Deformation of the Nucleon and Delta in Excited States

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A quark model for baryons, in which the valence quarks are moving in a deformable mean field, is considered. A q-q interaction, consisting of one-gluon-exchange and one-pion-exchange potentials, is diagonalized exactly in the model space of deformed orbitals. With four model parameters, the masses of both the even- and odd-parity states and the radiative transition amplitudes for the $L=0$ excited states are obtained in fair agreement with experiment.

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Baryon spectroscopy is a very active field of research with a wealth of available experimental data.[1] Rigorous nonperturbative QCD is not as yet able to confront this problem, and the best that can be done is to unravel the data by various phenomenological QCD-inspired quark models. The most successful of such models is that of Isgur and Karl,[2,3] where the constituent quarks move nonrelativistically in a harmonic confinement, and anharmonic residual interactions are treated in first-order perturbation theory. The hyperfine and tensor q-q interactions are taken from one-gluon exchange, and the spin-orbit part is suppressed. The model has the merit (over relativistic models) of eliminating the center-of-mass motion cleanly, and generating analytical wave functions for the excited states. Extensive comparison with experimental data is easily made. Recently, Forsyth and Cutkosky[4] have refined this model and examined the masses and elastic widths of all states up to $N=3$ oscillator excitations.

Despite the success of the spherical-oscillator model discussed above, we present in this paper an alternative model[5,6] in which the baryon acquires considerable deformation in excited states. What is the necessity of our introducing yet another model when the simpler scheme of Isgur and Karl is successful? To answer this, let us examine the role of the central residual interaction in the spherical-oscillator model. Its matrix element in the ground state is roughly of the same magnitude as the oscillator spacing. In first-order perturbation, the diagonal matrix element of this interaction brings down in energy the symmetric $L=0$, $N=2$ harmonic excitation close to the odd-parity $N=1$ state. This explains the observation of $N(1440)^+_{1/2}$ around the same energy as the low-lying odd-parity states. If, however, this interaction is exactly diagonalized in the truncated model space of the spherical model, the large off-diagonal element between the ground state and its nodal excitation nullifies this effect. In our view, the prescription of neglect of the off-diagonal matrix elements of the anharmonic interaction may be circumvented if the baryon is allowed to deform in the excited states. With the deformation increasing with excitation the excited states are naturally brought down in energy.

In this Letter, we demonstrate that a realistic q-q interaction, whose parameters are consistent with theory, may be exactly diagonalized in the model space of the deformed orbitals to yield energy levels in good agreement with the experimentally observed states. Moreover, a sensitive test of whether the excited-state orbitals are deformed or not is to calculate the transition amplitudes between the excited and the ground states. In the spherical-oscillator model, $N(1440)^+_{1/2}$ is mainly a nodal excited symmetric state with little admixture of the mixed-symmetry $N=2$ state. In our model, its wave function is very different with about 40% admixture of the mixed-symmetric component. The helicity amplitude $A_{1/2}$ for the electromagnetic transition $N(1440)\to N(940)+\gamma$ is known experimentally[1] to about 10% accuracy for the proton, and we find that our model fares better than the spherical one in this sensitive test. For $N(1710)^+_{1/2}$, the predictions of the two models are spectacularly different, and the experimental data, although rather poor, favor large mixing.

In our model, the three valence quarks are assumed to move in a deformable mean field. This deformation may be the result of quantum many-body effects, like the polarization of the sea-quark field with excitation energy, and is not due to the one-gluon-exchange two-body potential. In the nonstrange sector, the mean-field Hamiltonian is taken to be[6]

$$H_0=\sum_{i=1}^{3}[p_i^2/2m+\tfrac{1}{2}m(\omega_x^2x_i^2+\omega_y^2y_i^2+\omega_z^2z_i^2)],$$

where $\mathbf{p}_i$ and $\mathbf{r}_i$ denote the momentum and position of the ith quark, and $m=m_u=m_d$ is the constituent quark mass. After elimination of the center-of-mass part, the intrinsic energy is given by the motion of two uncoupled anisotropic oscillators in the intrinsic coordinates $\boldsymbol{\rho}=(\mathbf{r}_1-\mathbf{r}_2)/\sqrt{2}$ and $\boldsymbol{\lambda}=(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3)/\sqrt{6}$. The intrinsic energy is

