Nucleation of Cavitons in Strong Langmuir Turbulence

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Plasmon wave packets are shown to be nucleated in narrow density holes which are the remnants of previous generations of "burnt out" cavitons. The proximity of the nucleation length scales to those of dissipation precludes the possibility of a self-similar inertial range for a wide range of parameters. The nucleation mechanism may also arise in other contexts.

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In this Letter we show that the asymptotic state of a system developing according to the damped and driven Zakharov equations, under a wide range of conditions, is sustained by a novel local caviton-nucleation mechanism. Trapped Langmuir electric fields are nucleated in density cavities at spatial scales only slightly larger than the dissipation scales, precluding a developed inertial range with self-similar collapsing "solitons" as proposed by some authors.[1,2] This local nucleation mechanism has further novel implications for triggering Langmuir turbulence by quasineutral density fluctuations in the presence of large-scale driving mechanisms whose time dependence is near the mean plasma frequency.

Our analysis is based on the Zakharov equations which model the interaction between the Langmuir-wave envelope field E and the fluctuation, n, in the plasma density. In dimensionless form these equations are[3,4] for one spatial dimension

$$i(\partial_t+\nu_e)E+\partial_x^2E=nE+S(t), \tag{1}$$

$$(\partial_t^2+2\partial_t\nu_i-\partial_x^2)n=\partial_x^2|E|^2. \tag{2}$$

The units of time, distance, electric field, and density fluctuation are, respectively, $\frac{3}{2}(m_i/m_e)\omega_{pe}^{-1}$, $\frac{3}{2}(m_i/m_e)^{1/2}\lambda_{\text{D}e}$, $8(\pi n_0 Tem_e/3m_i),^{1/2}$ and $n_0(4m_e/3m_i)$, where n_0 is the dimensional background density. The Landau damping operators ν_e and ν_i are diagonal in k space (spatial Fourier transform); in these units the mass-ratio dependence enters only through ν_e and ν_i.[2] In a model appropriate for the description of laser-plasma interaction, $\overline{E}=\int_0^L E(x,t)\,dx/L$ is a constant E_0 which implies that the source S is given by $S=-\overline{nE}$; we call this the "clamped drive." [Note that $\nu_e(k=0)\equiv 0$.] To simplify the theoretical analysis of the stationary state we assume that L is large enough compared to a correlation length so that S is a constant in time. We will also consider the case where $S=0$ but $\nu_e(k\simeq 0)=-\nu_D<0$ corresponding to a beam unstable particle distribution with a high beam velocity.

The modulational instability of long-wavelength electrostatic or electromagnetic waves with frequencies near the electron plasma frequency, ω_p, has long been recognized as a source of strong Langmuir turbulence.[3,4] In Fig. 1 a time sequence of computer-generated solutions of the Zakharov equations in one dimension[5] (with periodic boundary conditions) are presented for the modulus $|E|^2$ of the Langmuir envelope field and the density perturbation n for a system driven by a clamped drive at wave number $k=0$ of intensity $E_0=1$ in the units described above. This system develops from a well defined sinusoidal global modulational instability whose peaks begin to steepen into cavitons. In Fig. 1 the electric field in one caviton is driven to a stage of collapse to small scales $(\sim 5\lambda_{\text{D}e})$ where it suffers strong Landau damping so that by $T=8.5$ the electric field envelope has substantially decayed (or "burned out") leaving a density cavity essentially unsupported by its ponderomotive force. The cavity then breaks up into left- and right-going sound pulses, each with about half the depth of the original cavity, shown at $T=9.0$, which continue to propagate through the system at the sound speed $(C_s=1$ in our units). This part of the scenario has been observed in several earlier simulation studies.[2,6]

