy theory is $N=1$ supersymmetric.[4] The size of Q is *a priori* unrelated to the string tension α', and so a unified string theory possesses two independent mass scales—M_{comp} and M_{string}—in addition to the phenomenological scale M_{Planck}. [The grand-unification-theory (GUT) scale M_{GUT} is not an independent scale: Existence of chiral fermions in four dimensions requires six-dimensional gauge fields to have nontrivial vacuum expectation values, so that E_8^2 or SO(32) is spontaneously broken at M_{comp}.] It will be shown that realistic unified string theories do not allow for hierarchies between these scales. Instead, one should have

$$M_{\text{GUT}} \simeq M_{\text{comp}} \sim M_{\text{str}} \sim M_{\text{Pl}} \tag{1}$$

in any realistic superstring model. Implications of (1) on superstring model buildings are discussed at the end of this paper.

Let us assume for a moment that contrary to (1), $M_{\text{comp}} << M_{\text{str}}$. In this case physics for energies between M_{comp} and M_{str} would be describable by a ten-dimensional field theory, namely, by an $N=1$ supersymmetric gauge theory coupled to supergravity (energy increases to the left):

$$(\text{superstring}) \rightarrow (d=10,\ N=1 \text{ supergravity}) \rightarrow (d=4 \text{ field theory}).$$

Note that it is not assumed that the gauge group is $E_8 \otimes E_8$ or that the string is heterotic or that the six-manifold Q is of Calabi-Yau type.

Let us consider quantum effects in the ten-dimensional theory. Their magnitude is controlled by $g_{10}^2\Lambda^6$ and $\kappa_{10}^2\Lambda^8$, where g_{10} and κ_{10} are coupling constants of gauge and gravitational sectors, respectively, and Λ is the cutoff; the dependence on Λ is explicit since the theory is not renormalizable. For sufficiently high Λ quantum corrections become large and we can no longer expect the behavior of the quantum theory to resemble the classical behavior. While our knowledge of strongly interacting ten-dimensional field theories is severely limited, our experience with four-dimensional theories suggests several alternatives.

(1) Quantum corrections never become large if the theory is effectively cut off at some scale $\Lambda << \Lambda_0 = \min(g_{10}^{-1/3}, \kappa_{10}^{-1/4})$; above Λ the theory is replaced by a different one. A four-dimensional example of such behavior is provided by the ψ^4 theory of weak interactions that becomes a gauge theory above M_W. Since there are no renormalizable field theories in ten dimensions, one must replace the $d=10$ supergravity directly with a string theory. Hence, this alternative corresponds to $M_{\text{str}} << \Lambda_0$.

(2) The spectrum of a strongly interacting theory may form Regge trajectories, with light particles corresponding to classical fields, but heavy particles having no classical analogs; the Regge slope α' should be $O(\Lambda_0)$. If $d=10$ supergravity actually behaves in this way, and if its massive states match those of a superstring theory, that would imply that supergravity is a low-energy manifestation of superstrings. In this case $M_{\text{str}} \simeq \Lambda_0$. Alternatively, massive states of strongly coupled supergravity may not resemble superstring excitations (e.g., Regge trajectories may be nonlinear). If this indeed happens, then quantum supergravity cannot be reconciled with superstring unification.

(3) Finally, a strongly interacting quantum theory may be so unlike its classical (or weakly coupled) counterpart that there is no correspondence between their spectra. A four-dimensional example of such behavior is provided by QCD. Obviously, quantum supergravity that behaves in this way cannot be reconciled with a perturbative string theory. Unfortunately, nothing is known about nonperturbative string theory, and so we are left with a loophole in our proof. However, even the low-energy behavior of quantum super-

gravity behaving in this way should be very unlike that of the classical theory, so that we cannot use the latter to study spontaneous compactification of the extra six dimensions. Thus, although this case is not rigorously proven to be inconsistent, it cannot be used for the building of unified models, since we would not be able to analyze them with currently available techniques.

