Nonlinear Driven Reconnection in the Reversed-Field Pinch

Tetsuya Sato and Kanya Kusano
Institute for Fusion Theory, Hiroshima University, Hiroshima 730, Japan
(Received 7 May 1984)

We propose a theory that a new type of magnetic reconnection, nonlinear driven reconnection, is triggered during the nonlinear development of an $m=1$ helical kink-mode instability. This reconnection process can well explain nonlinear reconnection observed in the previous simulation and can be a candidate for the self-reversal mechanism in the reversed-field pinch.

PACS numbers: 52.30.−q, 52.55.Dy, 52.55.Ez

In spite of its physical interest and practical importance in plasma confinement, the self-reversal process in the reversed-field pinch (RFP) remains unexplained. Several causes leading to toroidal field reversal have been suggested theoretically, numerically, and experimentally.[1–5] Among them, Sykes and Wesson's simulation work[3] was the first to successfully demonstrate that an $m=1$ kink mode could be a cause of the reversal. However, any satisfactory causal mechanism of how a reversed-field configuration develops has not yet been presented, although this is essential in order to understand the maintenance of the RFP configuration. The purpose of this paper is to present a consistent, dynamical model leading to reversal of the toroidal field, by proposal of a new type of reconnection.

There is no doubt that magnetic reconnection plays a key role in the relaxation and reversal process in the RFP. Reconnection causes a local enhancement of dissipation whereby the system can relax toward a minimum-energy state in a time scale shorter than the natural diffusion time. In addition, reconnection causes a topological change in the magnetic field configuration which is also essential in the obtaining of a different equilibrium configuration.

There are two types of reconnection. One is linear tearing-mode reconnection,[6] and the other is driven reconnection.[7] Linear reconnection is expected to occur if and only if an antiparallel field configuration exists locally in the initial state. Tearing-mode instability in tokamaks is such an example,[8] while driven reconnection can take place for a variety of magnetic field configurations.[9]

In this paper we propose a theory that the self-reversal of the toroidal field is caused by a new type of magnetic reconnection stimulated by a helical kink instability, which we call nonlinear driven reconnection.

For a better understanding of the following arguments, we first describe the necessary conditions for driven reconnection to take place. (1) The first condition is the existence of converging flows. Reconnection can take place at the converging point, if other conditions are satisfied. (2) The second condition is that antiparallel field components perpendicular to the converging flows be brought into the converging point by the flows. (3) The third condition is that there should be no field component parallel to the flows at the converging points. Existence of a field component perpendicular both to the converging flows and the antiparallel fields has nothing to do with reconnection. Once these conditions are fulfilled, then reconnection can be triggered at the converging (X) point and the antiparallel fields are converted into pairs of antiparallel fields which are parallel to the original converging flows. The converted fields are taken away from the converging point in the direction of the original antiparallel fields by the outgoing flows. Figure 1 shows a topological relation of the field and flow directions associated with reconnection.

Since we are concerned with an $m=1$ single helicity mode in a cylinder (r, θ, z), let us introduce a helical coordinate system (r, Λ, Z) where r is the usual radial coordinate, Z is the coordinate along the helical line which is the ignorable coordinate in the present helical symmetry problem $(\partial/\partial Z=0)$, and Λ is an angle coordinate which completes a right-handed coordinate system.

We give the initial axisymmetric force-free magnetic field and current in this helical coordinate system as follows:

$$\mathbf{B}_0=(0,B_{0\Lambda},B_{0Z}), \quad (1)$$

$$\mathbf{J}_0=(0,J_{0\Lambda},J_{0Z}). \quad (2)$$

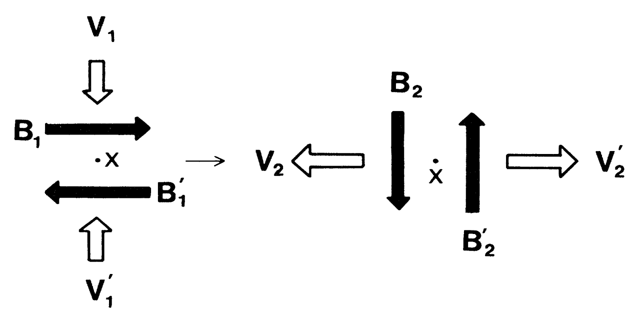

FIG. 1. Geometrical relation of the driving plasma flow and magnetic field in the driven reconnection process. Point X represents the reconnection point. In the present model the horizontal and vertical directions correspond, respectively, to the radial and Λ directions.

We note here that $B_{0\Lambda}/B_{0Z}=J_{0\Lambda}/J_{0Z}$ and that the unperturbed quantities with suffix 0 are all functions of r, although no expression is made explicitly.