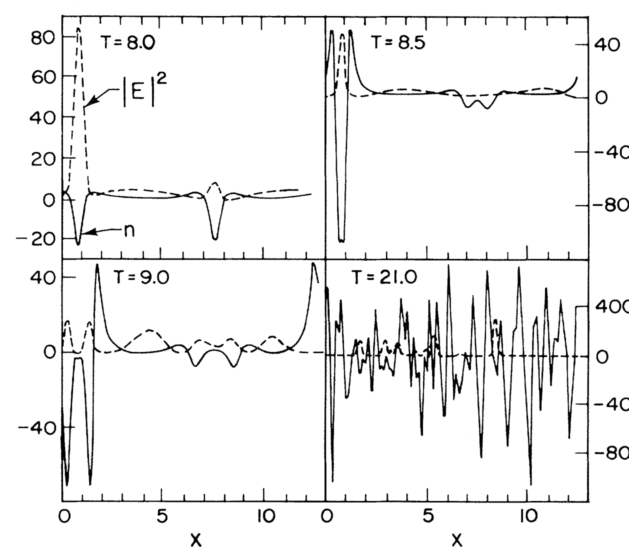

FIG. 1. Snapshots of $|E(x,t)|^2$ (dashes) and $n(x,t)$ (solid line) as functions of x for various times with a clamped drive $E_0=1$ and $m_i/m_e=1836$.

The mechanism which sustains *stationary* Langmuir turbulence has not been clearly elucidated. New localized or trapped electric field envelopes *are observed* to be nucleated in narrow but shallow density wells resulting from the interference or decay of cavities from previous burn-out events, creating another generation of burnt-out cavities and so on until a stationary state is reached. The damping of the ion waves is relatively weak if $T_e >> T_i$ and the stationary state is dominated by a high level of density fluctuations,[2] as in Fig. 1 at $T=21$, whose scale is near the dissipation scale set by the Langmuir Landau damping which produced the burn out.

The stationary turbulence state shows no sign of a global modulational instability. Indeed one expects short-scale density fluctuations to scatter long-wavelength Langmuir waves, producing an effective dissipation on the latter[1,2,7] which suppresses this instability.

In the stationary state for the various driving methods and for various electron-to-ion mass ratios we observe an essentially flat spectrum, $\langle |E(k)|^2\rangle \sim$ const, connecting the low-k driving regime[8] and the high-k dissipation regime. (Angular brackets denote temporal averages.) Several examples are shown in Fig. 2 for the clamped drive; for other methods of driving the spectra are essentially identical for equivalent values of the mean electrostatic energy density $\langle W\rangle$. A quantitative understanding of the physics is obtained from an energy-balance equation for the Fourier modes which follows from Eq. (1):

$$\partial_t |E_k|^2 = -2\nu_e(k)|E_k|^2 + \sum_{k'} T(k|k') \quad (k\neq 0), \qquad (3)$$

where $t(|k|k')$ is a transfer spectrum,

$$T(|k|k') = \{-iE_{k'}n_{k-k'}E_k^* + \text{c.c.}\}, \qquad (4)$$

which has the property $\sum_k \sum_{k'} T(k|k')=0$ where the sum is over $k,k'\neq 0$. It is useful to compare $\langle T(k|k'=0)\rangle$ (this is the energy injection spectrum when E_0 is clamped), which measures the transfer of Langmuir energy directly from the driven modes ($k=0$) to the mode k, with the average of the first term on the right-hand side of (3) which we call the dissipation spectrum. A typical case is shown in Fig. 3; we see that Langmuir energy is transferred directly from $k\simeq 0$ to high k; the transfer spectrum $\langle T(k|0)\rangle$ peaks at a value of $k=k_T$, only a factor of 2 or so lower than the peak in the dissipation spectrum, k_d, which is controlled by Langmuir Landau damping. There is insufficient range in k between k_T and k_d to develop an extended inertial range toward high k. The predictions of Refs. 1 and 2 of a power-law inertial range based on an extended regime of self-similar supersonic collapse are never verified in our simulations. The observed spectrum is consistent with the fact that the localized electric field packets are all nucleated at nearly the same scale. In contrast to what is observed, the self-similar collapse model of this stationary state[1] claims that the supersonic self-similar collapse of cavitons is the primary mechanism for the transfer of plasma-wave energy from the long wavelengths of modulational instability to short wavelengths (a few Debye lengths, λ_D) where there is efficient absorption of energy by particles.

