Band Termination at Very High Spin in ¹⁵⁸Yb

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The unusual yrast spectrum of ¹⁵⁸Yb at very high spins is interpreted in terms of a band termination at $I^{\pi} = 36^+$. A similar spectrum with band terminations is obtained in microscopic cranking calculations for this nucleus. A basic element of the theory is that the finiteness of rotational bands is connected with a gradual shape change until the rotation takes place around a symmetry axis.

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Universal or recurrent features of the interplay between collective and single-particle degrees of freedom in nuclei can often be derived from solvable models. A simple microscopic model to describe rotations of a many-fermion system is the rotating harmonic-oscillator potential for independent particles.¹ At deformations typical for nuclear ground states, this integrable model generates rotational bands which all behave in a similar fashion: With increasing spin the intrinsic shape evolves until it is symmetric around the axis of rotation. At this point the rotation is no longer collective; instead the wave function is a many-particle, many-hole configuration with the single-particle angular momenta aligned along the symmetry axis. The ground band of the ²⁰Ne nucleus, for example, behaves like a terminating band of the rotating harmonic-oscillator model.² The low-spin states of this nucleus are collective, but the ground band terminates at spin I = 8 with a noncollective state corresponding to oblate intrinsic shape. The latter state is generated by maximum spin alignment in the $d_{5/2}$ shell of the two valence protons and two valence neutrons outside a ¹⁶O core. In contrast, the lowerspin states arise in this picture² from collective rotation of a prolate $(I^{\pi} = 0^+)$ or slightly triaxial $(I^{\pi} = 2^+, 4^+, 4^+)$ and 6^+) shape.

This Letter presents evidence that analogous terminating bands also characterize the high-spin behavior of a much heavier nucleus with a large number of valence particles. The actual observation of a band termination is usually not feasible in heavy nuclei because in most cases the highest-spin states of the terminating bands lie far above the yrast line. However, favorable conditions have been anticipated³ in the region of nuclei with 10–12 valence particles outside a ${}^{146}_{64}Gd_{82}$ core, and it will be seen that recent experimental data⁴ on ${}^{158}_{70}Yb_{88}$ fulfill the theoretical expectations.

The model used in Ref. 3 is similar to the cranked harmonic-oscillator model mentioned above except that a more realistic Nilsson modified-oscillator potential was used and, furthermore, the average energy dependence on spin and deformation was renormalized to the rotating-liquid-drop behavior.⁵ In the calculations, a minimization with respect to shape is carried out separately for each configuration at each spin, and an effort is made to construct bands by tracing of the configurations through a sequence of spins. All particles are treated on the same footing, but for the purpose of the discussion it is convenient to distinguish between a ${}^{146}_{64}$ Gd₈₂ core and the valence particles in the $\pi(h_{11/2}, d_{3/2})$ and $\nu(f_{7/2}, h_{9/2}, i_{13/2})$ shells outside the core. It may be remarked that the ¹⁴⁶Gd core is broken by the ground-state deformation, but according to the calculations the core is restored in the high-spin configurations of interest so that ¹⁵⁸Yb has strictly twelve valence particles. It turns out that these particles form collective structures, but also that terminating states where the spin vectors are fully or almost fully aligned are competitive in energy.

Figure 1 puts some perspective on the experimental yrast excitation energies of ¹⁵⁸Yb by plotting them versus spin alongside other recent data⁶ for the considerably heavier isotope ¹⁶⁸Yb, and also the experimental levels^{7,8} of a lighter nucleus, ¹⁵²₆₆Dy₈₆, which has only six particles outside the ¹⁴⁶Gd core. The yrast energies are shown relative to a smooth rotational-like reference, 0.007I(I+1) MeV. The ¹⁵²Dy levels scatter around a roughly constant average value and thus abide by rigid rotational behavior in an average sense.



FIG. 1. Experimental yrast spectra of the nuclei 152 Dy, 158 Yb, and 168 Yb (Refs. 4 and 6–8) plotted relative to a rigid rotational reference curve. The intrinsic shapes which underlie these spectra according to theory are indicated with quadrupole shape coordinates as defined in Ref. 5.

The trends are smoother for ¹⁵⁸Yb and ¹⁶⁸Yb, which suggests collective band structure for these two nuclei. It is remarkable that the heavier and more collective nucleus, ¹⁶⁸Yb, *loses* energy linearly with increasing spin relative to the reference, while the lighter and less deformed nucleus, ¹⁵⁸Yb, *gains* energy. In fact, over the spin range I = 30-36 the nucleus ¹⁵⁸Yb gains ~ 500 keV of energy relative to the reference which consists of rotation of a rigid body with a fairly large quadrupole deformation $\epsilon \simeq 0.3$.

The different characteristic behaviors at high spin for nuclei with $N \leq 90$ and $N \geq 90$, respectively, have been predicted by the Nilsson-Strutinsky cranking model.^{3,5} Calculated results are presented for ^{158,168}Yb in Fig. 2 together with the experimental data, and can be found for ¹⁵²Dy, e.g., in Dossing, Neergård, and Sagawa.⁹ The fact that an overall agreement with experiment is found lends some credibility to the underlying microscopic phenomena in the theory. The yrast states of ¹⁵²Dy are obtained from the theory as a sequence of single-particle configurations with spins aligned with an oblate symmetry axis, those of ¹⁶⁸Yb arise from the collective rotation of an almost fixed prolate intrinsic shape, while in ¹⁵⁸Yb the energy



FIG. 2. Angular momentum vs E2 transition energy along the positive-parity yrast lines of ¹⁵⁸Yb and ¹⁶⁸Yb, from experiment (filled circles; Refs. 4 and 8) and from Nilsson-Strutinsky cranking-model calculations (open circles). For ¹⁵⁸Yb, the calculated collective bands shown here correspond to the configurations $\pi_4\nu_5$ or $\pi_4\nu_6$ of Fig. 3. They terminate at I = 36 and 42.

increment required to reach high spins is lowered relative to these cases because of a successive shape change from prolate to oblate (in ²⁰Ne an "upbend" was also calculated¹⁰ and observed experimentally at the termination of the ground band). We have carried out the present calculations using both the modifiedoscillator potential with parameters as in Ref. 3 and, alternatively, a Woods-Saxon single-particle potential,^{11,12} i.e., with no free parameters in the present application. Pairing correlations were neglected.

