Extension of Supersymmetry in Nuclear Structure

P. Van Isacker, J. Jolie, and K. Heyde
Instituut voor Nucleaire Wetenschappen, B-9000 Gent, Belgium

and

A. Frank
Centro de Estudios Nucleares, Universidad Nacional Autónoma de México, México 04510, Distrito Federal, México
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Supersymmetry, recently proposed in the context of the interacting-boson model, is extended to include simultaneously odd-even, even-odd, and odd-odd partners of a given even-even nucleus. The quartet of nuclei ^{196}Pt, ^{197}Pt, ^{197}Au, and ^{198}Au is shown to be well described within this framework.

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Since its formulation ten years ago, the interacting-boson model has proved to be remarkably successful as a unified description of nuclear excitations, both for even-even[1] and odd-A [2] nuclei. One of its most attractive features is its suitability for a group-theoretical treatment, which exploits the existence of dynamical symmetries and exact solutions. Recently, this unifying trend was advanced further by Iachello,[3] who proposed a *simultaneous* description of even-even and odd-even nuclei through the introduction of a superalgebra, energy levels in both nuclei belonging to the same (super)multiplet. Several such schemes have been studied in the last few years, the U(6/4) [3] and the U(6/12) [4] symmetries being the best examples to date, applied to the Ir-Au [5-10] and the W-Os-Pt regions,[11-16] respectively.

The basic ideas behind these symmetry schemes are the following: (a) The even-even nucleus is described in an s,d-boson space spanned by the irreducible representation (irrep) $[N]$ of $U^B(6)$. (b) The neighboring odd-A isotope *or* isotone is described in the interacting boson-fermion model within the boson-fermion space spanned by the irrep $[N]\times[1]$ of $U^B(6)\otimes U^F(\Omega)$, where $\Omega=\sum_j(2j+1)$ is the dimension of the single-particle space. (c) Supersymmetry enters the picture by embedding $U^B(6)\otimes U^F(\Omega)$ into the supergroup $U(6/\Omega)$, which gives rise to the simultaneous description of an even-even and an odd-A nucleus.

In this Letter we show that supersymmetry can naturally be generalized to simultaneously describe isotope *and* isotone neighbors of an even-even nucleus, within the same mathematical framework. In this extended supersymmetry a supermultiplet contains, in addition to these three nuclei, a fourth, odd-odd member.

The essential idea of the extension is to make a distinction between neutrons and protons. Apart from this distinction, extended supersymmetry arises in much the same way as the normal one: (a) The even-even nucleus is described in a boson space spanned by the irrep $[N_\nu]\times[N_\pi]$ of $U^B_\nu(6)\otimes U^B_\pi(6)$, where N_ν (N_π) is the number of neutron (proton) bosons. (b) Neighboring nuclei with odd numbers of neutrons and/or protons are described in a boson-fermion space associated with

$$U^B_\nu(6)\otimes U^F_\nu(\Omega_\nu)\otimes U^B_\pi(6)\otimes U^B_\pi(\Omega_\pi), \qquad (1)$$

where $\Omega_\rho=\sum(2j_\rho+1)$, $\rho=\nu,\pi$. The nucleus with an odd number of neutrons corresponds to the irrep $[\mathcal{N}_\nu-1]\times[1]\times[\mathcal{N}_\pi]\times[0]$ of (1), the nucleus with an odd number of protons to $[\mathcal{N}_\nu]\times[0]\times[\mathcal{N}_\pi-1]\times[1]$, and the odd-odd nucleus to $[\mathcal{N}_\nu-1]\times[1]\times[\mathcal{N}_\pi-1]\times[1]$. In these expressions $\mathcal{N}_\nu$ ($\mathcal{N}_\pi$) equals the number of neutron (proton) bosons of the neighboring *even-even* nucleus which may

$$
\begin{array}{c}
U_\nu(6/12)\times U_\pi(6/4)\\
\cup\\
U^F_\nu(12)\times U^B_\nu(6)\times U^B_\pi(6)\times U^F_\pi(4)\\
\cup\\
SU^F_\nu(2)\times U^F_\nu(6)\times U^B_{\nu+\pi}(6)\times SU^F_\pi(4)\\
\cup\\
SU^F_\nu(2)\times U^{B+F}_{\nu+\pi}(6)\times SU^F_\pi(4)\\
\cup\\
SU^F_\nu(2)\times \bar{O}^{B+F}_{\nu+\pi}(6)\times SU^F_\pi(4)\\
\cup\\
SU^F_\nu(2)\times O^{B+F}_{\nu+\pi}(6)\\
\cup\\
SU^F_\nu(2)\times O^{B+F}_{\nu+\pi}(5)\\
\cup\\
SU^F_\nu(2)\times O^{B+F}_{\nu+\pi}(3)\\
\cup\\
\mathrm{Spin}(3)
\end{array}
$$