A detailed understanding of the local cavity nucleation can be obtained in terms of the complete set of global eigenfunctions, $\phi(x,t)$, in the instantaneous density profile $n(x,t)$:

$$\lambda(t)\phi(x,t) = -\partial_x^2\phi(x,t) + n(x,t)\phi(x,t); \quad \langle\phi|\phi\rangle = 1. \qquad (5)$$

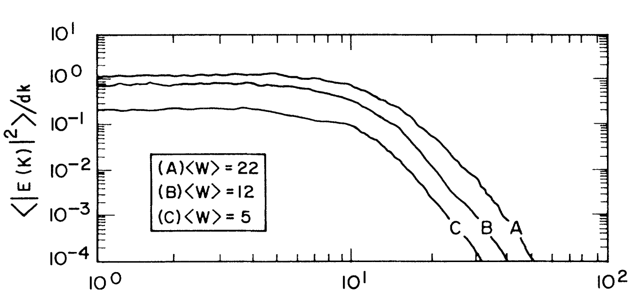

FIG. 2. Time-averaged $|E(k)|^2$ vs k. For clamped E_0 drive with (A) $E_0=1.0$, (B) $E_0=0.5$, and (C) $E_0=0.25$. The values of $\langle W\rangle = \langle |E|^2\rangle = \langle \sum_k |E(k)|^2\rangle$ are shown for each case. ($L=4\pi$, $m_i/m_e=1836$, $dk=2\pi/L=\frac{1}{2}$.)

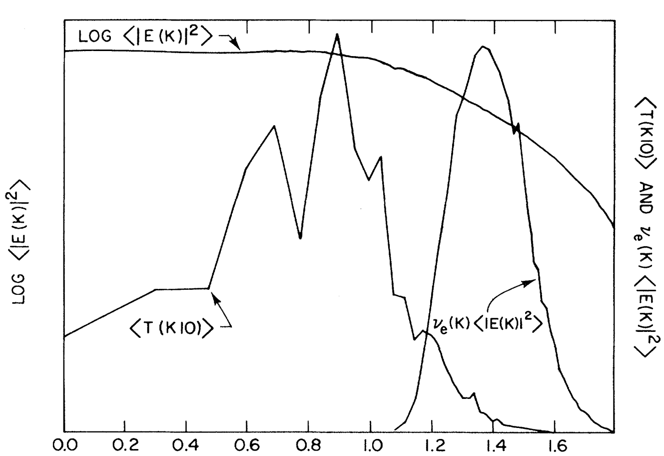

FIG. 3. Transfer spectrum $\langle T(k|0)\rangle$, dissipation spectrum $2\nu_e(k)|E(k)|^2$, and $\log\langle|E(k)|^2\rangle$ in arbitrary units vs $\log k$ for the conditions of Fig. 1.

(For clarity the eigenvalue index is suppressed. Also note that from here on in the text, brackets signify the Dirac notation for vector inner product.) It is a remarkable fact, found from the numerical solution of (5), that out of the complete set of ϕ's, $E(x,t)$ in a given cavity is usually dominated by a unique eigenfunction, ϕ_0, which is *localized* in that cavity and can be described as the local ground state. This localized eigenstate describes a trapped Langmuir mode in the cavity. Consider a time interval in which ϕ_0 and λ_0 evolve smoothly, and let

$$E(xt) = H(t)\phi_0(xt)\exp[-i\int_0^t \Lambda(t')\,dt'] + E_R(xt), \tag{6}$$

where $\Lambda(t) = \lambda_0(t) - i\Gamma_0(t)$ and $\Gamma_0(t) = \langle\phi_0|\nu_e|\phi_0\rangle$. Then the equation of motion of H is easily shown to be

$$i\partial_t H = \exp i\int_0^t \Lambda(t')\,dt' \int dx\,\{\phi_0(x,t)[S(t) - i\nu_e E_R(xt)] + i\partial_t\phi_0(x,t)E_R(xt)\}, \tag{7}$$

where $E_R(xt)$ is orthogonal to ϕ_0: $\langle\phi_0|E_R\rangle = 0$. The local Langmuir energy in the cavity is given by

$$w_0(t) = |H|^2\exp[-2\int_0^t dt'\,\Gamma_0(t')].$$

For an isolated cavity the terms involving the remainder field E_R are observed to be not significant.

For a given simulation of Eqs. (1) and (2), Eq. (5) and then Eq. (7) with the E_R terms omitted are solved numerically, yielding the function $w_0(t)$. Alternatively, $w_0(t)$ can be obtained by projecting ϕ_0 on the numerically determined $E(x,t)$ obtained directly from Eqs. (1) and (2) via the formula $w_0(t) = |\int dx\,\phi(xt) \times E(xt)|^2$. Quantitative agreement is found between these two expressions in the source case. Qualitative agreement is found in the beam case in the sense that both expressions for w_0 peak at the time when the local mode becomes stable; i.e., Γ_0 changes sign from negative to positive.

