Low-Lying Isovector Collective States and the Interacting-Boson Model

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Isovector low-lying collective 2^+ states are studied in the samarium isotopes in terms of the interacting-boson model. On the basis of $E2$ transition data, it is pointed out that the 2_3^+ states of ^{148,150}Sm are isovector. The first calculation including different proton and neutron boson charges is reported for the $E2$ transitions $0_1^+ \rightarrow 2_1^+$, 2_2^+, and 2_3^+ of ^{148,154}Sm, giving rise to the first comprehensive agreement with experiment. Some predictions of $B(M1)$ values are presented for further confirmation of the isovector properties.

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The recent discovery of the $J^\pi = 1^+$ magnetic collective state in ^{156}Gd demonstrates that isovector collective states can be quite low in energy.[1] The interacting boson model version 2 (IBM-2) treats explicitly the proton and neutron degrees of freedom[2,3] and is capable of describing this state which is isovector in the proton and neutron quadrupole degrees of freedom.[4,5] However, this state may not be the only low-lying isovector state. In this paper, we use IBM-2 to search for isovector states and properties, taking the samarium isotopes as an example.

The IBM-2 states are constructed from proton $J^\pi = 0^+ (s_\pi)$ and $2^+ (d_\pi)$ bosons, and neutron $J^\pi = 0^+ (s_\nu)$ and $2^+ (d_\nu)$ bosons.[2,3] The IBM-2 Hamiltonian[3] is taken to be

$$H = \epsilon_d n_d - \kappa Q_\pi Q_\nu + M, \tag{1}$$

where ϵ_d is the single d boson energy with respect to the s boson energy, n_d stands for the operator which counts the total number of d bosons, κ denotes a coupling constant, and Q_π and Q_ν are defined as

$$Q_\tau = P_\tau + \chi_\tau [d_\tau^\dagger \tilde{d}_\tau]^{(2)}, \qquad P_\tau = s_\tau^\dagger \tilde{d}_\tau + d_\tau^\dagger s_\tau, \quad \tau = \pi, \nu \tag{2}$$

with χ_τ being a coefficient.[3] The third term on the right-hand side of Eq. (1) is the Majorana interaction[2]; $M = \sum_{L=1,3} \xi_L [d_\pi^\dagger d_\nu^\dagger]^{(L)} [\tilde{d}_\nu \tilde{d}_\pi]^{(L)}$.

The IBM-2 eigenstates are obtained by diagonalizing the Hamiltonian in Eq. (1) for a set of N_π proton bosons and N_ν neutron bosons, which are equal to half of the number of valence protons and neutrons, respectively.[3,6] N_π is 6 for the Sm isotopes, since there are twelve protons outside the $Z = 50$ magic core. In the following, we consider ^{148}Sm–^{154}Sm, which correspond to $N_\nu = 2$–5, taking the $N = 82$ magic core. As in the standard IBM-2 phenomenological fit,[7] values of parameters ϵ_d, κ, χ_π, and χ_ν are searched so that calculated excitation energies reproduce experimental energies. The search was carried out for each isotope separately, although no drastic variation is expected as a function of N_ν. Because of the large number of valence nucleons, we assume that any effect of the $Z = 64$ subshell gap will either be wiped out or absorbed in the phenomenological parameters.

The ξ_2 term in the Majorana interaction[7] is set $\xi_2 = 0$ is this paper, which is one possible phenomenological choice, and seems to be justified by a microscopic theory.[8] The ξ_1 and ξ_3 are assumed to be equal to each other[5] and are taken to be 0.1, 0.2, 0.4, and 0.6 MeV for $N_\nu = 2$–5. The ξ_1 and ξ_3 terms do not have any notable effects on the $J^\pi = 2^+$ states which we are going to investigate.

