Axial Currents in Nuclei and the Skyrmion Size

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Combining the skyrmion picture of the nucleon and the axial Ward identities in nuclear medium, I show that in a baryon-rich environment, the coupling constants g_A and f_π are *fundamentally quenched* and the rms size of the nucleon consequently *enhanced.*

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It has long been suspected that the axial-vector current mediating Gamow-Teller transitions is intrinsically modified in baryon-rich media such as nuclei, resulting in an axial-vector coupling constant g_A and other weak quantities that are *fundamentally* different from the free-space values. There are some experimental indications[1] that g_A is indeed quenched in nuclei to a value of about 1 from its free-space value of 1.25. There are numerous[2] mechanisms invoked to explain this phenomenon, but none of them has seriously addressed the question of the fundamental nature of the medium renormalization. In this paper, I propose to interpret the phenomenon in terms of what (little) we know of low-energy properties of quantum chromodynamics. In particular, using the skyrmion model of the nucleon[3] and axial Ward identities,[4] I show that the quenching of g_A and the pion decay constant f_π in nuclear matter implies that the skyrmion size (or the "bag" size) which is found to satisfy the relation

$$r_{\rm rms} = {\rm const} \times \sqrt{g_A}/f_\pi \tag{1}$$

expands by as much as 40% as a result of the presence of neighboring nucleons.

To arrive at Eq. (1), consider g_A of the nucleon in the skyrmion description. Specifically, the skyrmion Lagrangian will be of the form

$$\mathscr{L}_{\rm sk} = -\tfrac{1}{4} f_\pi^2 \,{\rm Tr}[L_\mu, L_\mu] + \tfrac{1}{4}\epsilon^2 {\rm Tr}[L_\mu, L_\nu]^2$$

with $L_\mu = U^\dagger \partial_\mu U$, $U = f_\pi^{-1}[\sigma(r) + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(r)]$, and $U^\dagger U = 1$. It is convenient to use the variable (following the notation of Jackson and Rho[5]) $\tau = \ln(r/r_0)$, where r_0 is a redundant scale parameter. For the soliton configuration in the "hedge-hog" form,[3] $U_0 = \exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}\theta(r)]$, one has a unique solution for $\theta(\tau)$ for a given $A = 16\epsilon^2/f_\pi^2 r_0^2$. For any other value of A, say A', it follows from a simple scaling argument[5] that $\theta_{A'}(\tau) = \theta_A[\tau - \frac{1}{2}\ln(A'/A)]$. For a conserved axial current,[6] the g_A is determined by the asymptotic behavior of the soliton solution,

$$\theta(\tau) \rightarrow \alpha e^{-2\tau}, \quad \tau \rightarrow \infty, \tag{2}$$

where the coefficient α may depend upon the value of A. Specifically it is given by the residue of the pion pole[7] and so

$$g_A^h = 8\pi\alpha(f_\pi r_0)^2. \tag{3}$$

The physical g_A (appropriate for neutron beta decay) is related to g_A^h by an appropriate numerical (quantization) factor as described by Adkins, Nappi, and Witten.[5] This factor will not be needed for the calculation, since it will be cancelled out in the ratio calculated later.

The skyrmion size or equivalently the "bag" size is obtained simply from the anomalous baryon density (or equivalently the winding-number density[3]). For the baryon number $B = 1$, it is of the form[5]

$$r_{\rm rms}^2 \equiv I r_0^2 = -r_0^2 \pi^{-1} \int_{-\infty}^{\infty} e^{2\tau}[1 - \cos 2\theta(\tau)] \frac{d\theta(\tau)}{d\tau} d\tau. \tag{4}$$

Substituting Eq. (3) into Eq. (4), one arrives at Eq. (1) with the constant given by

$$C = (I/8\pi\alpha)^{1/2}. \tag{5}$$

A crucial observation to make here is that the ratio I/α is independent of A and hence of f_π and ϵ. This follows from the identity

$$I(A)/I(A') = A/A' = \alpha(A)/\alpha(A'), \tag{6}$$

which is a consequence of the scaling property of $\theta(\tau)$ and its asymptotic behavior (2). Thus C is a fixed constant.

