Conversion of Wave Energy to Magnetic Field Energy in a Plasma Torus

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The circuit equations for current drive are derived by the finding of appropriate response functions in the presence of an electric field. The effect of arbitrary wave-induced fluxes on runaway production and current generation can then be determined. An interpretation of recent remarkable experiments is now possible, and the favorable results appear to scale to reactor-grade devices.

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Recent current-drive experiments on the Princeton Large Torus (PLT)[1] have converted wave energy to poloidal field energy with the remarkable efficiency of 25%. Previous experiments[2–5] have concentrated more on maintaining an rf current ("steady state") than on increasing it ("rampup"). The PLT rampup experiments, however, have been in a new parameter regime, wherein the dc electric field dominates over collisions in influencing the hot, current-carrying electrons. In order to interpret these experiments, and in order to determine whether this high efficiency might be obtained also in larger tokamak experiments, such as TFCX, it is necessary to solve for the behavior of the plasma in the presence of both intense rf waves and a strong electric field.

The circuit equations that describe the rampup include Maxwell's equation, $dLI/dt = -V$, and a constitutive relation, e.g., $V = V(I,P)$, where V is the loop voltage, P is the input power, L is the tokamak inductance, and I is the toroidal current. To find the constitutive relation, which reflects the macroscopic properties of the plasma medium, we distinguish $I = I_B + I_d$, where I_B is independent of the rf power. The driven current, $I_d(t)$, contains the cumulative effects of rf-induced fluxes at time τ for all $\tau < t$.

To find I_d, we generalize a previously employed technique.[6] Let $j(t,\mathbf{v},E) = qv_\parallel$, where $v_\parallel$ is the velocity parallel to magnetic field $\mathbf{B}$ at time t of an electron that has initial velocity space coordinate $\mathbf{v}$ and is immersed in a dc parallel electric field E. Initially, $j(0,\mathbf{v},E) = qv_\parallel(t=0)$. In a torus of radius R_T, the contribution to the toroidal current I by this electron is $j/2\pi R_T$ (although we loosely refer to j as a current). In the absence of an electric field, $j \to 0$ at $t \to \infty$ because of collisions with the background plasma. In the presence of an electric field large enough to cause the electron to run away, $j \sim t$ for large t, and for $E > 0$, $j \to -qc$ as $t \to \infty$, where c is the velocity of light.

Suppose that power $P(\tau,\mathbf{v})$ is expended at time τ in pushing electrons with coordinate $\mathbf{v}$ in some direction $\mathbf{S}$ in velocity space to a nearby location. The current that results at some later time t may be expressed as

$$I_d(t) = \int_0^t \frac{P(\tau,\mathbf{v})}{2\pi R_T} \frac{\mathbf{S}\cdot\nabla j(t-\tau,\mathbf{v},E)}{\mathbf{S}\cdot\nabla\epsilon} d\tau, \qquad (1)$$

where the gradient operates in velocity space, and where $\epsilon = mv^2/2$ is the energy associated with the initial coordinates of the electrons pushed. The physics of the rf current drive is contained in the Green's function j, which we shall calculate numerically.

Before embarking on this program, we remark briefly on the major processes we expect to describe. An electron absorbing energy and momentum from a source of rf power subsequently slows down either by collisions or by the electric field. In the former instance, all the rf input energy goes into plasma heating so that the conversion efficiency of rf energy to poloidal field energy, given by the ratio P_{el}/P, where $P_{el} = -VI_d$, is zero. In the latter instance, the electron is decelerated by the field, so that all of its energy, including the incrementally added rf energy, must go into the field. Evidently, $P_{el}/P \to 1$. This points to a regime for efficient energy conversion except for two further effects that are worrisome.

First, if the electric field is very strong, the decelerated electron may eventually run away in the direction opposite to the one desired for current rampup. Such an electron, while initially giving up its initial energy to the field, serves as an immense drain on the field energy when it accelerates in the runaway direction. Second, even if no runaways were produced, the rf-driven electrons, being hot and relatively collisionless, tend to accumulate and to form a large, hot, plasma component. The conductivity of this hot component can be much larger than that of the background plasma. Since it is difficult to change the total plasma current in less than an L/R (inductance/resistance) time, the large, hot conductivity may significantly impede current rampup.[7]

It turns out that between the regimes of relatively high and relatively low electric field strength that appear to contain unfavorable effects, there exists a regime of intermediate field for which high efficiency is possible. Whether by serendipity or astute design, the remarkable PLT experiment apparently falls in the intermediate regime.