For the samarium isotopes the valence protons and neutrons are filling different shell-model orbits. Hence the states described in IBM-2 will all have the same isospin, the isospin of the ground state, $T = T_Z = \frac{1}{2}(N - Z)$. The term isovector used beforehand refers to transitions between states with the same isospin but for which the difference of the proton and neutron quadrupole matrix elements is much larger than their sum.[9] However, within IBM-2, there is an SU(2) group different from isospin which can classify the states with the same isospin according to their symmetry with respect to proton and neutron bosons. The quantum number is called F spin[2,10] which can range in value from $F_{\max} = \frac{1}{2}(N_\pi + N_\nu)$ to $F_{\min} = \frac{1}{2} \times |N_\pi - N_\nu|$. The IBM-2 Hamiltonian in Eq. (1) does not conserve F spin because the proton-neutron quadrupole interaction is assumed to be much stronger than the neutron-neutron or proton-proton quadrupole interaction. Nevertheless, the lowest states will be primarily composed of the states with $F = F_{\max}$, which have the most proton-neutron symmetry. For example the s boson condensate state

$$|F = F_{\max}, 0\rangle = |s_\pi^{N_\pi} s_\nu^{N_\nu}\rangle \tag{3}$$

has maximum F spin. Likewise all the states in which quadrupole bosons are created in a symmetric manner also have maximum F spin,[2]

$$|F = F_{\max}, l\rangle \propto (d_\pi^\dagger s_\pi + d_\nu^\dagger s_\nu)^l |F = F_{\max}, 0\rangle, \tag{4}$$

where l is an integer ($0 \leq l \leq N_\pi + N_\nu$) and all other quantum numbers are omitted for brevity. States in Eq. (4) are also referred to as totally symmetric states.

The IBM-1 treats only these totally symmetric states,[11] because proton and neutron bosons are not distinguished.[2] In the one d boson (d^1) configuration, the totally symmetric state is

$$|F=F_{\max},1\rangle=(N_\pi+N_\nu)^{-1/2}\{\sqrt{N_\pi}|d_\pi s_\pi^{N_\pi-1}s_\nu^{N_\nu}\rangle+\sqrt{N_\nu}|d_\nu s_\pi^{N_\pi}s_\nu^{N_\nu-1}\rangle\}, \tag{5a}$$

while the lower F-spin state is given by

$$|F=F_{\max}-1,1\rangle=(N_\pi+N_\nu)^{-1/2}\{\sqrt{N_\nu}|d_\pi s_\pi^{N_\pi-1}s_\nu^{N_\nu}\rangle-\sqrt{N_\pi}|d_\nu s_\pi^{N_\pi}s_\nu^{N_\nu-1}\rangle\}. \tag{5b}$$

The IBM-2 Hamiltonian with an attractive ($\kappa > 0$) $Q_\pi Q_\nu$ interaction pushes down the state in Eq. (5a) while pushing up the state in Eq. (5b). This is the basic reason why lower F-spin states are found to be at higher excitation energies.

The IBM-2 $E2$ transition operator is $T=e_\pi^B Q_\pi + e_\nu^B Q_\nu$ where e_π^B and E_ν^B are referred to as proton and neutron boson charges, and Q_π and Q_ν are defined in Eq. (2). The IBM-2 calculations performed so far assumed $e_\pi^B=e_\nu^B$ for the sake of simplicity. In this paper, however, we searched for proper values of e_π^B and e_ν^B separately. The $E2$ transition operator can be written as $T=e_s Q_s+e_v Q_v$ where Q_s is an F-spin scalar and Q_v is an F vector,

$$Q_{s,v}=P_\pi \pm P_\nu+\chi_{s,v}([d_\pi^\dagger \tilde{d}_\pi]^{(2)}\pm[d_\nu^\dagger \tilde{d}_\nu]^{(2)}), \tag{6}$$

where

$$e_{s,v}=\tfrac{1}{2}(e_\pi^B \pm e_\nu^B),$$

$$\chi_{s,v}=[(e_\pi^B\chi_\pi \pm e_\nu^B\chi_\nu)/(e_\pi^B \pm e_\nu^B)].$$

The transition between totally symmetric states is mainly due to the F-scalar term, while the transition between a state with $F=F_{\max}$ and a state with $F=F_{\max}-1$ is due solely to the F-vector term. Thus, if all low-lying states were totally symmetric as assumed in IBM-1, the F-vector term plays no significant role. The major purpose of this paper is to point out in which nuclei and at what energy one can find states with $F<F_{\max}$ like those in Eq. (5b), and to demonstrate where and how significantly the F-vector $E2$ operator contributes to experimental $B(E2)$'s.

