External Noise Can Suppress the Onset of Spatial Turbulence

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We show experimentally that for the electrohydrodynamic instability in nematic liquid crystals the threshold for the onset of spatial turbulence can be increased by at least a factor of 2 by superimposing noise on the applied voltage. We find that for sufficiently high noise intensities a direct transition from the spatially homogeneous state to turbulence via intermittency can be induced.

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The onset of spatial turbulence (i.e., the loss of coherence in both space and time) in fluid systems is at present under extensive investigations.[1-6] The most frequently studied configurations are the Taylor instability (in which a fluid between two concentric cylinders is subjected to an external torque) and the Bénard instability (a layer of a simple fluid is investigated under the influence of a vertical external temperature gradient) but many other systems are also being studied (cf., e.g., Ref. 1). Depending on the geometry and the aspect ratio of a specific experimental setup, one has observed—e.g., for the Bénard instability—a sequence of spatially periodic structures before the onset of turbulence or a direct transition.

Here we report for the first time how external multiplicative noise can alter the sequence of transitions. The system chosen is the electrohydrodynamic instability in nematic liquid crystals.[7] (See, for example, Kai and Hirakawa[8] for a review.) A thin layer of nematics is studied under the influence of an external electric field. An electrohydrodynamic instability was chosen because it is much easier to superimpose in a controlled way external noise on an electric field than, e.g., on a temperature gradient or a torque. We have found that for increasing intensity of the noise the threshold for the onset of the periodic structures and the turbulent regime increase by up to a factor of about 2. For very high noise intensities, a direct transition from the spatially homogeneous state to a turbulent state is observed. All the qualitative features are not sensitively dependent on the nature of the noise; for example, whether it is Gaussian white or binomial is of secondary importance. Furthermore, we observe that for Gaussian white noise, the onset time of the periodic spatial structures depends linearly on the strength of the noise—a surprisingly simple result which has been found to hold before only for spatially homogeneous models[9,10] and their experimental realization.[11]

A sample of the material MBBA [*N*-(*p*-methoxybenzylidene)-*p*-butylaniline] which forms a nematic phase at room temperature was sandwiched between two horizontal glass plates which are covered with conducting electrodes. The sample thickness is about 110 μm and the lateral dimensions are 8 mm each; this gives rise to an aspect ratio of 72; i.e., a cell with large aspect ratio is being investigated here. An ac voltage of 60 Hz and variable magnitude were applied across the sample and the response to the external electric field was monitored by observation in a polarizing microscope. Temperature during the experiments was controlled up to ± 0.05 deg. As one increases the applied (deterministic) voltage a sequence of electrohydrodynamic instabilities characteristic[8] for this order of magnitude of the applied frequency is observed. First one finds a roll pattern called the Williams domains (WD) (Ref. 8, Fig. 7), the analog of the rolls in the Bénard instability. As the voltage is increased, these rolls start to fluctuate in time [fluctuating Williams domains] and eventually a transition to the grid pattern (GP), which resembles the cross-roll instability of Bénard, takes place [Ref. 8, Fig. 9(b)].

As the applied voltage is stepped up further, the spatially incoherent regime, usually called dynamic scattering mode (DSM) is reached.

To study the influence of external noise, a noise voltage as produced by a random noise generator (NF Corp. WG-722) and amplified by a Pioneer amplifier (A-470), which can be used for voltages up to 500 V, was superimposed on the deterministic voltage. The nature (e.g., Gaussian white or binomial) and the correlation time of the noise could be varied. In Fig. 1, the results of our observations are plotted as a function of the intensity of the external noise voltage. To arrive at Fig. 1, Gaussian white noise of a correlation time of 30 μs has been used. This correlation time is much shorter than the characteristic time of the system (~ 10 ms) and thus guarantees a clearcut separation of time scales. By inspection of Fig. 1, several outstanding features are noted immediately. Over a large range of values for the strength of the external noise there is a linear relationship between the thresh-