To calculate the efficiency, we need the Green's function j, but it is unwieldy and impractical to solve completely for $j(t,\mathbf{v},E)$, whose arguments span a huge parameter space. Instead, by following separately particles with common characteristics (runaway or not), and making appropriate approximations for each group, we can characterize j by several functions of fewer arguments. This approach can be implemented by formulation of the normalized Langevin equations[8,9] for electrons undergoing collisions in the presence of a decelerating electric field, namely

$$\frac{du}{d\tau} = -\frac{1}{u^2} - \mu, \tag{2a}$$

$$\frac{d\mu}{d\tau} = -\frac{1+Z}{u^3}\mu + f(\tau) - \frac{1-\mu^2}{u}, \tag{2b}$$

where $\mu = v_\parallel / v$. We normalized $\tau = \nu_R t$, $u = v/v_R$, and defined

$$v_R^2 \equiv \nu_t v_t^3 m/|qE|, \quad \nu_R = \nu_t v_t^3/v_R^3,$$

$$v_t^2 = T/m, \quad \nu_t = \omega_{pe}^4 (\ln\Lambda)/4\pi n v_t^3,$$

and Z is the ion charge state. The runaway threshold velocity, v_R, is related to v_b defined by Dreicer[9] by $v_b = v_R(2+Z)^{1/2}$. At $|v| < v_R$, no electrons run away, and at $v > v_b$, almost all electrons run away (see Fig. 2). The term $f(\tau)$ is a stochastic source,[8] so that Eqs. (2) are equivalent to the Boltzmann equation in the high-velocity limit. We can use them to form a moment hierarchy and proceed to solve analytically. This can be done for E small to recover the steady-state efficiency[2] and the rf-enhanced conductivity.[7]

Note that the solution is determined by the three dimensionless parameters Z, $\mu(0)$, and $u(0)$, where 0 denotes initial location. For lower-hybrid current drive at high phase velocities we may restrict $\mu(0) \simeq \pm 1$, where + corresponds to rampup. Introducing a separate bookkeeping for runaway and nonrunaway (stopped) electrons, we write

$$j(t,\mathbf{v},E) = [1 - R(\mathbf{v},E)] j_S(t,\mathbf{v},E) + R(\mathbf{v},E) j_R(t,\mathbf{v},E), \tag{3}$$

where R is the probability that the particle runs away, j_S is the current due to the particles which are stopped by the background plasma ($v \to 0$, at $t \to \infty$), and j_R is the current due to particles which run away ($\mu v \to -\infty$, as $t \to \infty$). This system of bookkeeping facilitates further simplifications. Specifically, the function $j(t,\mathbf{v})$ may be adequately characterized by functions independent of time. For example, since stopped electrons contribute to the current within a slowing-down time, which is short compared to other times of interest, an excellent approximation is $j_S(\mathbf{v},t) \simeq \chi_S(\mathbf{v})\delta(t)$, where

$$\chi_S(\mathbf{v}) = \int_0^\infty j_S(t,\mathbf{v})\,dt. \tag{4}$$

Similarly, the runaway contribution may be characterized by $j_R = \chi_R \delta(t)$ plus a term describing the free acceleration of these electrons. The acceleration term requires that we define a fourth function, $v^{(0)}(\mathbf{v})$.

Characterization of the current j by the four time-independent functions R, χ_S, χ_R, and $v^{(0)}$ is adequate for the purposes of writing the circuit equations. It is also a great simplification of the problem. These functions are found by a Monte Carlo solution of Eq. (2), using 10 000 electrons at each initial condition.

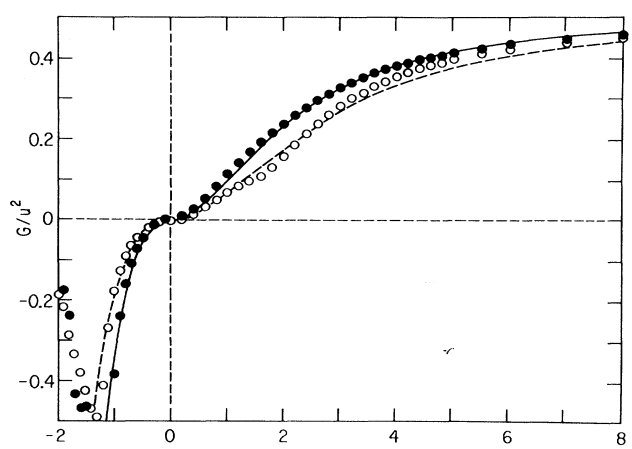

FIG. 1. The function $G(u,1)/u^2$ vs u and $G(u,-1)/u^2$ vs $-u$ for $Z=1$ (closed circles) and $Z=5$ (open circles). The points show the results of the Monte Carlo solution of the Langevin equations; the lines show the analytic fit, Eq. (8). Here, $u<0$ corresponds to initial conditions with $\mu = -1$.