For later convenience, we here express the unperturbed field components in terms of those in the original cylindrical coordinates:

$$B_{0\Lambda}=B_{0\theta}\cos\phi-B_{0z}\sin\phi=B_{0\theta}(1-nq)\cos\phi, \quad (3)$$

$$B_{0Z}=B_{0\theta}\sin\phi+B_{0z}\cos\phi, \quad (4)$$

where ϕ is the pitch angle of the helical mode under study, n is the toroidal (axial) mode number, and q is the radial q profile. In the following discussion we choose $B_{0\theta}>0$, $B_{0z}>0$, and $0<\phi<\pi/2$, without loss of any generality. With this choice it turns out that B_{0Z} is always positive but $B_{0\Lambda}$ is positive only for nonresonant modes. For resonant modes $B_{0\Lambda}$ changes sign at the resonance surface.

As an $m=1$ perturbed helical flux we assume $\psi_1(r)\cos\Lambda$, where the direction of the radial displacement is the $\Lambda=0$ direction. Then the perturbed magnetic field and current components are given by

$$B_{1r}(r,\Lambda)=-\psi_1\sin\Lambda/r, \quad (5)$$

$$B_{1\Lambda}(r,\Lambda)=-(\partial\psi_1/\partial r)\cos\Lambda, \quad (6)$$

$$B_{1Z}(r,\Lambda)=0, \quad (7)$$

$$J_{1r}(r,\Lambda)=0, \quad (8)$$

$$J_{1\Lambda}(r,\Lambda)=0, \quad (9)$$

$$J_{1Z}(r,\Lambda)=-\Delta_r\psi_1\cos\Lambda/\mu_0, \quad (10)$$

where $\Delta_r=\partial^2/\partial r^2+r^{-1}\partial/\partial r-r^{-2}$ and $\psi_1(r)$ is abbreviated by ψ_1.

Let us consider that the $m=1$ perturbation grows with the growth rate γ. Then the equation of motion gives the perturbed radial flow, which is the driving force for instability, as

$$\rho\gamma v_{1r}(r,\Lambda)=\xi(r)\cos\Lambda/r, \quad (11)$$

where ρ is the mass density, and

$$\xi(r)=r(\Delta_r\psi_1 B_{0\Lambda}/\mu_0+J_{0Z}\partial\psi_1/\partial r). \quad (12)$$

The first term on the right-hand side of Eq. (12) acts to stabilize this perturbation, whereas the second term can contribute to destabilize it. For this mode to grow ξ must be positive and peaked around the magnetic axis, because the radial displacement is assumed in the $\Lambda=0$ direction. Since v_{1r} is the driving source, the Λ component of the flow can be obtained by the assumption of incompressibility as

$$\rho\gamma v_{1\Lambda}(r,\Lambda)=-(\partial\xi/\partial r)\sin\Lambda. \quad (13)$$

We note here that $\partial\xi/\partial r\lesssim 0$.

Thus we have been able to express all quantities necessary for discussing driven reconnection in terms of the perturbed helical flux and unperturbed quantities. From Eqs. (11) and (13) it is seen that both v_{1r} and $v_{1\Lambda}$ have potentiality to drive reconnection, because they have points where the flow convergence condition, namely, condition (2) described earlier, is satisfied.

Let us first see reconnection associated with v_{1r}. The second condition given earlier, namely, condition (2), requires the existence of antiparallel Λ fields, since reconnection is possible only on the r-Λ plane. The Λ field has an unperturbed component, and so we shall throw away the perturbed part for the time being. In order to have reconnection, therefore, it is required that there exists a resonance surface ($r=r_0$). If $v_{1r}=0$ and $\partial v_{1r}/\partial r<0$ are satisfied in Eq. (11) at $r=r_0$, then reconnection could be triggered there. The $m=1$ tearing mode is such a case. However, here we are interested in a different reconnection process which is associated with the flow $v_{1\Lambda}$.

From Eq. (13), the convergence condition of the flow is satisfied at $\Lambda=\pi$ where $v_{1\Lambda}=0$ and $\partial v_{1\Lambda}/\partial\Lambda<0$. The magnetic field involved in reconnection for this flow is B_{1r}. Very interestingly, B_{1r} is antiparallel with respect to $\Lambda=\pi$; thus the field condition for reconnection, namely, condition (2), is also automatically satisfied for the $m=1$ kink instability under study. However, there is no guarantee yet that condition (3), namely, vanishing of the field component in the converging flow direction, can be satisfied. This condition is given by

$$B_{0\Lambda}(r)+B_{1\Lambda}(r,\pi)=0. \quad (14)$$

Thus, the final question is whether or not there appears a radial point on $\Lambda=\pi$ which satisfies Eq. (14).