$$E=(N_x+1)\hbar\omega_x+(N_y+1)\hbar\omega_y+(N_z+1)\hbar\omega_z,$$

where $N_x=(n_{\rho_x}+n_{\lambda_x})$, the number of oscillator quanta excited in the x direction, etc. An intrinsic state is specified by the occupied orbitals (N_x,N_y,N_z), with $N=N_x+N_y+N_z$. The shape of the potential in each intrinsic state is determined by minimization of the intrinsic energy E, with respect to variations in ω_x, ω_y, and ω_z, subject to the volume conservation condition $\omega_x\omega_y\omega_z=\omega_0^3$. This yields the equilibrium condition $\omega_x(N_x+1)=\omega_y(N_y+1)=\omega_z(N_z+1)$. For the ground state, $N_x=N_y=N_z=0$, and consequently, $\omega_x=\omega_y=\omega_z=\omega_0$. Thus, the ground state is spherical, but the excited states are deformed. It was shown in Ref. 6 that the states of axially symmetric prolate shape $(N=N_z)$ come down most in energy. The equilibrium parameters for this shape are given by $\omega_x=\omega_y=(N+1)\omega_z=\omega_0(N+1)^{1/3}$. The corresponding intrinsic wave functions of appropriate symmetry may be readily constructed, and are given in Ref. 6. The next step is to diagonalize the q-q interaction in this nonorthogonal basis.

In Ref. 6, the central residual q-q interaction, although weaker than in the spherical model, was still treated in first-order perturbation. The main objective of this paper is to show that a realistic QCD-inspired q-q interaction may be diagonalized without the neglect of the off-diagonal elements, and yet reproduce the data for both the even- and odd-parity states. Consider first the one-gluon-exchange potential (OGEP). The running coupling constant in QCD is

$$\alpha_s(q^2)=12\pi/[(33-2N_f)\ln(-q^2/\Lambda^2)],$$

where N_f is the number of flavors and Λ the scale parameter. At $-q^2=1$ GeV^2, and for $N_f=3$, we get α_s in the range of 0.3 to 0.4 for Λ between 0.1 and 0.2 GeV. The static part of the OGEP is taken to be[7]

$$V_{\rm OGEP}(\mathbf{r})=-\frac{2}{3}\frac{\alpha_s}{r}+\frac{2\pi}{3}\frac{\alpha_s}{m^2}\delta^3(\mathbf{r})+\frac{4\pi\alpha_s}{9m^2}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\delta^3(\mathbf{r}). \quad (1)$$

To be consistent with the QCD estimate, we choose $\alpha_s=0.35$ in Eq. (1). Such a small value of α_s yields somewhat less than half the observed N-Δ ground-state splitting. One alternative way out of this dilemma is to take α_s close to unity.[6] We found, however, that a better simultaneous fit to the N-Δ *and* particularly odd-parity spectra is obtained if a spin-isospin–dependent q-q force is postulated. Such a potential may arise if the pion field is regarded as elementary, from considerations of chiral symmetry, interacting with the valence quarks.[8] With the introduction of the pion, the tensor force and the spin-orbit force in particular coming from OGE are suppressed by a factor of 3 with further suppression coming from deformation. A detailed account of how this leads to a solution of the long-standing spin-orbit puzzle will be published shortly. In this paper, we do not treat the pions dynamically, but assume simply a one-pion-exchange potential (OPEP) between the quarks. With a pseudoscalar coupling the central part of the potential is

$$V_{\rm OPEP}(\mathbf{r})=-\frac{q_{qq\pi}^2}{4\pi}\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)}{12m^2}\left(4\pi\delta^3(\mathbf{r})-m_\pi^2\frac{e^{-m_\pi r}}{r}\right). \quad (2)$$

