Critical Localization in Two-Dimensional Landau Quantization

Hideo Aoki[a]
Department of Physics, University of Tokyo, Tokyo 113, Japan

and

Tsuneya Ando
Institute for Solid State Physics, University of Tokyo, Tokyo 106, Japan
(Received 2 July 1984)

The localization in two-dimensional disordered electron systems in strong magnetic fields is investigated by the finite-size scaling method. The scaling function numerically obtained for macroscopically long systems shows that the single-parameter is invalid. The inverse localization length is found to have a novel critical behavior near the center of the Landau band, in which the Landau-index–dependent critical exponent determines the behavior of conductivity.

PACS numbers: 71.55.Jv, 71.30.+h

The current interest in the quantized Hall effect[1] leads naturally to the question of how this quantum effect is related, in the single-particle picture, to localization in the two-dimensional (2D) system. It has been shown[2] that both localized and extended states are required for the effect, since the plateau in the Hall conductivity (σ_{xy}) as a function of electron concentration comes from the localization, while the progressive quantum steps in the conduction necessitate mobile carriers. From the scaling theory,[3] on the other hand, it has been shown that all the states are localized in the absence of magnetic fields in the lower critical dimensionality of 2. Thus the problem arises as to whether the change of the universality class of the system from orthogonal ($H=0$) to unitary ($H\neq 0$) by introduction of a magnetic field (H) can alter the behavior of localization.

The studies on localization in the Landau levels in strong magnetic fields with disorder include the Thouless-number study[4] and the gauge-transformation study.[5] These results suggest that the inverse localization length [$\alpha(E)$] in this system is continuous and touches $\alpha=0$ axis at a single energy, which is the center of Landau level. In this Letter, this anomalous situation, which is one of the critical manifestations of the peculiarity in 2D systems in magnetic fields, is studied in detail. Since no expansion parameters like $1/E_F\tau$ exist in the present system, the diagrammatic method is ineffective unless it is performed up to infinite order. The result[6] of a partial summation in fact differs qualitatively from the numerical results.

The finite-size scaling can be a powerful approach to the Anderson-localization problem. In the absence of magnetic fields, MacKinnon and Kramer[7] employed the method to derive scaling functions in two and three dimensions which are consistent with the theoretical prediction.[3] Here we employ the finite-size scaling idea to study localization in the Landau levels. Our method is a natural extension of the original formalism, which is restricted to tight-binding Hamiltonians with only nearest-neighbor interactions. By contrast, the Hamiltonian for the electrons in 2D continuous space is given by

$$\mathcal{H}=\sum_{NX}|NX\rangle\,(N+\tfrac{1}{2})\hbar\omega_c\,\langle NX|+\sum_{NXN'X'}\sum|NX\rangle\,\langle NX|V|N'X'\rangle\,\langle N'X'|.$$

with the Landau wave function $|NX\rangle$, ω_c-eH/mc, and a random potential V. If we consider the quantum limit, in which interactions between different levels can be neglected, $\mathcal{H}$ has matrix elements between $|NX\rangle$ and $|NX'\rangle$, for $|X-X'|\lesssim l_N$ within the Nth Landau level. Here $l_N=(2N+1)^{1/2}l$ with cyclotron radius $l=(c\hbar/eH)^{1/2}$, and the center of cyclotron motion, X, takes discrete values with a spacing $\Delta X=2\pi l^2/L_y$ for a system of width L_y. The finite-size scaling procedure for the present system may be summarized as follows. We start by dividing a long strip of width L_y into cells such that the next-nearest-neighbor intercell interactions can be neglected. We define the Green's function, $\hat{G}(n)=(z-H^{(n)})^{-1}$ with $g(n)=\langle n|\hat{G}(n)|n\rangle$ and $G(n)=\langle 1|\hat{G}(n)|n\rangle$, where $|n\rangle$ is the set of states belonging to the nth cell, and $H^{(n)}$ is the total Hamiltonian for the strip of n cells excluding the intercell Hamiltonian, $H_{n,n+1}$. We can then obtain a set of recursion formulas in a matrix form,

$$G(n+1)=G(n)H_{n,n+1}g(n+1),$$

$$g(n+1)=[z-H_{n+1}-H_{n+1,n}g(n)H_{n,n+1}]^{-1},$$

with intracell Hamiltonian H_n. Thus we can calculate the off-diagonal Green's function, $G(n+1)$, for arbitrary n by iteration starting from $G(1)=g(1)=(z-H_1)^{-1}$ with numerical procedure limited only by computation time. The inverse localization length may be obtained by