FIG. 1. Group-chain decomposition of $U_\nu(6/12)\otimes U_\pi(6/4)$ considered in this work.

be different from the number of neutron (proton) bosons in the even-odd, odd-even, or odd-odd nuclei [i.e., $\mathcal{N}_\rho = N_\rho + M_\rho$, where N (M) is the number of bosons (fermions)]. (c) Supersymmetry arises by embedding the direct product (1) into

$$U_\nu(6/\Omega_\nu) \otimes U_\pi(6/\Omega_\pi). \quad (2)$$

The irreps of (1), mentioned in (a) and (b), are contained in one irrep of (2), namely, in the direct product $[\mathcal{N}_\nu\} \times [\mathcal{N}_\pi\}$ of fully supersymmetric irreps.[17]

In Fig. 1 we show a particular chain of subgroups of (2) with $\Omega_\nu = 12$ and $\Omega_\pi = 4$, which corresponds to $j_\nu = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, and $j_\pi = \frac{3}{2}$, respectively, and where an O(6) dynamical symmetry is assumed for the bosons. The group chain is chosen such that for the even-even nucleus one obtains the F-spin symmetric O(6) limit of the neutron-proton interacting-boson model.[18,19] For the odd-A *isotope*, it reduces to the O(6) limit of U(6/12)[4,13,14] and, for the odd-A *isotone*, it reduces to the Spin(6) limit of U(6/4),[3,5,6] provided that one only considers the symmetric irrep$[\mathcal{N}_\nu + \mathcal{N}_\pi - 1]$ for $U^{B}_{\nu+\pi}(6)$. The latter is justified since nonsymmetric states are expected to occur above $E_x \simeq 3$ MeV. For the odd-odd nucleus the chain in Fig. 1 constitutes a new dynamical symmetry.

With neglect of terms contributing only to the binding energy, the Hamiltonian corresponding to the

