Mass and Anomalous Magnetic Moment of an Electron between Two Conducting Parallel Plates

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The configuration of the electromagnetic field between two conducting plates in the presence of a point source is studied. It is shown that the discretization of the field modes effectively decreases the radiative mass of the source as well as its anomalous magnetic moment. Both effects are finite, cutoff independent, and separable from continuum results and can be measured by precision experiments.

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In this paper it will be shown that the geometry of the measurement apparatus affects the mass and anomalous magnetic moment of an electron. These corrections are almost within the range of present accuracy and can be detected by precision experiments.

When two conducting plates are arranged parallel to each other the boundary condition imposed by them changes the electromagnetic field configuration in two ways[1–4]: It discretizes the field modes in the direction normal to the plates and imposes a lower limit on the wavelength, which cannot exceed twice the distance between the plates. Thus certain radiative corrections containing internal photon fields, in particular the electron self-energy and the electron anomalous magnetic moment, will be affected.[5,6]

In what follows the photon propagator between the plates is derived for use in covariant perturbation theory. A simple rule for calculation of the modified matrix elements is found. Then the results are applied to the calculation of the electron self-energy and its anomalous magnetic moment between two conducting plates. The plates do not impose boundary conditions on the electron field, which is unaffected. For the evaluation of radiative corrections the position of the electron, as well as its state, is of no significance.

For a derivation of the photon propagator I shall use the method of images.[4] If the plates are assumed to be normal to the x_1 axis and positioned at $x_1=0$ and $x_1=a$ (see Fig. 1), the photon propagator $G_B(x-x')$ can be written as

$$G_B(x_1-x_1')=\sum_{n=-\infty}^{\infty}[G(x_1-x_1'-2an)-G(x_1+x_1'-2an)]. \tag{1}$$

Here, only the x_1 arguments have been explicitly given, since all other directions are not affected. G stands for the usual photon propagator in unbounded space-time and G_B is a sequence of multiple reflections between mirrors placed at $x_1=0$ and a. This is equivalent to the imposition of Dirichlet boundary conditions,[7,8]

$$A_\mu(x_1=0)=A_\mu(x_1=a)=0, \tag{2}$$

on the vector potential. Generally it is impossible to utilize the exact form of the boundary condition, if we take into account all microscopic constituencies of the plate or set the normal component of the magnetic field and the tangential component of the electric field to zero. However, as has been pointed out previously[4,6,8] the electromagnetic field between parallel conducting plates is approximated well by (2). The Fourier transformation of $G_B(x-x')$ is given by

$$G_B(x-x')=2i\sum_{n=-\infty}^{\infty}\frac{1}{a}\int\frac{d^3\bar{k}}{(2\pi)^3}\,\frac{\exp[i\bar{k}(x-x')+ik_nx_1]\sin(k_nx_1')}{k_0^2-k_n^2-k_2^2-k_3^2-i\epsilon}, \tag{3}$$

where $k_n=n\pi/a$. The form of (3) implies that energy-momentum conservation is only approximate in the x_1 direction and, for outgoing k and q and for incoming p, leads to

$$\frac{\sin[(p_1-k_n-q_1)a/2]}{\pi(p_1-k_n-q_1)}=\delta_a(p_1-k_n-q_1)\underset{a\to\infty}{\longrightarrow}\delta(p_1-k_n-q_1). \tag{4}$$

This is due to the fact that energy and momentum can be transferred to the plates which have been assumed rigid. In what follows it is assumed that these transfer processes are negligible and an exact energy-momentum conservation holds. Hence, the only effect of boundary conditions will be that in momentum space the continuous integration in the x_1 direction is changed to a summation over a discrete momentum spectrum such that the following

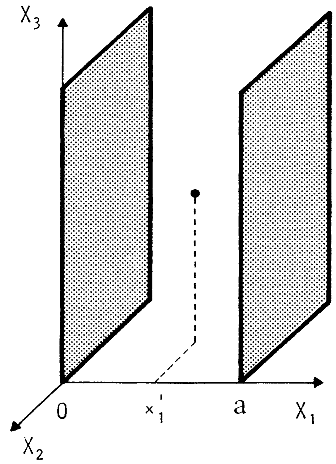

FIG. 1. An electron at x_1' between two conducting plates normal to the x_1 axes at $x_1=0$ and $x_1=a$.

substitution can be performed:

$$\int dk_1 \to \frac{\pi}{a}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}, \quad k_1 \to k_n = \pi n/a. \tag{5}$$

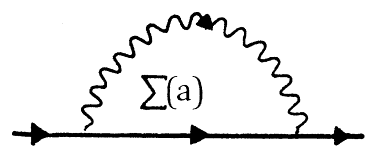

FIG. 2. Lowest-order electron self-energy diagram.