$$|\alpha(E,L_y)=-\lim_{n\to\infty}[2(n-1)]^{-1}\ln[\mathrm{Tr}|G(n)|^2].$$

We have performed the calculation for the first two ($N=0,1$) Landau levels for long strips along the x direction for the width ranging from $\tilde{L}_y=\frac{1}{4}$ to 16 with dimensionless $\tilde{L}_y \equiv L_y/(2\pi l)^{1/2}$ in periodic boundary conditions along y. The length of the sample, which is taken to be $51\,300l$, is a macroscopic size (~ 0.4 mm) much larger than real MOSFET systems for typical $H=10$ T with $l=81$ Å. This length is sufficient for which the value of α converges within an error of 2%. Note that the only relevant parameter in our problem is the energy for a given Landau band in contrast to the case of usual random systems, in which the degree of disorder as well as energy are the relevant parameters. This comes from the fact that, if we consider the case of dense, short-range scatterers as the random potential (white-noise limit), electronic structures of the Landau level broadened by disorder are similar once the energy is normalized by the broadening, Γ_0.[8] In the present study, we consider this case with equal number of attractive and repulsive scatterers (of concentration $40/2\pi l^2$) to make the electronic structure symmetric about the band center. We normalize the energy by the band width, Γ, for every L_y.

The L_y dependence of $\alpha(E,L_y)L_y$ is plotted in Fig. 1 for various E. It is clearly seen that, in the present system, there is no single-parameter scaling relation, $\alpha(E,L_y)L_y=f_1(\alpha_\infty(E)L_y)$, where $\alpha_\infty(E)=\lim\alpha(E, L_y\rightarrow\infty)$ and f_1 is a hypothetical function independent of E.

It is not surprising that an introduction of magnetic fields should destroy the single-parameter scaling, because the limits $L_y\rightarrow 0$ and $L_y\rightarrow\infty$ in fact represent two different universality classes, i.e., orthogonal and unitary, respectively. In the limit $L_y\rightarrow 0$, the Hamiltonian, which has nonvanishing matrix elements only between nearest-neighbor basis functions ($\langle X|V|X+\Delta X\rangle$), can be cast into a real one for a one-dimensional chain. We can then obtain the exact asymptotic form for $\alpha(E,L_y)$ for $L_y\rightarrow 0$. From the exact formula[9] for the localization length in 1D systems, we have

$$\alpha L_y=\pi/2+(L_y^2/2\pi l^2)F(E),$$
$$F(E)=\int dE'\,D_0(E')\ln|2(E-E')/\Gamma|, \tag{1}$$

where $D_0(E)=(2/\pi)^{1/2}\Gamma^{-1}\exp[-2(E/\Gamma)^2]$ is the Gaussian density of states in the limit $L_y\rightarrow 0$. For $L_y\rightarrow\infty$, on the other hand, we have $\alpha(E,L_y)L_y\rightarrow\alpha_\infty(E)L_y$. Thus, as is shown in the result, αL_y cannot be a monotonic function of L_y when $|E/\Gamma|<0.64$ [and $\alpha_\infty(E)>0$], since, although $F(E)$ is negative for these energies, $\alpha(E,L_y)L_y$ for $E\neq 0$ eventually starts to increase for sufficiently long L_y. This explains why studying too small sample sizes[10] could mislead to a two-branched scaling function with false mobility edges. Breakdown of the conventional single-parameter scaling[3] in the present system has also been discussed by Pruisken and co-workers,[11,12] who point out that, in the unitary nonlinear σ model, the origin of the second parameter lies in the σ_{xy} term similar to the θ vacuum in the Yang-Mills theory.

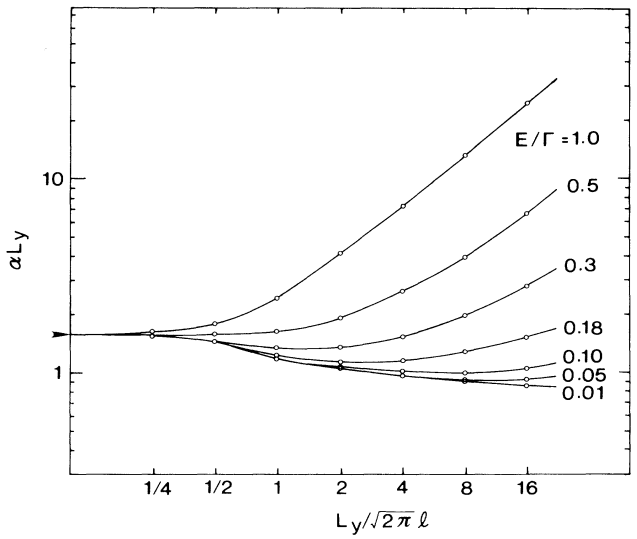

FIG. 1. Log-log plot of numerically obtained $\alpha(E,L_y)L_y$ vs L_y for $N=0$. Errors in the ensemble-averaged value of α, which are typically 1% for $\tilde{L}_y=16$ and 0.1% for $\tilde{L}_y=\frac{1}{4}$, are smaller than the size of each datum point. The arrow indicates the asymptotic value of $\alpha L_y=\pi/2$ for $L_y\rightarrow 0$. Curves are guide to the eye.