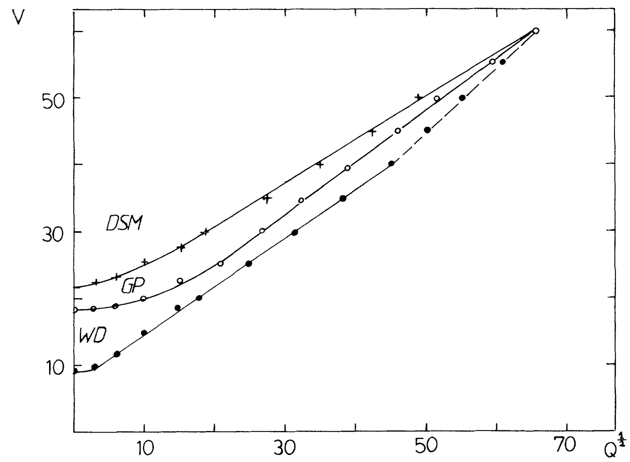

FIG. 1. Phase diagram showing the domains of existence of the various patterns as a function of external voltage. The noise used to get this graph was Gaussian white with a correlation time $\tau \approx 30\ \mu s$.

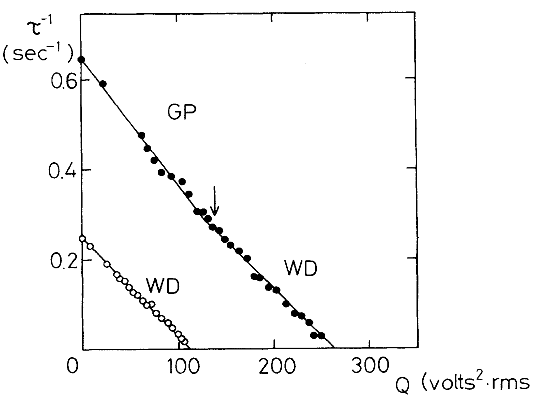

FIG. 2. Inverse of the onset time for the initial formation of the periodic spatial pattern (WD and GP) as a function of the strength of the external noise. Lower curve: $V_1 = 14$ V, upper curve: $V_2 = 18$ V.

old voltage for the transitions and the noise amplitude and, most importantly, higher noise always leads to a stabilization of the periodic structure thus suppressing the onset of an instability. This behavior has been noted mainly for the onset of the WD pattern (see Kai *et al.* and Kawakubo, Yanagita, and Kabashiman[12] and it was proposed first by Brand and Schenzle[13] that multiplicative noise is important for the interpretation of the experimental results. Here we find that external noise not only stabilizes the transitions between spatially periodic structures but also the transition from GP to spatial turbulence (DSM). In contrast to usual noise effects which are small and lead only to a "smearing out" of the sharpness of the transitions, the consequences of external noise reported here are drastic and of comparable order of magnitude as the purely deterministic field effects.

Finally, we note a most fascinating feature at very high strength of the external noise intensity: A direct transition from spatial homogeneity to spatial turbulence is observed without the buildup of any stable periodic structure. In this regime of Fig. 1, the visual observations can be described as follows. As one increases slowly the deterministic voltage at fixed noise intensity, one observes intermittent bursts of spatially incoherent structures embedded in a homogeneous background. The location of these bursts varies irregularly as a function of time. As the external deterministic voltage is increased the frequency of the bursts becomes higher as does their duration and the fraction of the area filled with this spatially disordered structures. Eventually the whole field of view is filled with a spatially disordered structure which also shows strong fluctuations in intensity as a function of time.

In order to check whether the phase diagram depends qualitatively on the nature of the noise we have carried out analogous experiments for other types of external noise, like binomial or uniform noise, and we have found only quantitative changes whereas the qualitative features are not modified. A detailed discussion of these investigations will be given elsewhere.[14] In addition, we have checked how the correlation time of the noise influences the structure of the phase diagram.[14] These measurements can be summarized in saying that—as one might have suspected—the qualitative overall features remain the same as long as the correlation time for the noise is very small compared to the characteristic time scales of the system. Furthermore, one finds that the direct transition to turbulence appears at higher intensities of the noise for longer correlation times. Another feature worth mentioning is the fact that although the threshold for going turbulent is monotonically increasing with increasing noise voltage, the difference between the onset value for WD and DSM is monotonically decreasing and eventually vanishes completely leaving one with the direct transition to turbulence described above.