We turn now to the calculation of the electron self-energy and its anomalous magnetic moment. The separation of the finite change of radiative corrections can be done as follows: If $m(a)$ is the renormalized mass dependent on the separation of the conducting plates and $\Sigma(a)$ is the self-energy contribution, then

$$\begin{aligned} m(a) &= m(\infty)+\delta m+\Sigma(\infty)+\Sigma(a)-\Sigma(\infty)\\ &= m(\infty)+0+\Delta m(a). \end{aligned} \tag{6}$$

As a consequence of this subtraction procedure $\Delta m(a)$ is finite and well defined.

For explicit calculations it is assumed that the momentum of the electron in the x_1 direction is small compared to its rest mass, $p_1 << m$. By application of substitution (5) for the k_1 integration and evaluation of all other integrals by assumption of cylindrical coordinates, the mass shift $\Delta m(a)$ defined in (6) is obtained from the calculation of the first-order self-energy diagram (see Fig. 2):

$$\begin{aligned} \Delta m(a) &= \frac{\alpha}{a}\left(\int_0^\infty dn\, F(n) - \sum_{n=1}^{\infty} F(n)\right),\\ F(n) &= \ln\left(\frac{m+[(\pi n/a)^2+m^2]^{1/2}}{\pi n/a}\right) + \frac{[(\pi n/a)^2+m^2]^{1/2}-\pi n/a}{m}. \end{aligned} \tag{7}$$

With application of the Euler-MacLaurin formula and the assumption that $m >> \pi/a$, the leading term is found to be

$$\Delta m(a) = -(\alpha/2a)\ln(ma). \tag{8}$$

For the calculation of the electron anomalous magnetic moment Δg or $g-2$ similar considerations apply. The lowest-order contribution to the anomalous magnetic moment of an electron between conducting plates can be computed by evaluation of the diagram shown in Fig. 3. The change $\delta g(a)=\Delta g(a)-\Delta g(\infty)$ due to boundary conditions is given by

$$\begin{aligned} \delta g(a) &= -\frac{2\alpha}{a}\left(\int_0^1 dn\, K(n) - \sum_{n=1}^{\infty} K(n)\right),\\ K(n) &= \frac{1}{m}\ln\left(\frac{m+[(\pi n/a)^2+m^2]^{1/2}}{\pi n/a}\right) - \frac{2[1+(\pi n/am)^2]}{[(\pi n/a)^2+m^2]^{1/2}} + \frac{2\pi n}{am^2}. \end{aligned} \tag{9}$$

Again, $\delta g(a)$ can be evaluated and assumes a simple form for $m >> \pi/a$:

$$\delta g(a) = -(\alpha/ma)\ln(ma). \tag{10}$$

Both corrections to the electron mass and its anomalous magnetic moment are cutoff independent. Finally it

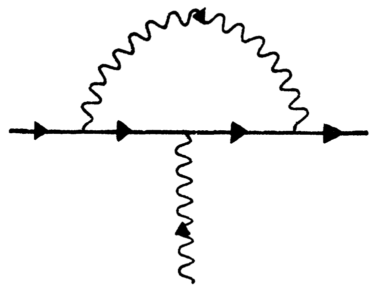

FIG. 3. Lowest-order diagram contributing to the electron anomalous magnetic moment.

should be noted that with substitution (5) at hand it is easy to calculate matrix elements between conducting plates. Experimental tests of the predictions of $m(a)$ and $\Delta g(a)$ are encouraged. Whereas mass deviations are small [for instance, $\Delta m_{\mathrm{el}}(1\ \mathrm{cm})/\ m_{\mathrm{el}} \simeq -10^{-11}$], measurements of the changes of the electron anomalous magnetic moment look more promising [for instance, $\delta g_{\mathrm{el}}(1\ \mathrm{cm}) \simeq -10^{-11}$], since the distance of the Penning-trap electrodes used in recent precision experiments[9] is of the order of 1 cm.

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