Having found that the single-parameter scaling is invalid, we seek the full description of the finite-size scaling function with a second variable involving the magnetic length, l. We have determined α as a function of E and L_y by fitting the numerical result by Padé approximation (Fig. 2). We have first fitted the ratio, $\gamma(E,L_y)\equiv\alpha(E,L_y)L_y/\alpha(0,L_y)L_y$, which is monotonic, and then fitted $\alpha(0,L_y)L_y$, which may be approximated by $\alpha(0.05\Gamma,L_y)L_y$ within the numerical accuracy. We note that absence of a single-parameter scaling for γ as seen from Fig. 2 implies absence of a separable scaling, $\alpha L_y=h_1(\alpha_\infty(E)L_y)h_2(L_y/l)$. In fitting $\gamma(E,L_y)$ and $\alpha(0,L_y)L_y$, we can specify their correct $L_y\rightarrow 0$ asymptote [Eq. (1)] and $L_y\rightarrow\infty$ asymptote. For large L_y, $\gamma(E,L_y)$ becomes proportional to $\alpha_\infty(E)$, while $\alpha(0,L_y)L_y$ can be assumed to approach a constant from the numerical result. This is consistent with the result that $\alpha_\infty(E)$ reaches zero at $E=0$, since αL_y is shown to converge to a constant at such a critical energy.[7] We have confirmed by a series of Padé formulas [e.g., $(3,2),\ldots,5,4$) Padé approximants for γ] that the least-squares fitted value for $\alpha_\infty(E)$ shows no systematic dependence on the model function employed.

From the result for $\alpha_\infty(E)$ for $N=0$ (Fig. 3) with

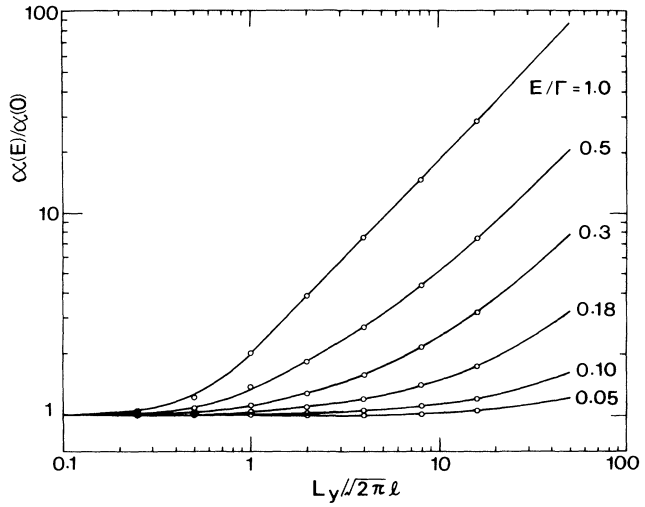

FIG. 2. Log-log plot of $\alpha(E,L_y)/\alpha(0,L_y)$ vs L_y for $N=0$. The curves are the least-squares-fitted results by the (3,2) Padé approximation. Here $\alpha(E=0)$ is approximated by $\alpha(0.01\Gamma)$.

considerably reduced error bars emerges a novel critical behavior near $E=0$,

$$\alpha_\infty(E) \propto |E|^s, \tag{2}$$

with $s \lesssim 2$, i.e., we have indeed an anomalous situation, in which the localization length is finite everywhere except at the center ($E=0$) of the $N=0$ Landau level. Similar analysis for the second ($N=1$) Landau level[13] is consistent with the power law, $\alpha_\infty(E) \propto |E|^s$, with $s \lesssim 4$ and E measured from the center of this band, although we cannot rule out a mobility edge at a small but finite $|E|$ due to an uncertainty in the form of $\alpha(0,L_y)$ for $N=1$.

