Observation of a Relativistic, Bistable Hysteresis in the Cyclotron Motion of a Single Electron

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A bistable hysteresis in the cyclotron motion of a single electron has been observed which is so pronounced that it provides the best signal-to-noise ratio ever observed with a single elementary particle in a trap. The effect is due entirely to the relativistic mass increase despite very low kinetic energies between 0.016 and 10 eV. During these experiments, a single electron was trapped continuously for more than 10 months in a Penning trap.

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The relativistic mass increase of electrons was first measured by Kaufmann,[1] before special relativity was formulated, with use of electrons of velocity $v \approx c$ in a cathode-ray tube. The explanation is provided by special relativity, according to which an electron (with charge $-e$ and effective mass m, in an electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$) obeys the equation of motion[2]

$$d(m\mathbf{v})/dt = -(e/c)\mathbf{v}\times\mathbf{B} - e\mathbf{E} + \mathbf{F}_r. \tag{1}$$

The extra force $\mathbf{F}_r$ represents damping and $\mathbf{F}_r = -m\gamma_c/\mathbf{v}$ for cyclotron motion. The damping rate or "width" γ_c associated with the motion is due to radiative decay in this experiment. The effective mass m of an electron with kinetic energy K is related to its rest mass m_0 by

$$m = \gamma m_0 = (1 + K/mc^2)m_0. \tag{2}$$

For the observed kinetic energies between 0.016 and 10 eV, the effective mass differs from the rest mass by only 0.05 to 20 ppm but, nonetheless, produces a very large and dramatic bistability and hysteresis when the electron's cyclotron motion is driven. This possibility was discussed recently in this journal by Kaplan.[3] He pointed out that the interaction of an electron, the simplest of microscopic objects, and an electromagnetic field is already intrinsically bistable and exhibits a unique hysteresis. All other observed and studied bistabilities depend upon macroscopic nonlinear properties of a medium.

In the absence of special relativity, an electron in a magnetic field orbits a magnetic field line with the cyclotron frequency, $\omega_{c0} = eB/m_0c$. The relativistic mass increase makes the cyclotron frequency ω_c weakly linear dependent on kinetic energy K:

$$\omega_c = \omega_{c0}(1 - K/mc^2). \tag{3}$$

Free-electron cyclotron motion thus exhibits anharmonic properties similar to those which occur when the linear restoring force in a spring has an additional cubic term.[4] The response to an external driving force at frequency ω with power P is given by

$$K = \frac{P}{\gamma_c}\frac{\gamma_c^2/4}{(\omega-\omega_c)^2+\gamma_c^2/4}. \tag{4}$$

For fixed cyclotron frequency ω_c, this is the familiar Lorentzian response associated with a driven harmonic oscillator. Inclusion of Eq. (3) for the relativistically shifted cyclotron frequency in Eq. (4), however, yields a modified dependence of kinetic energy upon drive frequency which is illustrated by the solid curve in Fig. 1. The important point here is that the resonance line shape has a triple-valued region when the maximum cyclotron frequency shift is more than several times the linewidth γ_c. The cyclotron motion of an electron which is driven at a frequency in the multivalued region is actually bistable, because the middle branch is unstable.[5] The arrows and dashed curve in Fig. 1 illustrate the resulting hysteresis by contrasting the large

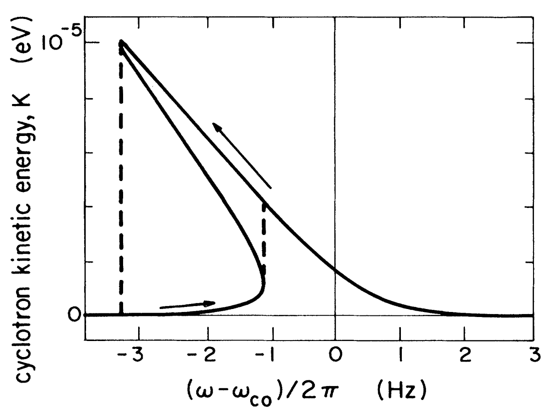

FIG. 1. Illustration of the resonance line shape (solid curve) of an anharmonic oscillator described by Eqs. (3) and (4). The observed line shapes depend upon the direction (indicated by arrows) in which the driving frequency is swept through resonance and include discontinuous jumps (dashed segments).

excitation produced when the drive frequency is swept down in frequency through resonance with the much smaller excitation which occurs when the drive frequency is instead swept upward.