For ¹⁶⁸Yb the observed transition energies follow a rotational sequence (Fig. 2) with a moment of inertia about 85% of the rigid-body value at the calculated equilibrium deformation. For ¹⁵⁸Yb and spins I = 20-36, on the other hand, the transition energies stay roughly constant around $E_{\gamma} \sim 0.75$ MeV. This equidistant spacing of levels ceases abruptly above I = 36. The next levels are found considerably higher in energy, suggesting a band termination at I = 36. The transition energies above this apparent band termination are still not as high as the transition energies at the same spins in ¹⁶⁸Yb.

The theoretical calculations reproduce the observed behavior of ¹⁵⁸Yb qualitatively. A fully quantitative agreement cannot be expected *a priori* for a transitional nucleus like ¹⁵⁸Yb, considering the uncertainty in the overall prolate-oblate energy difference and singleparticle level spacings, and the possible effect of residual pairing in some configurations. Nevertheless, it is instructive to take a closer look in order to obtain a more precise scenario for what happens at the band termination. Figure 3 shows the detailed structure of the positive-parity yrast line for ¹⁵⁸Yb as it emerges from theory. The calculated ¹⁵⁸Yb yrast states for spins I = 24-38 have noncollective character and would give rise to an irregular yrast line like the one observed in ¹⁵²Dy (Fig. 1). That possibility is, how-



FIG. 3. Positive-parity levels along the yrast line of ¹⁵⁸Yb from a Woods-Saxon-Strutinsky cranking-model calculation, plotted as in Fig. 1. The solid curves represent bands. Band-terminating levels are indicated by dots, which are encircled for those cases where the spin vectors of all twelve valence particles are aligned. Proton (π) and neutron (ν) valence configurations are defined in the figure. The dashed line shows the yrast line from the calculation, and the dot-dashed line shows the yrast line that would result if the energy of the configuration ν_4 were somewhat lower.

ever, excluded by the smoothness of the ¹⁵⁸Yb experimental yrast sequence which suggests instead a band structure for $I \leq 36$, maybe with band crossings at $I \approx 26$ and $I \approx 30$ (see Fig. 1). The experimental situation can in fact be recovered from Fig. 3 by the assumption that the energy of the configuration v_4 was calculated a few hundred kiloelectronvolts too high relative to the configurations v_1 and v_3 . Thus, if the configuration v_4 is arbitrarily lowered a few hundred kiloelectronvolts, the band $\pi_2 \nu_4$ which terminates with the I = 36 yrast level comes down through the shaded region in Fig. 3 and becomes yrast also for a range of lower spins. The calculated downslope of the band $\pi_2 \nu_4$ in Fig. 3 is more than enough to account for the anomalous downslope of the experimental ¹⁵⁸Yb yrast line in Fig. 1. Lowering the I = 36 band termination in Fig. 3 also accounts for the feeding of this state by higher-energy transitions as observed in experiment.

The Woods-Saxon and modified-oscillator results are found to be encouragingly similar. One difference is that the modified oscillator inverts the ordering of the $\pi_2 \nu_4$ and $\pi_4 \nu_4$ bands which both terminate at I = 36. The state which terminates the band $\pi_3 \nu_2$ at I = 44 is predicted to be the most favored of all configurations with positive parity and even spin. The existence of such favored configurations beyond the I = 36 band termination would explain why transition energies above I = 36 in ¹⁵⁸Yb are still not as high as in ¹⁶⁸Yb (cf. Fig. 2). It is a worthwhile challenge for experiment to confirm such ultimate yrast band terminations where the spin vectors of the twelve valence particles all point in the same direction.

In summary, the recently measured high-spin vrast levels of ¹⁵⁸Yb do not fit into any conventional picture. However, the cranking model, which accounts for the widely different rotational behaviors of the lighter and heavier nuclei in this mass region, is shown to provide an explanation for the peculiarities of the level spacings. Over the spin range I = 26-40 the ¹⁵⁸Yb nucleus steadily gains energy relative to gross rotational behavior, which indicates a gradual change of the internal structure. The theory describes this change as a shape transition associated with the evolution of finite rotational bands to their points of termination. The observed spectrum of ¹⁵⁸Yb indicates that the I = 32-36 levels and possibly the I = 30 level belong to such a band which terminates at I = 36. It is suggested by the rotating harmonic-oscillator model that this type of evolution is a general feature of rotational bands, but there has previously been no experimental evidence from discrete-line spectroscopy in nuclei with as many valence particles as ¹⁵⁸Yb. It may be remarked that the low transition energies in connection with band terminations (Figs. 1-3) are not a universal signature of terminating bands, as illustrated by the harmonicoscillator model itself, but they are a predicted³ signature in certain nuclei around ¹⁵⁸Yb.

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