In the case with a finite driving source S, Eq. (7) shows that the amplitude H for a given cavity excitation behaves as a source-driven oscillator; if $\lambda_0(t)$ were independent of t the largest response occurs when the cavity eigenfrequency is at resonance with the source frequency which is $\lambda_0 \simeq 0$ for the cases considered here. In a relatively deep well of depth n_0 and width d_0, λ_0 is strongly negative ($|\lambda_0|d_0 >> 1$) and far from resonance; starting with a near zero initial value H will have an oscillatory behavior with a maximum amplitude $\sim |S/\lambda_0|d_0^{1/2}$ which will be too small to perturb significantly the cavity density if $|S/\lambda_0|^2(d_0\lambda_0)^{-2} < n_0$. Note that in dimensional units (denoted by tilde), $d_0\lambda_0 = (\tilde{d}_0/\tilde{C}_s)\tilde{\lambda}_0$ is just the product of the sound transit time across the cavity times the eigenfrequency. Such mild oscillatory behavior is widespread throughout the system and actually accounts for most of the energy dissipation observed in the simulations even though there is a relatively small dissipation in each cavity and such cavities do not "burn out." A more dramatic but far less frequent behavior occurs if $|E|^2$ can grow to a large magnitude before the density has evolved much, i.e., in times $t < d_0$ appropriate for supersonic behavior. For example, when $|d_0\lambda_0| > 1$ and $|S/\lambda_0|^2(\lambda_0 d_0)^{-2} >> n_0$ then a supersonic-driven collapse will follow. If $|d_0\lambda_0| < 1$, then collapse requires $S^2d_0^2 >> n_0$. In all cases the cavities initially respond reactively where the source can add or take away energy depending on its phase; in the case evolving to supersonic-driven collapse, the later stage is dominated by dissipation through Langmuir Landau damping. This is demonstrated by comparing in Fig. 4 the integration of (7) (with the E_R terms omitted) with a similar equation with no Landau damping and finding that the position of the peak in $H(t)$ as a function of t is almost unchanged; however, since by this time n_0 and hence $|\lambda_0|$ have increased so that the oscillation amplitude $|S/\lambda_0|d_0^{1/2}$ has substantially decreased, the subsequent rapid decrease in H is mainly due to Landau damping. Since d_0 starts near the dissipation length scale, it cannot contract very much during collapse before the caviton energy $w_0(t)$ begins to quickly decrease by Landau dissipation. As a result, the evolution of $|E|^2$ closely resembles that of $w_0(t)$ contrary to the theory of self-similar collapse[1,2] in

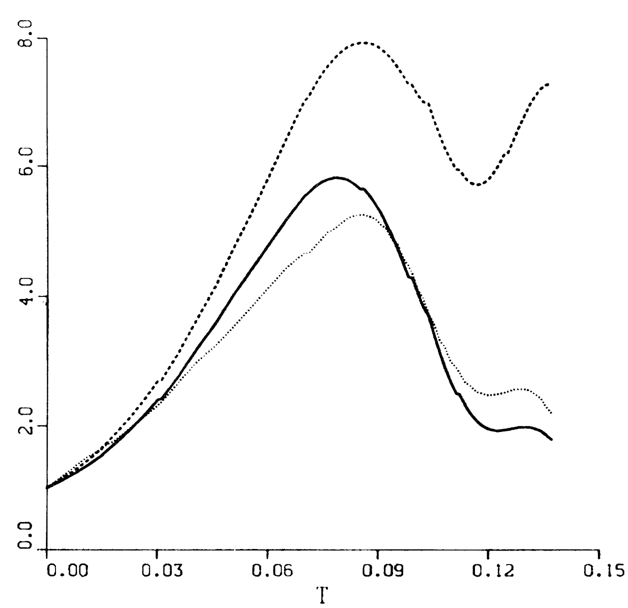

FIG. 4. Typical collapse for source $S = 156$. Solid curve, $w_0(t)/w_0(0)$; dashed curve, $w_0(t)/w_0(0)$ as determined by Eq. (7) with E_R terms removed and $\Lambda = \lambda_0$; dotted curve, $|E(t)|^2/|E(0)|^2$ at peak of local mode.

which w_0 is assumed constant while $|E|^2$ is exploding. The overall scenario for the beam-driven case is similar to the source drive but differs in detail: In the beam case the local cavity fields can be unstable[9] because $\Gamma_0 < 0$ in the exponential factor of Eq. (6).[10]

The large but infrequent burn-out events are responsible for sustaining the high level of density fluctuations. For $T_e >> T_i$ burn out produces deep density wells or rarefaction pulses preceded by a compressional pulse of nearly equal amplitude (see Fig. 1); the relatively shallow wells which are efficient nucleation centers are mainly produced by the interference of rarefaction pulses with compressional pulses.