As can be easily understood from the above argument, the process we are now interested in is nonlinear in the sense that both the corresponding flows and fields are perturbed quantities. In the resonant-mode case, as we are already discussed, the $m=1$ tearing-mode reconnection would be linearly excited, so that the situation would become complicated. For the time being, therefore, we shall forget the resonant case, but focus on the nonresonant case where no linear reconnection takes place. Since the radial displacement is in the $\Lambda=0$ direction and the plasma is bounded, it is obvious that the Λ component is compressed in the front part of the displacement ($\cos\Lambda>0$) and rarefied in the back ($\cos\Lambda<0$), namely, $\partial\psi_1/\partial r<0$ in Eq. (6) and $B_{1\Lambda}$ is negative at $\Lambda=\pi$ except in the central region. Thus it becomes possible that Eq. (14) is satisfied, especially when the perturbation grows into a large amplitude and the field configuration is largely distorted.

Let us next consider whether or not the reversed flux can be generated in the outer part of the plasma column by this reconnection process, that is, whether or not a proper electric field is generated along the reconnection line (along the Z axis). At the reconnection line, there exists no Λ component of the magnetic field, as we discussed above. Thus, the only possible driving electric field is associated with $v_{1\Lambda}$ and B_{1r}. This electric field is given by

$$E_{DZ} = \psi_1 (\partial \xi / \partial r) \sin^2 \Lambda. \tag{15}$$

Obviously, this electric field is directed in the negative Z direction. In driven reconnection[7,9] this field penetrates into the X reconnection line in the presence of resistivity, this giving the reconnection rate. The negative-Z electric field generated at the X line acts to take away the reconnected positive-Λ field radially outwards and the negative-Λ field inwards (see Fig. 1). In other words, as the present nonlinear driven reconnection process proceeds, the positive-Λ flux is generated in the outer region of the reconnection point and the same amount of negative flux in the inner region. The generated flux is proportional to the square of the perturbation amplitude. Since the Λ component produces the axial component which is given by $B_z = -B_\Lambda \sin\phi$, it is concluded that a negative axial (toroidal) field is generated in the outer region by the proposed new nonlinear reconnection process, and hence, a reversal configuration can be realized. Incidentally, it is interesting to point out that the Z electric field associated with the tearing-mode reconnection in the RFP is positive, so that no generation of reversed flux in the outer region is expected for this case.

The nonlinear reconnection process observed by the previous simulation[4] for the nonresonant mode can well be explained by the present theory. The second reconnection process observed for the resonant mode can also be explained along the same line, since it appeared after the resonant surface was completely removed and a helically distorted configuration was achieved by the $m = 1$ tearing mode.

The proposed reconnection process has a marked contrast with the conventional tearing-mode reconnection process in that this is a nonlinear driven process and requires no resonance.

We are grateful to Dr. L. Chen, Dr. T. Hayashi, and Dr. M. Tanaka for their valuable comments and wish to express our gratitude to Professor K. Nishikawa for his encouragement throughout the course of this work. This work is supported by a Grant-in-Aid from the Ministry of Education, Science and Culture in Japan and by the Nissan Science Foundation in Japan.

[1]D. A. Baker and W. E. Quinn, in *Fusion,* edited by E. Teller (Academic, New York, 1982), Vol. 1, Part A, pp. 437–475; H. A. B. Bodin and A. A. Newton, Nucl. Fusion **20**, 1255 (1980).

[2]J. B. Taylor, Phys. Rev. Lett. **33**, 1139 (1974).

[3]A. Sykes and J. A. Wesson, in *Proceedings of the Eighth European Conference on Controlled Fusion and Plasma Physics, Prague, Czechoslavakia, 1977* (Czechoslovak Academy of Sciences, Prague, Czechoslovakia), Vol. 1, p. 80.

[4]E. J. Caramana, R. A. Nebel, and D. D. Schnack, Phys. Fluids **26**, 1305 (1983).

[5]A. Y. Aydemir and D. C. Barnes, Phys. Rev. Lett. **52**, 930 (1984).

[6]H. P. Furth, J. Killeen, and M. N. Rosenbluth, Phys. Fluids **6**, 459 (1963).

[7]T. Sato and T. Hayashi, Phys. Fluids **22**, 1189 (1979).

[8]B. B. Kadomtsev, Fiz. Plazmy **1**, 710 (1975) [Sov. J. Plasma Phys. **1**, 389 (1975)].

[9]T. Sato, Hiroshima University Report No. HIFT-97, 1984 (to be published).