Consider now a skyrmion inside a nuclear medium. What happens to its bag size? To see what happens, we will first assume[8] that as long as the skyrmions do not overlap, the baryon-number density of a single skyrmion is still given by a winding-number density whose normalization remains the same but whose profile may be modified. According to arguments based on large-N_c QCD[9] (where N_c is the number of colors), the skyrmion size is $O(1)$ while $g_A \sim O(N_c)$ and $f_\pi \sim O(N_c^{1/2})$. The medium effects, e.g., quark loops which are higher order in $1/N_c$ and hence suppressed at large N_c, would modify g_A to $\tilde{g}_A(\rho)$ and f_π to $\tilde{f}_\pi(\rho)$ where ρ is the nuclear density. Of course, vacuum bubbles, such as quark-antiquark loops, also will contribute higher corrections in $1/N_c$, but we need not worry about them in considering the modification caused by nuclear matter. In what follows, they will be ignored. There will also be contributions of higher order in $1/N_c$ that cannot be incorporated into $\tilde{g}_A(\rho)$ or

$\tilde{f}_\pi(\rho)$. These will be corrections to the *quasiparticle* description that we will adopt here, so will be ignored in the calculation. They will be considered in a future paper. Note that within the approximation, the constant C remains unchanged. We are thus led to a formula describing the size of a "quasiskyrmion" (a skyrmion modified by the hadronic background),

$$\tilde{r}_{\rm rms}(\rho) \simeq C[\tilde{g}_A(\rho)]^{1/2}/\tilde{f}_\pi(\rho). \qquad (7)$$

An intriguing feature of Eq. (7) is that the hadronic size is principally determined by the weak-interaction constants. To know the size, it thus suffices to have experimental data on $\tilde{g}_A$ and $\tilde{f}_\pi$ for various nuclei. Unfortunately, nothing is known from measurements on how $\tilde{f}_\pi$ behaves in nuclear matter, although there have been some suggestions for measuring it in neutrino processes.[10] I will instead calculate them, albeit approximately, using two different models.

To put the arguments in the present context, I first extend the notion of the skyrmion model such that it can be applied to nuclear matter. When the skyrmion is properly quantized as discussed by Adkins *et al.*,[5] the resulting theory is believed to be equivalent to nucleons (baryons in general) coupling to pions in a way consistent with chiral symmetry (e.g., current algebra[11]). Generalizing this in the sense of large-N_c QCD,[9] one may consider a Lagrangian which contains other mesons than pions such as the ϵ meson (which will be denoted as σ), ω, ρ, $A_1, \ldots,$ etc. Let us assume that the theory is renormalizable in the conventional sense. Suppose now we choose to renormalize the theory on the free *nucleon mass shell.* The medium effects will then be a finite renormalization of the type known in nuclear physics as *exchange currents.* In this approach, $\tilde{g}_A$ and $\tilde{f}_\pi$ are predominantly renormalized by the virtual excitation of the $\Delta(1232)$ in the medium. Let us use the Δ-hole model[12] used previously for $\tilde{g}_A$. In this model both $\tilde{g}_A$ and $\tilde{f}_\pi$ are modified by the same mechanism; one finds[10,12]

$$\frac{\tilde{g}_A}{g_A} = \left[1 + (\tfrac{2}{3})^2 \frac{f^{*2}}{m_\pi^2}\frac{2}{\omega_R}\rho g_0'\right]^{-1}, \qquad (8)$$

$$\frac{\tilde{f}_\pi}{f_\pi} = 1 - (\tfrac{2}{3})^2 \frac{f^{*2}}{m_\pi^2}\frac{2}{\omega_R}\rho\left[\frac{\tilde{g}_A}{g_A}\right]. \qquad (9)$$

Here g_A and f_π are free-space quantities, f^* is the $\pi N\Delta$ coupling constant with the quark-model value $f^*/f = (72/25)^{1/2}$, $f^2 \simeq 1$, $\omega_R = m_\Delta - m_N \simeq 2.1 m_\pi$, ρ is the nuclear density, and g_0' is the Landau-Migdal parameter which I will take to be 0.6 (the currently accepted value is between 0.5 and 0.7). For the nuclear matter density $\rho \simeq 0.46 m_\pi^3$, we find $\tilde{g}_A/g_A \simeq 0.76$, $\tilde{f}_\pi/f_\pi \simeq 0.60$, and thus $\tilde{r}_{\rm rms}/r_{\rm rms} \simeq 1.45$.