We now return to the samarium isotopes. We determined the IBM Hamiltonian in Eq. (1) so that the experimental spectrum is reproduced well. The following values of the parameters in Eq. (1) are obtained, for $N_\nu=2$, 3, 4, and 5, respectively; $\kappa=0.140$, 0.090, 0.075, and 0.085 MeV; $\epsilon_d=1.0$, 0.7, 0.5, and 0.38 MeV; $\chi_\pi=-0.8$, -0.8, -1.0, and -1.0; $\chi_\nu=-0.8$, -0.5, -0.9, and -0.75. We obtained a good agreement between IBM-2 and experimental low-lying spectra. Both experimental and theoretical energies show the transition from the vibrational spectrum of ^{148}Sm to the rotational spectrum of ^{154}Sm.

We shall now study the wave functions of ^{148}Sm. The d^1 component of the 2_1^+ wave function, which is the dominant component, is a linear combination of the states in Eqs. (5a) and (5b) with amplitudes 0.815 and -0.145, respectively. Thus, the d^1 component primarily consists of the totally symmetric state in Eq. (5a). Relatively large overlaps with totally symmetric states are found for higher configurations as well, making this 2_1^+ state predominantly a totally symmetric state. Since the ground (0_1^+) state is also primarily totally symmetric, the $E2$ transition $0_1^+ \rightarrow 2_1^+$ is dominated by the F-scalar term in Eq. (6).

The situation is completely reversed in the 2_3^+ state. The d^1 component, which is dominant, is a linear combination of the states in Eqs. (5a) and (5b) with amplitudes 0.076 and 0.814, respectively. The 2_3^+ state out to be primarily the lower F-spin state in Eq. (5b). The $E2$ transition $0_1^+ \rightarrow 2_3^+$ is carried mainly by the F-vector term in Eq. (6).

To test the structure of these wave functions, one can look at $E2$ transitions. As already mentioned, there are two boson charges e_π^B and e_ν^B. These are determined so that the two largest experimental $B(E2)$ values of three transitions $0_1^+ \rightarrow 2_1^+$, 2_2^+, and 2_3^+ are reproduced.

In the case of ^{148}Sm, we took the transitions $0_1^+ \rightarrow 2_1^+$ and 2_2^+,[12] obtaining $e_\nu^B=0.057$ e b and $e_\pi^B=0.128$ e b. With use of these values, other transitions[12, 13] are reproduced remarkably well as shown in Fig. 1, which implies that the wave functions and boson charges are reasonably good. Note that $e_\pi^B/e_\nu^B \sim 2$, which is naturally expected from the shell model where the proton effective charge is larger by a factor 2–3 than the neutron effective charge.

Heavier Sm isotopes are also studied in the same way. The boson charges are shown in Fig. 2 in terms of the F-scalar and F-vector charges, e_s and e_v. The g-boson renormalization increases the F-scalar charge and decreases the F-vector charge from their shell model values,[14] and most likely will also explain their mass dependence.

With use of the boson charges in Fig. 2, all transitions in Fig. 1 are nicely reproduced.[12, 13] The ratio $B(E2{:}2_1^+ \rightarrow 4_1^+)/B(E2{:}0_1^+ \rightarrow 2_1^+)$ is the most important measure as to what extent the system is vibrational or rotational. This ratio is calculated without any adjustable parameter, giving rise to an almost perfect agreement with experiment. The transitions $0_1^+ \rightarrow 2_2^+$ and 2_3^+ are also reproduced very well. These transitions are weaker and more sensitive to isovector properties. In order to see this, the ratio between matrix