It is concluded that a consistent superstring model should have

$$g_{10} \lesssim M_{\rm str}^3 \quad \text{and} \quad \kappa_{10} \lesssim M_{\rm str}^4. \qquad (2)$$

Now consider coupling constants of the four-dimensional theory. For gauge symmetries which are not broken by the compactification, their gauge fields are constant on Q; the same is true for the $d=4$ graviton. Thus, given the volume of Q, we can compute g_4 and κ_4 in terms of the ten-dimensional couplings as follows:

$$g_4 = g_{10} M_{\rm comp}^3, \quad \kappa_4 = \kappa_{10} M_{\rm comp}^3 \qquad (3)$$

($M_{\rm comp}$ is normalized such that the volume of Q is $M_{\rm comp}^{-6}$). Combining (2) and (3) together we arrive at $g_4 \lesssim (M_{\rm comp}/M_{\rm str})^3$ and $\kappa_4 \lesssim M_{\rm comp}^3/M_{\rm str}^4$, i.e.,

$$\alpha_4 \leqslant O((M_{\rm comp}/M_{\rm str})^6), \qquad M_{\rm Pl} \geqslant O(M_{\rm str}^4/M_{\rm comp}^3). \qquad (4)$$

We see that if $M_{\rm comp} << M_{\rm str}$ then $\alpha_4 << 1$ and $M_{\rm str} << M_{\rm Pl}$. Note that the powers of $M_{\rm comp}/M_{\rm str}$ involved in (4) are so large that it does not take a large hierarchy between $M_{\rm str}$ and $M_{\rm comp}$ to impose a ridiculously low upper limit on α_4. For example, $M_{\rm str} \approx 10 M_{\rm comp}$ would require $\alpha_4 \lesssim 10^{-6}$!

Formula (3) involves classical, i.e., tree-level couplings. In a quantum theory it should be interpreted as a relation between effective (running) couplings as evaluated at $M_{\rm comp}$. Since $M_{\rm comp}$ is also $M_{\rm GUT}$ we should interpret α_4 as $\alpha_{\rm GUT}$ and use conventional renormalization-group analysis to compute the latter from the low-energy gauge couplings. Given $\Lambda_{\rm QCD} = O(100 \text{ MeV})$ and $M_{\rm GUT} \leqslant M_{\rm Pl}$, $\alpha_{\rm GUT}$ cannot be made lower than $O(\frac{1}{100})$ even if allowance is made for an extended color group at intermediate energies. Hence, a realistic model should have $M_{\rm comp}/M_{\rm str} \gtrsim (\frac{1}{100})^{1/6} \approx 0.46$, which is clearly inconsistent with the assumption of $M_{\rm comp} << M_{\rm str}$. Therefore, that assumption must be wrong and *realistic unified string models must have* $M_{\rm comp} \sim M_{\rm str}$.

A slightly modified version of the above arguments should be used if the compactification of six extra dimensions proceeds in stages. For example, two of the six dimensions may be compactified at some scale M_c below the compactification scale M_C of the other four. In such a case one should use a six-dimensional field theory[5] for energies above M_c and below M_C. The same arguments that gave us (4) before now yield $\alpha_4 < O((M_c/M_{\rm str})^2 (M_C/M_{\rm str})^4)$ with the result that $M_{\rm str}/M_c < O(1/\sqrt{\alpha_{\rm gut}}) \lesssim 10$ even if M_C is as big as $M_{\rm str}$. Thus, while we may have a substantial difference between $M_{\rm str}$ and the lowest compactification scale M_c, we cannot have a large hierarchy between the two.

Now let us prove the other part of (1), namely, the relation between $M_{\rm str}$ and $M_{\rm Pl}$. In the type-I superstring theory $M_{\rm str}$ is related to the ten-dimensional couplings as $g_{10} \sim M_{\rm str}^{-3} G$ and $\kappa_{10} \sim M_{\rm str}^{-4} G^2$, where G is the dimensionless string coupling. Taking (3) into account we see that $G \sim g_4 (M_{\rm str}/M_{\rm comp})^3$ and $\kappa_4 \sim G g_4/M_{\rm str}$, i.e., $M_{\rm str} \sim M_{\rm Pl} g_4 G$ and $M_{\rm comp} \sim M_{\rm Pl} \times g_4^{4/3} G^{2/3}$. Combining this with the limit $g_4 \lesssim (M_{\rm comp}/M_{\rm str})^3 \lesssim 1$ we arrive at $g_4 \lesssim G \lesssim 1$ and

$$O(g_4^2) M_{\rm Pl} < M_{\rm str} < O(g_4) M_{\rm Pl}, \qquad O(g_4^2) M_{\rm Pl} < M_{\rm comp} < O(g_4^{4/3}) M_{\rm Pl}. \qquad (5)$$

If we allow for reasonable numerical factors, (5) should be interpreted as $M_{\rm comp} \sim M_{\rm str} \sim 10^{18 \pm 1}$ GeV.