We now turn to an evaluation based closely on the axial Ward identity.[4] (The above evaluation is also a result of the Ward identity described below.) Consider a Lagrangian $\mathscr{L}^{\rm eff}$ consisting of N, π, σ, and ω. In the spirit of the relativistic mean-field approach to nuclear matter,[13] we first take into account the condensation in the medium of the (fluctuating) meson fields σ and ω: $\langle\sigma\rangle \neq 0$, $\langle\omega_0\rangle \neq 0$. We then calculate fluctuations around these mean fields. Denote the full unrenormalized axial-current–N–N vertex in the medium by ${}^5\Gamma_\mu^i$, with i the isospin index, and the corresponding single-nucleon Green's function by S, and distinguish the renormalized quantities by a tilde, i.e., ${}^5\tilde{\Gamma}_\mu^i$, $\tilde{S}$, etc. Defining the (infinite and density-dependent) renormalization constants Z by ${}^5\Gamma_\mu^i = Z_A^{-1}\,{}^5\tilde{\Gamma}_\mu^i$, $S = Z_2\tilde{S}$, we can write the axial Ward identity[4,6] as

$$q^\mu\,{}^5\tilde{\Gamma}_\mu^i(p+q,p) = \tilde{g}_A^{-1}[\tilde{S}^{-1}(p+q)\tfrac{1}{2}\tau^i\gamma_5 + \tfrac{1}{2}\tau^i\gamma_5\tilde{S}^{-1}(p)], \qquad (10)$$

where we have identified $\tilde{g}_A = Z_2/Z_A$ and the density dependence is suppressed. Equation (10) is ultraviolet finite. Taking the limit $q^\mu \to 0$, one sees that the right-hand side of Eq. (10) is nonzero if the nucleon has a nonzero scalar mass and hence there must be a zero-mass pole on the left-hand side, as dictated by the Goldstone theorem.[14] Let us denote the pole part ${}^5\tilde{\Gamma}_\mu^P$ and the nonpole part ${}^5\tilde{\Gamma}_\mu^R$. The former must be of the form

$$\left(\frac{Z_A}{Z_p}\right)\tilde{f}_\pi\frac{-q_\mu}{q^2+i\epsilon}\tilde{P}(p+q,p)$$

where Z_p is the renormalization constant for ${}^5\Gamma_\mu^P$, and $\tilde{P}$ is the renormalized πNN vertex in the medium.

We must now specify the renormalization prescription. The renormalization will be made at the "mass shell" defined by

$$(\bar{p} - m^*)u(p) = 0, \qquad (11)$$

$$\bar{p} \equiv (p_0 - \Sigma_v, \mathbf{p}), \quad m^* \equiv m_N + \Sigma_s,$$

where Σ_s is the scalar self-energy proportional to $\langle\sigma\rangle$, and Σ_v is the vector self-energy proportional to $\langle\omega_0\rangle$. Denoting the mass-shell condition as $\bar{p} \to m^*$, the prescription is $\tilde{S}^{-1}(p) \to 0$, ${}^5\tilde{\Gamma}_\mu^{Ri} \to \frac{1}{2}\tau_i\gamma_\mu\gamma_5$, and $\tilde{P}^i \to \tilde{g}_r\tau^i\gamma_5$ (with $\tilde{g}_r$ the renormalized πNN coupling constant in medium) as $\bar{p} \to m^*$, $q_\mu \to 0$. Sandwiching Eq. (10) between $\bar{u}(p+q)$ and $u(p)$ and taking the limit $q_\mu \to 0$, one obtains the Goldberger-Treiman relation[4]

$$m^* = \tilde{f}_\pi(\tilde{g}_r/\tilde{g}_A), \qquad (12)$$

with $Z_p = Z_2$. To obtain a relation for $\tilde{g}_A$, we do *not* go on the mass shell $\bar{p} \to m^*$, but instead first expand (10) in powers of q_μ and then set $\bar{p}^2 = m^{*2}$. To zeroth order in q_μ, one obtains again the Goldberger-Treiman relation (12). To first order in q_μ, however, we get a nontrivial relation (which is essentially the same as the partial conservation of axial-vector current result of

Ref. 4),

$$\tilde{g}_A = 1 + \delta\tilde{C} + y\tilde{f}_\pi, \tag{13}$$

where $\tilde{S}^{-1}(p) = (\bar{p} - m^*)(1 + \delta\tilde{C})$, $\delta\tilde{C}(\bar{p} = m^*) = 0$, and

$$\left.\frac{\partial}{\partial q^\mu}\tilde{P}^i(p+q,p)\right|_{q^\mu \to 0} \equiv y\frac{\tau^i}{2}\gamma_\mu\gamma_5. \tag{14}$$

An important point to note here is that because of the anticommutation with γ_5, only the scalar mass m^* contributes to Eqs. (12) and (13): The vector self-energy Σ_v does not contribute. This indicates that both $\tilde{g}_A$ and $\tilde{f}_\pi$ are intrinsically relativistic quantities.