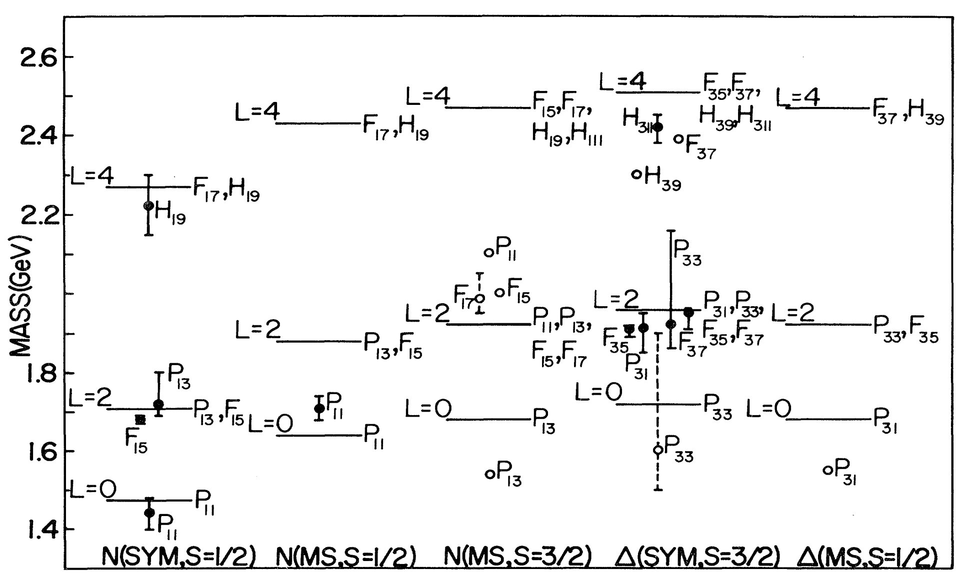

FIG. 1. Even-parity $N=2$ nucleon and delta spectra. The permutation symmetry (SYM or MS) refers only to the main component in the wave function. The nominal masses (and the spreads) of well-defined states are shown by filled circles (solid lines), while the masses of weak states are shown by open circles. The uncertainty in the mass spread of weak states, where it is known, is shown by dashed lines.

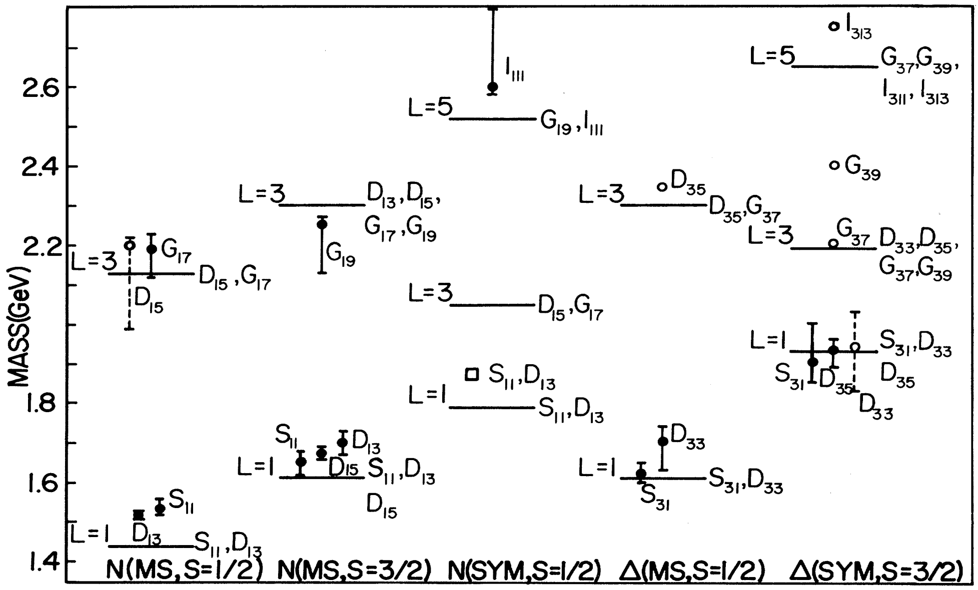

FIG. 2. Odd-parity $N=1$ (MS) and $N=3$ (SYM) nucleon and delta spectra. The notation is the same as in Fig. 1. In addition, the state denoted by the open square corresponds to the phase-shift analysis used in Ref. 4.