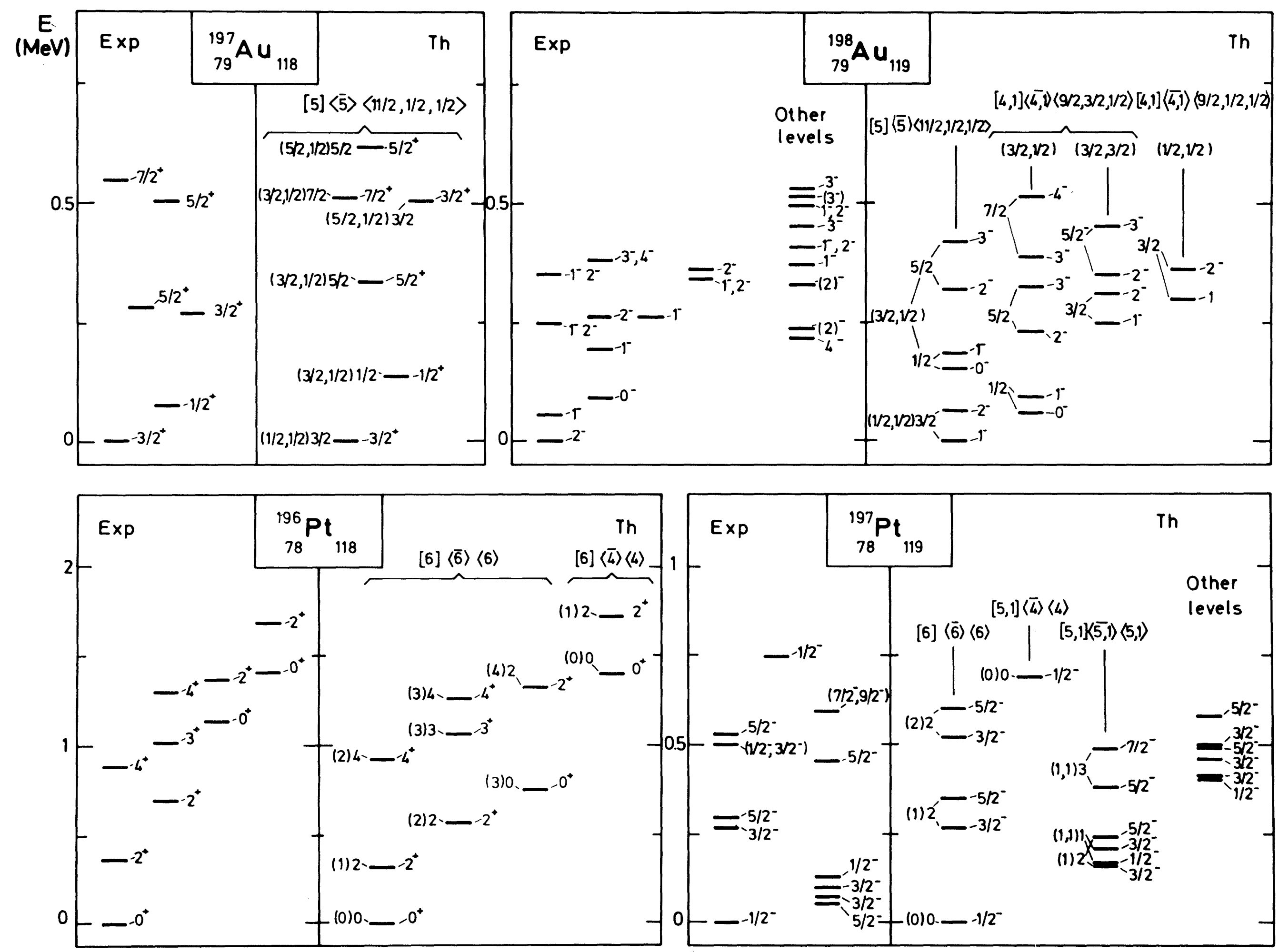

FIG. 2. Comparison between the experimental (Refs. 12 and 20) and the theoretical energy spectra of ^{196}Pt-^{197}Pt-^{197}Au-^{198}Au. These nuclei are contained in the irrep $[4\} \times [2\}$ of $U_\nu(6/12) \otimes U_\pi(6/4)$ and are calculated with use of Eq. (4), with the following parameters (in kiloelectrovolts): $A = 59$, $\bar{B} = 47$, $B = -97$, $C = 42$, $D = 9$, and $E = 16$. The theoretical levels are characterized by the quantum numbers of Eq. (3), with L shown at the left of the level and J at the right.

chain in Fig. 1 reads

$$H = AC_2(\underset{[N_1,N_2,N_3]}{\mathrm{U}^{\mathrm{B+F}}_{\nu+\pi}(6)}) + \bar{B}C_2(\underset{\langle\bar{\sigma}_1,\bar{\sigma}_2,\bar{\sigma}_3\rangle}{\bar{\mathrm{O}}^{\mathrm{B+F}}_{\nu+\pi}(6)}) + BC_2(\underset{\langle\sigma_1,\sigma_2,\sigma_3\rangle}{\mathrm{O}^{\mathrm{B+F}}_{\nu+\pi}(6)}) + CC_2(\underset{(\tau_1,\tau_2)}{\mathrm{O}^{\mathrm{B+F}}_{\nu+\pi}(5)}) + DC_2(\underset{L}{\mathrm{O}^{\mathrm{B+F}}_{\nu+\pi}(3)}) + EC_2(\underset{J}{\mathrm{Spin}(3)}), \quad (3)$$

where below each group we have indicated the quantum numbers labeling their irreps. The corresponding energy formula is

$$E = A[N_1(N_1+5)+N_2(N_2+3)+N_3(N_3+1)] + \bar{B}[\bar{\sigma}_1(\bar{\sigma}_1+4)+\bar{\sigma}_2(\sigma_2+2)+\bar{\sigma}_3^2] + B[\sigma_1(\sigma_1+4)+\sigma_2(\sigma_2+2)+\sigma_3^2] + C[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)] + DL(L+1) + EJ(J+1). \quad (4)$$

To obtain the energy spectra of the nuclei belonging to the supermultiplet, one further needs the decomposition of the irreps of the chained subgroups in Fig. 1, which is a standard group-theoretical problem. This, together with a discussion of nuclear properties other than energies, will be presented in a forthcoming paper. In this Letter we concentrate on the energy formula (4) and its applicability to specific examples.