Equations (12) and (13) can be evaluated approximately in nuclear matter. To do so, let us first assume that $\tilde{g}_r/\tilde{g}_A \approx g_r/g_A$. This is reasonable since both the axial vertex and the pseudoscalar vertex satisfy the same Schwinger-Dyson equation. (It might also be that $\tilde{g}_A/\tilde{g}_r \lesssim g_A/g_r$, as it is claimed to be the case in the presence of temperature[15].) Then

$$\tilde{f}_\pi/f_\pi \simeq m^*/m_N \simeq 0.62, \tag{12a}$$

with use of the value $\Sigma_s \simeq -350$ MeV currently favored in nuclear-matter calculations.[13,16]

For $\tilde{g}_A$, more work is needed: We have to calculate the pseudoscalar vertex (14) and the nucleon self-energy, i.e., $\delta\tilde{C}$, at least to one-loop order. We will do this using the linear σ model,[17] with the basic assumption that quantum fluctuations built on the mean-field background are sufficiently weak. To make the calculation as meaningful as possible, we rewrite Eq. (13) as

$$\tilde{g}_A = 1 + (g_A - 1)\delta(x^*)/\delta(x), \tag{15}$$

where $x^* = m_\sigma/m^*$, $x = m_\sigma/m_N$, and m_σ is the scalar-meson mass taken to be ~ 630 MeV in accordance with a recent QCD-based calculation.[18] To one loop, the result is[19]

$$\delta(x) = \frac{g_r^2}{16\pi^2}\left(4 - \tfrac{5}{3}x^2 - x^2(3 - \tfrac{5}{6}x^2)\ln x^2 - (\tfrac{16}{3} - \tfrac{10}{3}x^2)\lambda\{\tan^{-1}[(1 - \tfrac{1}{2}x^2)/\lambda] + \tan^{-1}(\tfrac{1}{2}x^2/\lambda)\}\right), \tag{16}$$

where $\lambda = (x^2 - \frac{1}{4}x^4)^{1/2}$. To arrive at Eq. (16) with x^*, we have dropped the explicitly density-dependent (second) term in the Green's function,

$$S_0(p) = (\bar{p} + m_0^*)\{(\bar{p}^2 - m_0^{*2} + i\epsilon)^{-1} + 2\pi i\delta(\bar{p}^2 - m_0^{*2})\theta(\bar{p}_0)n(p)\},$$

the reason being that it gives rise to fine-detail nuclear-structure dependence and hence *must* be calculated explicitly for individual nuclear systems. It is now clear that covariance is preserved and the limit $q_\mu \to 0$ can be taken in the way discussed above. It is perhaps worth noting that the results (8) and (9) correspond to setting $\Sigma_s = \Sigma_v = 0$ and ascribing dominant medium effects to the Δ-hole Lindhard function.

Numerical calculations of Eq. (16) show that if g_r is taken to be independent of density as befits the loop calculation, then $\delta(x^*)/\delta(x)$ is almost linear in m^*/m_N. Near $m^*/m_N \simeq 0.51$, at which $\delta(x^*)$ vanishes, the ratio can be well approximated by $\delta(x^*)/\delta(x) \simeq 3(m^*/m_N - 0.51)$. Thus we find that (with $g_A - 1 = 0.25$) $\tilde{g}_A/g_A \approx 0.86$ for $m^*/m_N \approx 0.6$ and 0.80 for $m^*/m_N \approx 0.5$. This suggests that we use $\tilde{g}_A/g_A \approx 0.8$ for nuclear matter (consistent with the experimental data[1] in nuclei). Combining with Eq. (12a) we have $\tilde{r}_{\rm rms}/r_{\rm rms} \simeq 1.44$. In view of the crudeness of the model (e.g., higher loops and other mesons may not be negligible), this result cannot be taken too seriously. Nevertheless, the two methods give very similar results, suggesting that the qualitative picture of a growing skyrmion size in nuclear medium must be correct.

There are some interesting implications of the dilated skyrmion. First of all, this may have some connection to a similar phenomenon observed in deep-inelastic lepton scattering from nuclei[20] (e.g., "EMC effects"). Secondly, it is perhaps teaching us how the nucleons in nuclear medium "prepare" themselves for an eventual chiral phase transition expected at some higher density. Lastly, it provides (to the author) the only plausible explanation known of the observation that the soft-pion exchange currents so important in light nuclei become rapidly suppressed—and soft-pion theorems become powerless—in heavier nuclei.[21]

It remains to be shown explicitly, but it seems most plausible, that the increase of the skyrmion size is in fact the very cause for the quenching of $\tilde{g}_A$ and $\tilde{f}_\pi$. This problem will be addressed elsewhere in terms of the chiral bag model.

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