In the $\mathbf{q}\to 0$ limit, $g_{qq\pi}=\frac{3}{5}(m/m_N)g_{NN\pi}$, where $g_{NN\pi}$ is the pion-nucleon coupling constant and m_N is the nucleon mass. From the above relation, taking $g^2_{NN\pi}/4\pi=14.6$, $m=330$ MeV, one gets $g^2_{qq\pi}/4\pi=0.65$. In Eq. (2), the zero-range term contributes dominantly, and should get suppressed for nonzero $-q^2$ because of vertex modification by the pion form factor. For simplicity of calculation of the matrix elements, however, we retain the form (2), but allow for a reduced value of $g^2_{qq\pi}$ to account for the suppression. With $\alpha_s=0.35$, the N-Δ splitting is reproduced if $g^2_{qq\pi}/4\pi$ is taken to be 0.35.

The intrinsic Hamiltonian, H_0, plus the interactions (1) and (2) are diagonalized in the nonorthogonal intrinsic basis, and the energies of good angular momenta are projected out, as described in Ref. 6. The results of this calculation are displayed in Figs. 1 and 2. The parameters chosen for this calculation are given by $m=330$ MeV, $\hbar\omega_0=550$ MeV, $\alpha_s=0.35$, and $g^2_{qq\pi}/4\pi=0.35$. In addition, there is an overall constant (present in every constituent-quark model), which is taken to be -1259 MeV to set the mass of $N(940)$. Note that the number of parameters we need is the same as in the spherical model, but we are also able to explain the observed odd-parity states in the N and Δ around 2000 MeV. Forsyth and Cutkosky[4] have stressed that an additional parameter is needed in the spherical model to obtain these $N=3$ excitations.[9]

Finally, to compute the helicity amplitudes for radiative transitions, wave functions of good angular momentum were projected out from the deformed intrinsic states. This could be done analytically in our model. The $N=2$ states in our model are considerably deformed, with $\omega_x=\omega_y=3\omega_z=(3)^{1/3}\omega_0$. When expressed in terms of spherical orbitals of oscillator parameter ω_0, they contain about 15% admixtures of $N=4$ and a couple of percent of $N=6$ shells. As emphasized earlier, we find that the projected wave function of $N(1440)^+_{1/2}$ has about 40% admixture of the mixed-symmetry component, and similarly $N(1710)^+_{1/2}$ is about 40% symmetric. The computed helicity amplitudes from these states are shown in Table I, where the experimental values, as well as the results of the Isgur-Karl model, are shown.[10] The computed helicity amplitudes depend sensitively on the structure of the wave functions of the excited states. With deformed orbitals, the interference of various terms enhances the transition amplitudes $A_{1/2}$ from $N(1440)^+_{1/2}$ to the ground state, and they cancel for $N(1710)^+_{1/2}$ to yield a near-zero value. For the ground-state N and Δ, our wave functions are very similar to the Isgur-Karl model, and the transition amplitudes for $\Delta\to N+\gamma$ are almost the same. In all three cases, our model calculation yields only about 60% of the experimental value for the amplitudes—for the spherical quark model the situation is worse. In the same context, note that the radius of the nucleon ground state is only about 0.6 fm in our model, and this underestimation of the radius is also a common feature of the constituent-quark models. A dynamic treatement of the pion cloud[8] rectifies this shortcoming, and should improve the results of the computed helicity amplitudes.

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TABLE I. The measured γN decay amplitudes (in units 10^{-3} $GeV^{-1/2}$) and the results in our model. For comparison we also show, in the last column, the results obtained by the Isgur-Karl model (Ref. 10).

Resonance	Helicity	Experiment	Our model	I-K model
$N(1440)P_{11}$	$A^p_{1/2}$	-69 ± 7	-38	-24
	$A^n_{1/2}$	37 ± 19	24	16
$N(1710)P_{11}$	$A^p_{1/2}$	-5 ± 16	3	-47
	$A^n_{1/2}$	-5 ± 23	3	-21

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