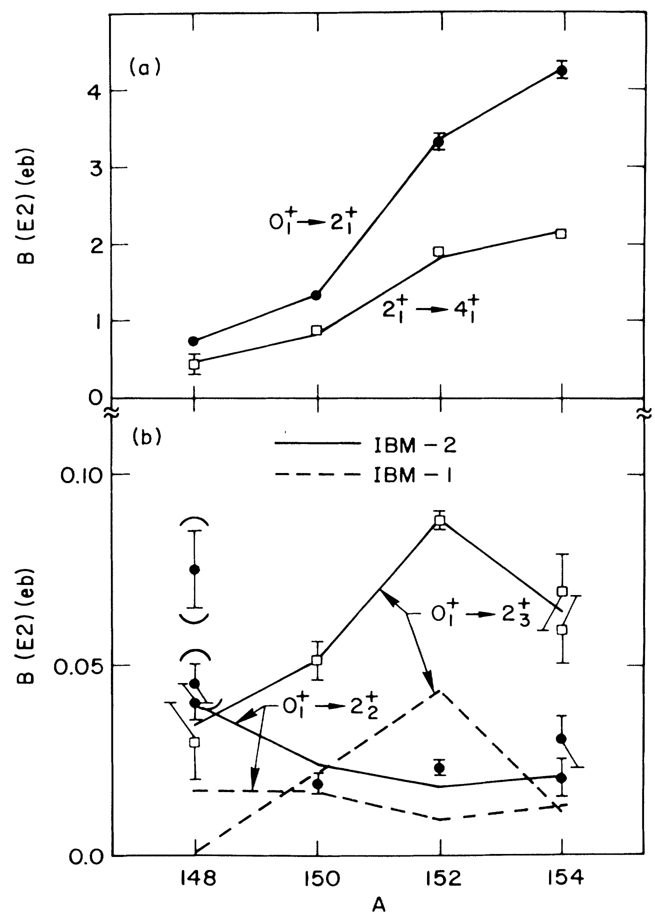

FIG. 1. Calculated (lines) and experimental (symbols) $B(E2)$ values for four transitions of ${}^{A}_{62}$Sm. The solid lines are obtained by the present IBM-2 calculation, while the dashed lines are obtained by an IBM-1 calculation (Ref. 15).

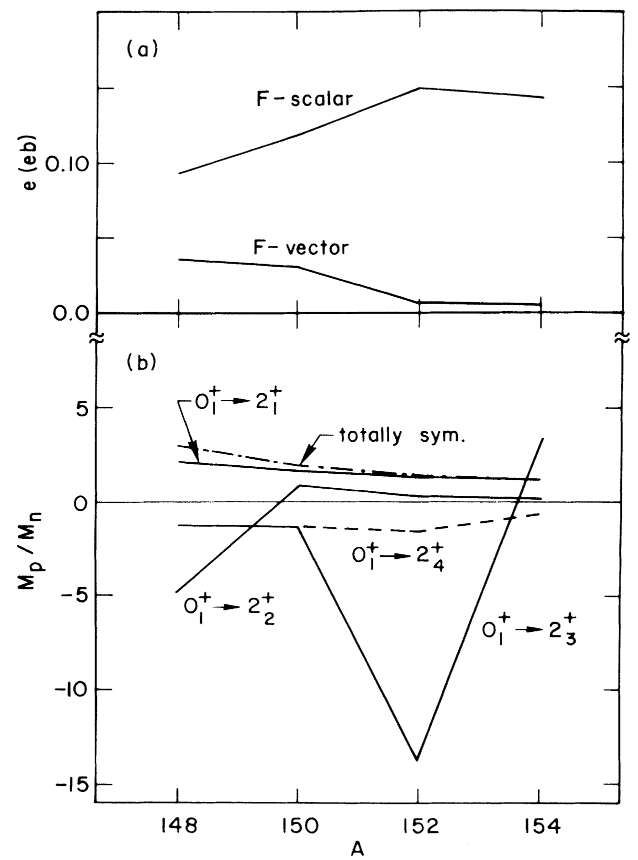

FIG. 2. (a) The F-scalar and F-vector boson charges for ${}^{A}_{62}$Sm. (b) The proton-neutron ratio $M_p/M_n = \langle 2^+ \| Q_\pi \| 0^+ \rangle / \langle 2^+ \| Q_\nu \| 0^+ \rangle$. The dash-dotted line indicates the ratio for totally symmetric states. The dashed line shows the $0_1^+ \rightarrow 2_4^+$ transitions in ^{152,154}Sm.

element of Q_π over that of Q_ν is shown as M_p/M_n in Fig. 2. The $0_1^+ \rightarrow 2_1^+$ transition follows the totally symmetric values, particularly in the deformed region. On the other hand, the ratio is -1 for the transition from the state in Eq. (3) to the state in Eq. (5b). Similar values are found in Fig. 2 for transitions $0_1^+ \rightarrow 2_3^+$ of ^{148,150}Sm and for $0_1^+ \rightarrow 2_4^+$ of ^{152,154}Sm, indicating that these 2^+ states contain a considerable amount of components with $F < F_{max}$.