In the heterotic case, expressions for g_{10} and κ_{10} involve equal powers of the string coupling G. Therefore, $M_{\rm str} \sim g_{10}/\kappa_{10} = g_4/\kappa_4 \cong g_4 M_{\rm Pl}$. Specifically, Gross *et al.*[6] give us $g_{10} = 2\kappa_{10}$ in the string units. String units are defined such that $\alpha' = \frac{1}{2}$; if we define $M_{\rm str}$ as the mass of lightest massive string excitation, then in string units $M_{\rm str}^2 = 8$. Hence $g_{10} = \kappa_{10} M_{\rm str}/\sqrt{2}$, $g_4 = \kappa_4 M_{\rm str}/\sqrt{2}$, or $\alpha_4 = M_{\rm str}^2/M_{\rm Pl}^2$. Here $\alpha = 4$ is the tree-level gauge coupling normalized such that the E_8^2 (or D_{16}) Lie algebra has roots of length $\sqrt{2}$; comparing this with the usual normalization of $\alpha_{\rm GUT}$ we find $\alpha_{\rm GUT} = 2\alpha_4$. Thus,

$$M_{\rm str} = M_{\rm Pl} (\alpha_{\rm GUT}/2)^{1/2} \approx (1.5\text{–}5) \times 10^{18} \text{ GeV}, \qquad (6)$$

since most supersymmetric GUT's have $0.03 < \alpha_{\rm GUT} < 0.3$.

What are the implications of (1)? First of all, we can no longer analyze the physics of spontaneous compactification in terms of the conventional $d=10$, $N=1$ supergravity Lagrangean,[7] or the anomaly-free Lagrangean of Ref. 2. In order to make this point obvious, note that integrating out the heavy string modes results in operators of arbitrarily high dimension. The relative magnitude of these operators is controlled by powers of $\mu/M_{\rm str}$, where μ is the energy scale of a process under consideration. The energy scale of the compactification is of course $M_{\rm comp}$; when $M_{\rm comp}/M_{\rm str} = O(1)$, higher-dimensional operators are no longer small and have to be taken into account. The only consistent way to do so is to ignore the $d=10$ field theory altogether and to study superstrings in nontrivial backgrounds. "Equations of motion" that determine the shape of Q and its size can be obtained from the requirements of the conformal invariance of the world sheet.[8]

In the case of $M_{\rm comp} << M_{\rm str}$ preservation of $N=1$ supersymmetry in four dimensions required Q to be Kähler and Ricci flat, i.e., to be a Calabi-Yau manifold.[4] The proof does not hold for $M_{\rm comp} \sim M_{\rm str}$ and there may be supersymmetric solutions that cannot be continuously deformed into the flat ten-dimensional space. Unlike Calabi-Yau manifolds, such solutions may allow for the dynamical breaking of supersymmetry without destabilization of the vacuum.[9,10] Whether such solutions actually exist and whether they yield realistic four-dimensional theories remains to be seen.

From the four-dimensional point of view the most important aspect of (1) is $M_{\rm GUT} \sim M_{\rm Pl}$—well above conventional GUT scales. An immediate implication of a high GUT scale is slow proton decay: As long as $M_{\rm GUT} > 10^{17}$ GeV, proton's lifetime is longer than about 10^{37} yr. (It is assumed that proton decay is not controlled by an intermediate mass scale,[11] since that would make the decay unrealistically fast.) Thus, *unified string theories predict a proton decay rate that is too slow to be experimentally observable* at present.

Finally, apart from its immediate implications on proton stability, a high GUT scale is a constraint on superstring model building. Going through the list of Calibi-Yau–based models of Ref. 11 and comparing phenomenological values of $M_{\rm GUT}$ with (1), one sees that the latter favors models A4 $[\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)^2]$ and B2 $[\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)^2]$ with extended flavor group and excludes models C1 and C2 $[\mathrm{SU}(4)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)^{(2)}]$ with extended color group. Minimal extensions of the standard model $[\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \cdots]$ have $M_{\rm GUT} \simeq 10^{17}$ GeV and their status *vis-à-vis* (1) cannot be determined without further analysis.[12] Of course, different gauge groups might be favored by (1) if the internal manifold is not of the Calabi-Yau type.

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[12]One has to consider numerical factors in (1) as well as higher-loop corrections to renormalization-group equations for $M_{\rm GUT}$; the latter are especially important for the four-family models.