An observation of bistable hysteresis in the cyclotron motion of a single electron is shown in Fig. 2 for $\omega_c/2\pi = 164$ GHz and $\gamma_c/2\pi = 0.5$ Hz. The kinetic energy in the cyclotron motion is plotted on the right vertical scale, but with increasing kinetic energy going down, and the horizontal scale is the frequency of the driving force. (We shall presently discuss the vertical scale on the left and the way that the energy is measured.) The radiative linewidth γ_c is so narrow that even an excitation of only one quantum level involves a relativistic frequency shift which is many linewidths by Eq. (3). The observed hysteresis for the much larger excitations which we can detect is therefore much more pronounced than that which is represented in Fig. 1. A very large excitation, to a kinetic energy of 0.8 eV, is observed when the drive frequency is swept downward through resonance [Fig. 2(a)], but no resonance at all is observed on this scale when the drive frequency is instead swept upward [Fig. 2(b)]. Excitations more than 10 times larger than that shown in Fig. 2 have been observed, to kinetic energies as high as 10 eV, and the signal-to-noise ratio is much larger than has previously been observed with a single, trapped elementary particle. The pronounced bistable hysteresis is nonetheless entirely due to the extremely

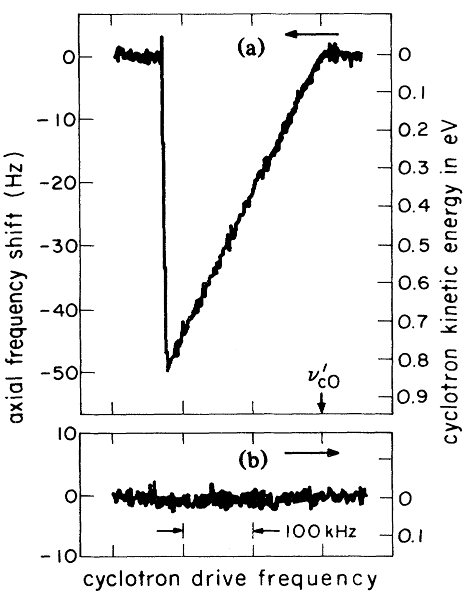

FIG. 2. Observed axial frequency shift (left vertical scale) as a function of the frequency of the driving force when this frequency is swept (a) downward and (b) upward through resonance. The right vertical scale is the kinetic energy in the cyclotron motion in electronvolts.

small relativistic mass increase which occurs for kinetic energies less than 10 eV.

The method for measurement of the kinetic energy in the cyclotron motion is itself quite unusual insofar as it relies upon a second consequence of the relativistic mass increase. The relatively weak quadrupole potential of a Penning trap,[6] which is superimposed upon the strong magnetic field to keep the electron from drifting away along a magnetic field line, changes the cyclotron restoring force and frequency by less than 70 parts per 10^9 and does not complicate our previous discussion of this motion in any essential way. However, the quadrupole potential makes the electron oscillate harmonically along the direction of the magnetic field. This axial oscillation is perpendicular to the cyclotron motion and its frequency, $\omega_z/2\pi = 62$ MHz, is inversely proportional to the square root of the mass. The relativistic mass increase thus shifts the axial frequency just like the cyclotron frequency in Eq. (3) except for a factor of 2 which is due to the square root,

$$\omega_z = \omega_{z0}(1 - K/2mc^2). \quad (5)$$

The axial frequency shift is monitored continuously with a sensitivity of 1 Hz and thus kinetic energies larger than 16 meV can be measured. (This corresponds to a cyclotron quantum number $n \geqslant 24$ so that the classical arguments we have been employing are entirely appropriate. At the same time, Fig. 1 clearly represents an unrealistically low kinetic energy insofar as $\hbar\omega_c = 7\times10^{-4}$ eV. This choice was only made so that the single and multivalued regions of the resonance curve could be seen on the same plot.)