The transitions from the ground to the other 2^+ states are in intermediate situations. As shown in Fig. 2, the ratio M_p/M_n tends to become positive as deformation evolves, implying more totally symmetric components in the wave function. The ratio, however, never becomes that of totally symmetric states. Including such complicated cases, all the transitions $0_1^+ \rightarrow 2_1^+$, 2_2^+, and 2_3^+ are nicely reproduced by the present IBM-2 calculation, whereas the IBM-1 calculations attempted so far[12,15] have failed by a factor of 2 in the whole region of the Sm isotopes (see Fig. 1). This suggests to us the importance of the explicit treatment of the proton and neutron degrees of freedom as already implemented in IBM-2.

The $M1$ transition is also sensitive to F-vector properties. The $M1$ operator in IBM-2 is defined as[5] $T^{(M1)} = (\sqrt{3}/4\pi)(g_\pi L_\pi + g_\nu L_\nu)$ where $L_{\pi(\nu)}$ denotes the proton (neutron) boson angular-momentum operator, and $g_{\pi(\nu)}$ is the proton (neutron) boson g factor. We assumed $g_\nu = (-0.35, -0.30, -0.25, -0.20)\mu_N$ for $N_\nu = 2$–5, respectively, while g_π is kept constant at $g_\pi = 0.85\mu_N$. As a result of the difference between g_ν and g_π, there is a strong F-vector term in $T^{(M1)}$. Table I shows predictions of g_R factors of the 2_1^+ state, $B(M1{:}2_{2,3}^+ \rightarrow 2_1^+)$ and $B(M1{:}0_1^+ \rightarrow 1_1^+)$. The table also shows experimental g_R factors of the 2_1^+ states[13,16] which are reasonably well reproduced. We point out that a considerably large $B(M1)$ is obtained for the transitions $2_3^+ \rightarrow 2_1^+$ of ^{148,150}Sm. Although the Weisskopf estimate of $B(M1)$ is $1.8\mu_N^2$, this value mainly comes from the g_s factor which has a small ef-

TABLE I. Calculated g_R factor of the 2_1^+ state and three $B(M1)$'s of ^{148,154}Sm. The experimental g_R factors are also shown after the slash (Refs. 13 and 16).

A	g_R (μ_N) 2_1^+	$B(M1)$ (μ_N^2) $2_3^+ \rightarrow 2_1^+$	$2_2^+ \rightarrow 2_1^+$	$0_1^+ \rightarrow 1_1^+$
148	0.40/0.27(5)	0.46	0.049	0.61
150	0.38/0.36(4)	0.47	0.003	0.75
152	0.37/0.42(3)	0.06	0.007	1.46
154	0.38/0.39(2)	0.02	0.007	1.80

fect in quadrupole collective states.[5] By neglecting the g_s factor and averaging over proton and neutron values, one obtains the single-particle estimate $\sim 0.1\mu_N^2$. The $B(M1{:}2_3^+ \rightarrow 2_1^+)$ values of ^{148,150}Sm exceed this estimate, implying some collective effects, whereas in the other cases $B(M1{:}2_{2,3}^+ \rightarrow 2_1^+)$ values are as small as this single-particle estimate. The large $B(M1)$ values are due to the F-vector character of the 2_3^+ state of ^{148,150}Sm, and should be measured as a test of our conjecture.

In the pure vibrational limit, the $B(M1)$ from the ground state vanishes.[10] However, the calculated $B(M1{:}0_1^+ \rightarrow 1_1^+)$ in Table I is still sizable in spherical nuclei like ^{148}Sm. The ground-state correlation plays a crucial role here, whereas it is ignored in the usual SU(5) estimate. We note also that in the deformed samarium isotope the calculated $B(M1)$ is comparable to that measured in ^{156}Gd, $B(M1)=(1.3 \pm 0.2)\mu_N^2$ (Ref. 1).

We summarize this Letter by mentioning the following: (i) There can be isovector $J^\pi = 2^+$ collective states below 2 MeV. (ii) The IBM-2 can describe such states utilizing its F-vector properties. (iii) Even in spherical nuclei the magnetic dipole transition strength from the ground state can be collective, $\sim \frac{1}{2}$ that of deformed nuclei. More studies, particularly experimental, are strongly urged on this intriguing problem.

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