For $T_e \simeq T_i$ with strong ion wave damping, the density wells decay quickly; but since nucleation in small wells is favored, the local nucleation process is still observed to be dominant. In this regime nucleation occurs directly in the burnt-out density wells before interference effects can operate. The nucleation is at relatively larger scales compared to the dissipation scales than in the $T_e >> T_i$ case since the decaying wells spread out quickly, but again we do not observe any power-law inertial range. In general, the asymptotic state for a given drive can be reached without ever involving a global modulational instability by starting with an initial level of density fluctuations sufficient to suppress the modulational instability but with the correct range of amplitudes and spatial scales to favor the nucleation process.

For drives much weaker than considered above a variety of other scenarios may be possible including one in which the turbulence may be sustained by intermittent, long-wavelength modulational instability.

It is believed that only for spatial dimension ≥ 2 may cavitons collapse to a singularity in a finite time if dissipation is neglected. Preliminary results on a modified set of "Zakharov" equations which exhibit this property in one spatial dimension indicate that the nucleation mechanism is unchanged.[11] Again the small length scales associated with nucleation preclude self-similar evolution. In higher dimensions the linear propagation of ion acoustic waves has a different geometry and both the nucleation and collapse of cavitons are intrinsically anisotropic. The fact that shallow density wells are most effective for nucleation should carry over to higher dimensions and this indicates that this mechanism might continue to be dominant provided that a supersonic collapse condition is satisified. Particle-in-cell simulations corresponding to the one-dimensional system with the clamped drive have been carried out and exhibit qualitative agreement with the Zakharov model in the case of weak drive ($E_0 < 1$) when the hot electrons accelerated by the cavitons are cooled at the boundaries.[12] In other one-dimensional particle-in-cell simulations[12] driven by a long-wavelength transverse wave we have seen Langmuir cavitons nucleated in the density ripples induced by stimulated Brillouin scattering. We expect that this nucleation process is quite ubiquitous in plasma physics since there are many sources of density fluctuations (e.g., beam-plasma instabilities) which can provide nucleation centers for Langmuir cavitons when the necessary long-wavelength sources of free energy are present with frequencies near the mean plasma frequency.

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[5]This is a case considered by Degtyarev *et al.*, Ref. 2; their $E_0 = 2$ case. Except for factor-of-2 differences in the underlying units our equations are the same as those in Ref. 2. In Eq. (12) of Ref. 2 a factor of k^{-3} is apparently missing in the Landau damping formula. For stronger drives, $E_0 > 2$, which are considered in Ref. 2, we find, in apparent agreement with their numerical results, that $\delta n/n_0 > 1$ and the Zakharov equations are invalid.

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[8]A study of the mass-ratio dependence shows that the k dependence of various spectra scales roughly as $(m_i/m_e)^{1/2}k$ which means that in physical units the spectra do not depend sensitively on m_i/m_e.

[9]The possibility of local unstable modes, in a different context, is discussed by D. F. Escande and B. Souillard, Phys. Rev. Lett. **52**, 1296 (1984).

[10]If we take $\nu_e(k=0) = -\nu_D$ and $\nu_e(k>0) = \nu_e^L(k)$ (Landau damping), we find $\Gamma_0 = -(\nu_D/L)|\int_0^L dx\, \phi_0|^2 + \langle \phi_0 | \nu_e^L | \phi_0 \rangle$. The growth term is favored and the damping term suppressed by shallower cavities (considering d_0, the density cavity scale, fixed) since $|\int dx\, \phi_0|^2 \sim l_0$, the scale of the trapped wave function, which may be somewhat larger than d_0 for weakly bound states, i.e., for shallower wells. In this case, the criterion for subsequent supersonic-driven collapse is $|H_0|^2 \exp(2\Gamma_0 d_0)/(\Gamma_0 d_0)^2 d_0 >> n_0$ with $|d_0 \Gamma_0| >> 1$ and H_0 the initial amplitude.

[11]D. A. Russell, private communication.

[12]C. Aldrich and B. Bezzerides, private communication.