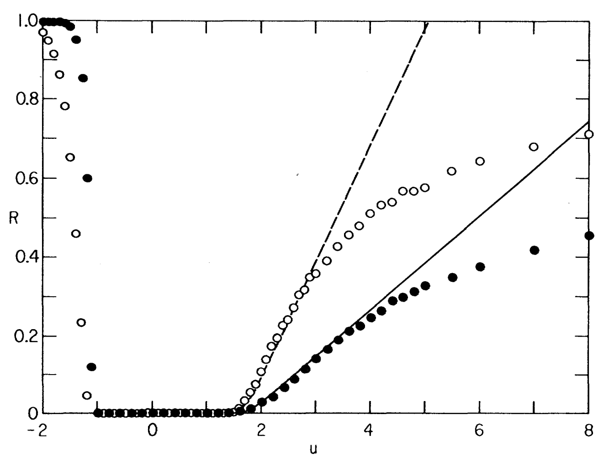

FIG. 2. The runaway fraction $R(u,1)$ vs u and $R(u,-1)$ vs $-u$ for $Z=1$ (closed circles) and $Z=5$ (open circles). The points show the results of the Monte Carlo method, the curves show approximate analytic fits near the "turn-on" region $\mu=1$, $u \gtrsim 1$. For $u > b$, the form of the curves is $R = a[(u-b)^4 - c^4] - ac$; for $u \leq b$, $R=0$. For $Z=1$, we have $a=0.12$, $b=1.4$, $c=0.4$. For $Z=5$, we have $a=0.3$, $b=1.3$, $c=0.4$.

Depending on their time-asymptotic behavior, the electrons are then classified as stopped or runaway. For our purposes here, however, we need only R and χ_S; a discussion of χ_R and $v^{(0)}$ will be reserved for a more lengthy report. To calculate the efficiency, we use a function $G(u,\mu) = \nu_R \chi_S / q v_R$, where the arguments are now understood to mean initial position. In Fig. 1 we plot G/u^2, where G is fitted approximately by

$$G[u,\mu=1] = \frac{u^4}{5+Z+[2u^2+2(5+Z)^2/3(3+Z)]u^2/(u^2+1)}, \tag{5a}$$

$$G[u,\mu=-1]\frac{u^4}{5+Z} - \frac{2u^6}{3(3+Z)}. \tag{5b}$$

This is a somewhat arbitrary fit to the data, chosen to reduce to the correct limits. This analytical expression also gives us a reliable estimate for derivatives of G, something not available directly from the Monte Carlo data. For $\mu=1$, as $u \to \infty$, $G(u,1) \to u^2/2$ and $P_{\rm el}/P \to 1$, but this efficiency applies only to power absorbed by stopped electrons; for large u, the runaway contribution dominates.

In Fig. 2 we show the runaway fraction $R(u,\mu = \pm 1)$. For rampup with $u \to \infty$, $R \simeq 60\%$ for $Z=1$ and $R \simeq 85\%$ for $Z=5$. Note that $R=0$ for u less than some threshold and the transition to finite R is abrupt. As pointed out by Valeo and Eder,[10] even if only a small fraction ($R \simeq 1\%$) of the resonant electrons run away, there may be a significant diminishing of the efficiency if these electrons are not lost. This can be seen as follows: If there are many Dreicer times ($1/\nu_R$) over the duration T of the experiment, then $j_R \simeq -qc$ and we may approximate

$$\frac{P_{\rm el}}{P} \equiv \frac{-VI}{P} \simeq (1-\eta_R)\frac{\mathbf{S}\cdot\nabla G}{\mathbf{S}\cdot\nabla u^2/2}, \tag{6}$$

where $\eta_R \simeq (c/v_R)\nu_R {\rm Tr}/G$. Successful startup[11] or rampup[1] experiments on PLT have been in the regime $\nu_R T \simeq 30$, $c/v_R \simeq 3$, and $G \sim O(1)$. Thus, $R \sim 1\%$ can seriously affect the efficiency if the runaways are confined. If the runaways are lost in time τ_c, where $1 \lesssim \tau_c \nu_R << T\nu_R$, then η_R must be reduced in Eq. (6) by about τ_c/T.