To get additional insight into the phenomenon reported we supplemented the evaluation of the phase diagram by dynamic observations. We measured the nonlinear onset time τ necessary for establishing a specific spatially periodic or incoherent structure after a step change of voltage from a value corresponding to spatial homogeneity up to a selected value as a function of the strength of the external, spatially homogeneous noise. For a deterministic voltage of $V_1 = 14$ V and $V_2 = 18$ V our results are shown in Fig. 2. As a striking result we find that both for the buildup of the WD pattern and the GP a linear dependence of the relaxation rate τ^{-1} on noise intensity Q is observed with slope for WD different from GP. For the DSM mode we find a more complicated nonlinear relation between onset time and noise intensity but again τ^{-1} decreases monotonically as a function of noise intensity.

In conclusion, we have demonstrated that external multiplicative noise has a drastic influence on the transition to turbulence for the electrohydrodynamic instability in liquid crystals. This is in contrast with the effects generated by additive noise which is always present and usually leads to a broadening of the transition regions. For moderate intensities of the external noise we have found that the transition to spatial turbulence can be suppressed. For high intensities we found that a direct transition from the spatially homogeneous state to turbulence can be induced; this short circuits any spatially periodic structures.

Finally, our experiments clearly show that the importance of state-dependent noise is not restricted to the spatially homogeneous systems studied previously experimentally and theoretically in coupled nonlinear oscillators,[11] for the two-mode laser in quantum optics[15,16] and other fields.[17–19]

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[1]*Hydrodynamic Instabilities,* edited by H. L. Swinney and J. B. Gollub (Springer, New York, 1981).

[2]G. Ahlers and R. Behringer, Phys. Rev. Lett. **40**, 712 (1978).

[3]G. Ahlers, D. S. Cannell, and M. A. Dominguez-Lerma, Phys. Rev. Lett. **49**, 368 (1982).

[4]G. P. King and H. L. Swinney, Phys. Rev. A **27**, 1240 (1983).

[5]J. P. Gollub and J. F. Steinmann, Phys. Rev. Lett. **47**, 505 (1981).

[6]S. Fauve, C. Laroche, A. Libchaber, and B. Perrin, Phys. Rev. Lett. **52**, 1774 (1984).

[7]A nematic liquid crystal is an anisotropic liquid of organic molecules which shows no positional order but which has, on average, a preferred direction about which orientational symmetry is broken [cf., e.g., P. G. de Gennes, *The Physics of Liquid Crystals* (Clarendon, Oxford, 1982), 3rd ed.].

[8]S. Kai and K. Hirakawa, Progr. Theor. Phys. Suppl. **64**, 212 (1978).

[9]A. Schenzle and H. Brand, Phys. Rev. A **20**, 1628 (1979).

[10]H. Brand and A. Schenzle, Phys. Lett. **A81**, 321 (1981).

[11]S. Kabashima, S. Kogure, T. Kawakubo, and T. Okada, J. Appl. Phys. **50**, 6296 (1979).

[12]S. Kai, T. Kai, M. Takata, and K. Hirakawa, J. Phys. Soc. Jpn. **47**, 1379 (1979); T. Kawakubo, A. Yanagita, and S. Kabashima, J. Phys. Soc. Jpn. **50**, 1451 (1981).

[13]H. Brand and A. Schenzle, J. Phys. Soc. Jpn. **48**, 1382 (1980).

[14]S. Kai and H. R. Brand, to be published.

[15]K. Kaminishi, R. Roy, R. Short, and L. Mandel, Phys. Rev. A **24**, 370 (1981).

[16]R. F. Fox, G. E. James, and R. Roy, Phys. Rev. Lett. **52**, 1778 (1984).

[17]P. deKepper and W. Horsthemke, C. R. Acad. Sci. (Paris) **C287**, 251 (1978).

[18]J. M. Sancho, M. SanMiguel, H. Yamazaki, and T. Kawakubo, Physica (Utrecht) **A116**, 560 (1982).

[19]J. Smythe, F. Moss, and P. V. E. McClintock, Phys. Rev. Lett. **51**, 1062 (1983).