Comment on "Observation of a Noise-Induced Phase Transition with an Analog Simulator"

Recently[1] analog simulations for a multiplicative stochastic process have been described. To characterize the observed static results the notation of *noise-induced phase transition* has been introduced. I point out that one should be cautious when introducing this term for the phenomena observed. In 1970 two groups[2,3] pointed out a broad range of analogies between the single-mode laser, a system far from thermal equilibrium, and the mean-field description of the paramagnetic-ferromagnetic phase transition in thermal equilibrium.[4] Here I will contrast the physical properties of those two systems with the properties of a "noise-induced phase transition."

The multiplicative stochastic process

$$\dot{x} = \tfrac{1}{2} - x + \tilde{\beta} x (1-x) \tag{1}$$

where $\tilde{\beta}$ is the sum of the deterministic term β and noise term ξ, studied in Ref. 1, was proposed by Arnold, Horsthemke, and Lefever,[5] and the idea to use the distance between the location of the maximum of the stationary probability density in the limit of very small variance of the noise and for strong noise intensity as an order parameter was proposed by Lefever and Horsthemke[6]. At the same time[7] all physically accessible quantities of a symmetrized version of Eq. (1),

$$\dot{x} = -x + (1-x^2)\xi, \tag{2}$$

where $\beta = 0$, were studied; i.e., static moments as well as upper and lower bounds for the lowest nonzero eigenvalue were investigated for the Fokker-Planck equation associated with Eq. (2). As is well known (cf. Ref. 8) this lowest-lying discrete eigenvalue of the Fokker-Planck equation determines completely the long-time behavior of both the time-dependent moments and the correlation functions [e.g., $\langle x(t) \times x(t+\tau) \rangle$].

In an equilibrium phase transition (as, e.g., paramagnet-ferromagnet) the moments of the probability density are macroscopic observables, i.e., they have a finite value in the limit $N \rightarrow \infty$ (with N the number of degrees of freedom). And these are the macroscopic quantities which show nonanalyticities at the transition as a function of temperature, say. Similarly a nonanalyticity is found for the moments[2,3] in some nonequilibrium systems at threshold as, e.g., for the single-mode laser.

In the experiments described by Smythe, Moss, and McClintock the moments are perfectly smooth at the transition. It is these moments which are analogous to macroscopic observables in the thermodynamic system. The probability distribution, on the other hand, while it is observable in the nonequilibrium system (2) having a single macroscopic degree of freedom, does not correspond to an observable in a thermodynamic system with $N \rightarrow \infty$. Therefore, it seems to me misleading to call the changes in the probability distribution of X a "phase transition" as in thermodynamics or statistical mechanics. For the dynamics, one arrives at a similar conclusion.

In summary, I note that physical quantities such as the stationary probability distribution, which show bifurcations in certain models of multiplicative stochastic processes, do not correspond to macroscopic observables in thermodynamic systems. Thus the term "noise-induced phase transitions" to describe these bifurcations seems to me unjustified.

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