With use of Figs. 1 and 2, an interpretation of the PLT data is possible. The high efficiency implies that η_R is small, either because the spectrum is restricted to u small or because runaways are not confined. Taking $\eta_R = 0$ in Eq. (5a) gives the efficiencies shown in Table I. Interpretation of the data now depends on the assumption concerning runaways. If runaways are confined, then to explain a 25% efficiency, we must restrict $Z \simeq 1$ and require a spectrum extending from $u \simeq 0.5$ to $u \simeq 1.4$. If runaways are not confined, then $Z=5$ is allowed, but the spectrum must extend to $u \sim 2$. This estimate takes into account other losses and the unlikelihood that all of the rf power is absorbed exactly in the favorable region $1 \lesssim u \lesssim 2$. Conventional wisdom, which says that $Z \simeq 1$ is unlikely, should then predict that the runaways are not long confined.

That the PLT experiment, with either explanation, is in the regime $u \simeq 1$ comports well with other experimental data. The reported rampup of 120 kA/s at a density of 2×10^{12} cm^{-3} and at a temperature of about 1 keV corresponds to $6v_t \simeq v_R \simeq c/4$. A spectrum of parallel phase velocities extending from the electron tail (say $3v_t$) to $c/2$ is consistent both with substantial absorption and with the waveguide phasing. Several theories exist for precisely why the spectrum should be so broad.[12,13] What is important here, however, is that such a spectrum corresponds exactly to u extending from 0.5 to 2.

Note that the efficiency of converting wave energy to poloidal field energy depends only on u, so that the favorable regime on PLT is available on TFCX ($L=8$ μH) or other large experiments if we keep the ratio v_R/v_T constant, and employ a similar spectrum of waves. For example, to ramp the current to 10 MA in 30 s, take $E=0.6$ V/m, $n=5\times10^{12}$ cm^{-3}, and $T=1$ keV, so that both v_R and v_R/v_T are unchanged from the PLT experiment. Also, take $Z=1$. With the assumption of, then, a 33% efficiency, an average rf power of about 40 MW would be required.

There are several tradeoffs involved here: At lower density, efficient but slower rampup may be achieved with less power. At higher density, faster waves may be used, but the plasma must be proportionately hotter to absorb these waves. There is a danger, however, of significant energy loss by Joule heating, which is proportional to $\sigma_{sp}E^2$, where σ_{sp} is the bulk conductivity. To avoid this loss, the plasma must be kept resistive.

TABLE I. Efficiency $P_{\rm el}/P$ upon pushing an electron from some location u_1 to u_2.

$u_1 \to u_2$	$Z=1$ (%)	$Z=5$ (%)
$1.4 \to 2$	61	47
$1 \to 1.4$	43	30
$0.5 \to 1$	24	16
$0.5 \to 1.4$	35	24

In the above case, this would imply $T \lesssim 2$ keV, which can be maintained if the bulk energy confinement time is short (~ 30 ms).

In the above calculation, care is taken not to produce runaways. If some method of removing these runaways were possible, then even higher efficiencies might be obtained by taking $\omega/k_{\parallel} v_R \sim 2$.

Note that much of our intuition derived from efficiency calculations in the steady state[14] is not suitable for rampup. For example, the steady-state efficiency, I/P, is inversely proportional to the density, whereas for quick rampup, a high efficiency is possible at high density; in fact, too low a density may be undersirable. Also, whereas in the steady state, electron-cyclotron waves are about as efficient in producing current as are lower-hybrid waves,[6] in rampup these waves would appear to be a poor current driver, because runaway production is more likely, while less parallel energy flows into the strong electric field.

Although the estimates made here are crude, it is clear that more precision is easily available within the framework of this analysis. The level of precision attempted here, however, is sufficient to show that the PLT results are amenable to interpretation and extrapolation. It is reasonable to expect efficient and swift conversion of wave energy to field energy in reactor-grade tokamaks if they are designed so that the PLT parameter regime of v_R/v_t and $\omega/k_{\parallel} v_R$ is kept.

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