We used oxygen-free, high-conductivity copper for the electrodes of the Penning trap to avoid the magnetic field inhomogeneity produced by the paramagnetism of earlier molybdenum electrodes.[7] The slope of the observed cyclotron resonances for high kinetic energies, such as in Fig. 2(a), provides a sensitive test of the residual magnetism of the trap. The most important consequence of residual magnetism is to add a small axial gradient, $\Delta B = bz^2$, to the strong magnetic field. This addition modified Eq. (5) for the axial frequency by the addition of a small constant ϵ so that

$$\omega_z = \omega_{z0}[1 - (1+\epsilon)K/2mc^2], \quad (6)$$

while the cyclotron frequency shift in Eq. (3) is not modified at all. Thus

$$\frac{\Delta\omega_c/\omega_c}{\Delta\omega_z/\omega_z} = \frac{2}{1+\epsilon}. \quad (7)$$

The measured slope is always observed to be very stable and such that the normalized ratio in Eq. (7) is close to 2. The largest possible currents through the so-called z^2 shim of the superconducting magnet used to provide the strong magnetic field change ϵ from a value of -0.2 to 0.4. By carefully adjusting the shim current we made $|\epsilon| < 5\times10^{-3}$ which is a reduction of this "magnetic bottle" inhomogeneity by a factor of 4000 compared to that used in measurements of the magnetic moment of an electron.[6]

A cyclotron driving signal with high spectral purity is absolutely crucial for the observations of the relativistic hysteresis. In fact, we had already observed a cyclotron resonance with one electron via the relativistic shift of the axial frequency discussed above,[8] several years before Kaplan's study[3] was published. We also had proposed[9] and were attempting to observe the bistable hysteresis, taking our cue from the performance of synchrocyclotrons at energies which are orders of magnitude higher. However, we observed no hysteresis at all until recently when a high-purity microwave system was completed, whereby a very low noise crystal oscillator is multiplied very cleanly up to 164 GHz in a long multiplication chain involving diode doublers, a step-recovery diode, and a GaAs diode, along with appropriate filters, amplifiers, and matching components. The 164-GHz microwaves are transferred to the Penning trap in liquid helium via a system of teflon lenses. It is difficult to obtain a spectrally pure microwave signal at 164 GHz insofar as even an ideal frequency multiplication[10] from 10 MHz to 164 GHz (which the present microwave system closely approximates but the earlier systems certainly did not) increases the power in the noise sidebands of the 10-MHz oscillator by the very large factor of 3×10^{8}. The relativistically anharmonic cyclotron oscillator is particularly sensitive to the broad noise pedestal of the microwave drive signal because a component of the noise is always exactly on resonance at the shifted cyclotron resonance frequency, ω_c, in Eq. (3). A coherent drive at frequency ω, however, generally shifts the resonance frequency to a lower value $\omega_c < \omega$ and thus the coherent response is largely off resonance. The power in the noise sidebands must be low enough so that the resonant noise excitation is smaller than the off-resonance coherent excitation. Otherwise, the noise excitation dephases the coherent response and the excitation jumps from the upper branch to the lower branch. In the language used by accelerator designers, the power in the noise sidebands must be low enough so that the electron is not excited out of the stable synchrotron bucket.

Now that we have observed the bistability and hysteresis which is present in the cyclotron motion of a single electron because of the relativistic mass shift, we hope to use this effect to measure the magnetic moment of an electron to improved accuracy without the magnetic field inhomogeneity previously employed. It turns out that we must therefore refine our technique to the point where we can measure the cyclotron frequency ω_{c0} in Eq. (3), which corresponds to the lowest quantum state, to an accuracy exceeding 218 Hz out of 164 GHz, or 1 part per 10^9. With such improved sensitivity, we will probe the quantum properties of the relativistic quantum oscillator,[9] which cannot be adequately described by classical arguments such as those given in Ref. 3. The signal to noise observed here is so good that we are even entertaining the notion of deliberately adding a magnetic field inhomogeneity such that $\epsilon = -0.9$ in Eq. (7) to further decouple the cyclotron frequency from thermal fluctuations in the axial motion. On the other hand, with higher microwave powers, it should be possible to make a much larger cyclotron excitation. Such a highly excited electron is required for a proposed technique to phase-lock optical and radio frequency sources[11] and our observations thus comprise a first step toward this goal. Finally, we note that a similar bistability and hysteresis might be observable for electrons in a semiconductor.[12]

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