The critical behavior, Eq. (2), is crucial in determining the physical quantity including transport properties. Specifically, we can show that s determines the low-temperature behavior of the dc conductivity (σ_{xx}). Consider $\sigma_{xx}(T)$ at temperature T given by an integral of $(-\partial f/\partial E)\sigma_{xx}(E)$. In the presence of inelastic scatterings, the integrand $\sigma_{xx}(E)$ has appreciable magnitude in an energy interval ΔE for which the localization length exceeds the inelastic scattering length, L_ϵ. The limiting value, $\sigma_{xx}(T \to 0)$, at the band center is finite or vanishes like $T^{\lambda-1}$ according as $\lambda \leqslant 1$ or $\lambda > 1$ when $\Delta E \sim T^\lambda$. We can estimate L_ϵ by $L_\epsilon^2 \sim D^*\tau_\epsilon$, where τ_ϵ is the inelastic scattering time and $D^* \sim (2N+1)l^2/\tau$ is the diffusion constant in the absence of localization effects with relaxation time $\tau \sim \hbar/\Gamma$. We can estimate another cutoff length, for which the average level spacing, $2\pi l^2/D(E_F)L_\epsilon^2$, is comparable with the inelastic level broadening, $\hbar/\tau_\epsilon$. For either estimate we end up with $L_\epsilon \sim (\tau_\epsilon/\tau)^{1/2}$ with the second cutoff length being more effective for higher N. Thus, when $\tau_\epsilon \sim T^{-p}$ at low temperatures, $\Delta E \sim T^{p/2s}$. Since $p=2$ for Fermi liquids for $H=0$ and $p=1$ in the dirty-metal limit,[14,15] p is considered to be close to unity in the present system in which momentum conservation is unimportant. Then σ_{xx} is finite at the band center and zero elsewhere for $T \to 0$ in the lowest Landau level for which $s \cong 2$ and $p/2s < 1$. Note that the Hall conductivity, σ_{xy}, on the other hand, is always a step function at the band center for $T=0$ as long as delocalized states exists.

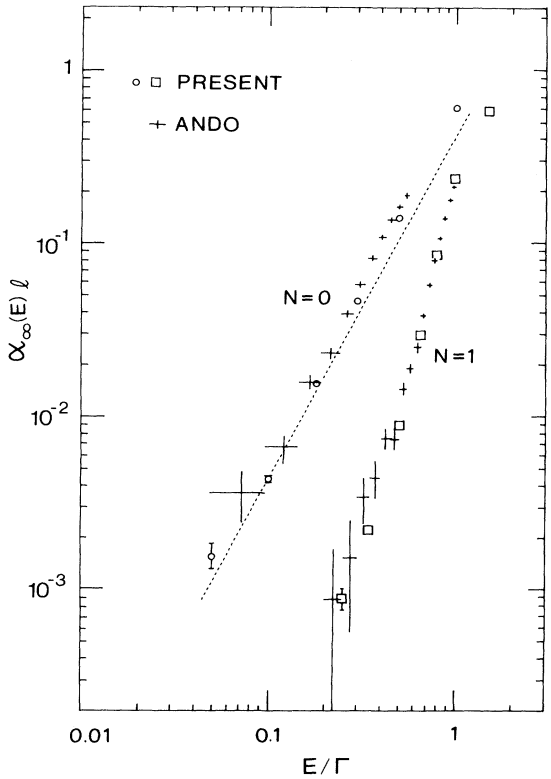

FIG. 3. Log-log plot of $\alpha_\infty(E)$ vs E (measured from each band center) determined by the Padé fit for $N=0$ and 1. The error bars represent ± twice the standard deviation, and the broken line represents $\alpha_\infty(E) \propto E^2$. The crossed error bars are the result of the Thouless-number study (Ref. 4).

This work was supported in part by a Grant-in-Aid for Special Promotion of Science from the Ministry of Education.

[a]Present address: Institute of Materials Science, University of Tsukuba, Sakura, Ibaraki 305, Japan.

[1]K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).

[2]H. Aoki and T. Ando, Solid State Commun. **38**, 1079 (1981).

[3]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[4]T. Ando, J. Phys. Soc. Jpn. **52**, 1740 (1983).
[5]H. Aoki, J. Phys. C **16**, 1893 (1983).
[6]Y. Ono, J. Phys. Soc. Jpn. **51**, 2055 (1982).
[7]A. MacKinnon and B. Kramer, Phys. Rev. Lett. **47**, 1546 (1981).
[8]T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **36**, 959 (1974).
[9]D. J. Thouless, Phys. Rep. **13**, 93 (1974).
[10]L. Schweitzer, B. Kramer, and A. MacKinnon, J. Phys. C **17**, 4111 (1984).
[11]A. M. M. Pruisken, Nucl. Phys. B **235**, [FS11], 277 (1984).
[12]H. Levine, S. B. Libby, and A. M. M. Pruisken, Phys. Rev. Lett. **51**, 1915 (1983).
[13]Note that, for $N \geqslant 1$, $F(E)$ in Eq. (1) is replaced by

$$\int dE' D_0(E') \ln(f_N |E - E'|/\Gamma) + 2N \ln \tilde{L}_y$$

with

$$f_N = 2[(4N-1)!!]^{1/2}/(2\pi)^N.$$

[14]B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitsky, J. Phys. C **15**, 7367 (1982).
[15]H. Fukuyama and E. Abrahams, Phys. Rev. B **27**, 5976 (1983).