At present, the best possible choice seems to be the quartet ^{194}Pt-^{195}Pt-^{195}Au-^{196}Au [contained in the irrep $[5\} \times [2\}$ of (2)], since the first three nuclei have been studied in the context of U(6/12)[4,11-14] or U(6/4),[3,5-10] with reasonably good agreement. Unfortunately, not much is known at present about the odd-odd nucleus ^{196}Au. Therefore, we turned our attention to the quartet ^{196}Pt-^{197}Pt-^{197}Au-^{198}Au (contained in $[4\} \times [2\}$), of which the odd-odd member is better known experimentally.[20] In Fig. 2 the results of the calculation for the $[4\} \times [2\}$ quartet are compared with the experimental data.[12,20] This calculation shows that the simultaneous fit to the four nuclei is of the same quality as the fits obtained separately in U(6/4)[6] and U(6/12).[14] Because of uncertain spin assignments it is difficult to associate the observed levels in ^{198}Au uniquely with calculated levels. Instead of making a level-by-level comparison for ^{198}Au, we show in Fig. 3 the observed and calculated negative-parity level density. This gives information about the average strength of the residual neutron-proton interaction, since this strength predominantly determines the density of levels, particularly at low excitation energy. Note that the knowledge of the explicit expressions for the Casimir operators in Eq. (3), together with the values for the coefficients A through E, implies a well-defined neutron-proton interaction between the fermions. From Fig. 3 we conclude that at least the average strength is well predicted by extended supersymmetry.

Finally, we mention a problem which exists for the odd-odd member of the quartet. In the example of Fig. 2 the neutron orbits have negative parity and the proton orbits have positive parity. Hence, only negative-parity levels are calculated in ^{197}Pt and ^{198}Au and only positive-parity levels in ^{197}Au. However, in ^{198}Au negative-parity states might also originate from the positive-parity $1i_{13/2}$ neutron orbit ($82 \leq N \leq 126$) and the negative-parity $1h_{11/2}$ orbit ($50 \leq Z \leq 82$). Since (a) the 12^- isomer at[20] 812 keV is of this nature, (b) other levels belonging to the $\nu 1i_{13/2} \times \pi 1h_{11/2}$ multiplet are not expected[21] to lie below the 12^- state because of the hole-hole character of the multiplet, and (c) the mixing between $\nu 1i_{13/2} \times \pi 1h_{11/2}$ states and the $(\nu 3p_{1/2}, \nu 3p_{3/2}, \nu 2f_{5/2}) \times \pi 2d_{3/2}$ states is weak, the assumption of unmixed $(\nu 3p_{1/2}, \nu 3p_{3/2}, \nu 2f_{5/2}) \times \pi 2d_{3/2}$ states below $E_x \simeq 500$ keV is justified in ^{198}Au.

To summarize, we have proposed an extension of supersymmetry to simultaneously describe quartets of nuclei. This formalism conceivably could be used to *predict* the properties of odd-odd nuclei, on the basis of those of neighboring even-even, even-odd, and odd-even nuclei. Besides the $\mathrm{U}_\nu(6/12) \otimes \mathrm{U}_\pi(6/4)$ struc-

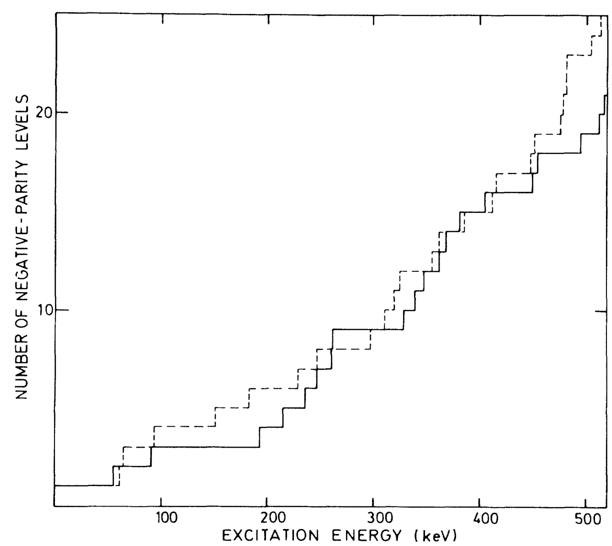

FIG. 3. The observed (solid line) and calculated (dashed line) negative-parity level density in ^{198}Au.

ture, discussed in this Letter, also other schemes [notably $U_\nu(6/12) \otimes U_\pi(6/12)$] could be investigated in a similar way.

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