Heterotic String

David J. Gross, Jeffrey A. Harvey, Emil Martinec, and Ryan Rohm
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544
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A new type of superstring theory is constructed as a chiral combination of the closed $D=26$ bosonic and $D=10$ fermionic strings. The theory is supersymmetric, Lorentz invariant, and free of tachyons. Consistency requires the gauge group to be $\mathrm{Spin}(32)/Z_2$ or $E_8\times E_8$.

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Recent interest in superstring unified field theories has been sparked by the discovery of Green and Schwarz[1] that nonorientable (type I) open and closed superstrings[2] with $N=1$ supersymmetry are finite and free of anomalies if the gauge group is SO(32). Previously the only consistent, anomaly-free[3] superstring theory was that of orientable (type II) $N=2$ supersymmetric closed strings. The new theory has the advantage of already containing a large (and unique) gauge group. It is much easier to contemplate this theory producing the low-energy gauge group, as well as families of chiral massless fermions, upon compactification of the original ten dimensions. Witten has discussed some of the phenomenology of this theory and has shown that it is easy to imagine compactifications that yield an SU(5) theory with any number of standard fermionic generations.[4]

The anomaly cancellation mechanism of Green and Schwarz is based on group theoretical properties of SO(32) which are shared by only one other semisimple Lie group, namely $E_8\times E_8$. Such a group, however, cannot appear in the standard form of open-string theory, in which gauge groups are introduced by attaching quantum numbers to the ends of the string and Chan-Paton factors[5] to the scattering amplitudes. This procedure yields only the gauge groups SO(N) and Sp($2N$).[6] The correspondence between the low-energy limit of existing supersymmetric string theories with anomaly-free, $D=10$, supergravity field theories suggests the existence of a new kind of string theory whose low-energy limit would have an $E_8\times E_8$ gauge group. Eschewing the Chan-Paton route to gauge groups for open strings, one might try to obtain $E_8\times E_8$ by compactifying a higher-dimensional closed-string theory. An important clue to how such a theory might arise is provided by the work of Frenkel and Kac.[7]

In this Letter we shall outline the construction of a new kind of closed-string theory, whose low-energy limit is $D=10$, $N=1$ supergravity coupled to supersymmetric Yang-Mills theory with gauge group $\mathrm{Spin}(32)/Z_2$ *or* $E_8\times E_8$. This theory is constructed as a hybrid of the $D=10$ fermionic string and the $D=26$ bosonic string, which preserves the appealing features of both. We show that the orientable, closed heterotic[8] string has an $N=1$ supersymmetric spectrum of states of positive metric, is free of tachyons and is Lorentz invariant. The requirement that gravitational and gauge anomalies be absent necessitates the compactification of the extra sixteen bosonic coordinates of the heterotic string on a maximal torus of determined radius, in a way that produces gauge groups $\mathrm{Spin}(32)/Z_2$ or $E_8\times E_8$. We further argue that the heterotic loop diagrams are free of all infinities—thus yielding new consistent candidates for a unified field theory.

The construction of the heterotic string is based on the observation that the states of the first quantized type-II closed strings, fermionic or bosonic, are essentially direct products of left- and right-moving modes. The physical degrees of freedom of the bosonic string are the 24 transverse coordinates $X^i(\tau-\sigma)$ and $\tilde{X}^i(\tau+\sigma)$ which describe right- (left-) moving two-dimensional free fields, with periodic boundary conditions on the circle $0\leq\sigma\leq\pi$. The fermionic string contains eight transverse coordinates as well as eight right- and left-moving two-dimensional real fermions, $S^a(\tau-\sigma)$ and $\tilde{S}^a(\tau+\sigma)$ $(a=1,\ldots,8)$ which are Majorana-Weyl ten-dimensional light-cone spinors.[1] The right- and left-handed components of the string are tied together by the constraint that the total momentum and position of each component be identical. Thus the bosonic coordinates are given by the operators (we choose units in which the slope parameter is $\alpha'=\frac{1}{2}$)

$$X^i(\tau-\sigma)=\tfrac{1}{2}x^i+\tfrac{1}{2}p^i(\tau-\sigma)+\frac{i}{2}\sum_{n\neq0}\frac{1}{n}[\alpha_n^i e^{-2in(\tau-\sigma)}],$$

$$\tilde{X}^i(\tau+\sigma)=\tfrac{1}{2}x^i+\tfrac{1}{2}p^i(\tau+\sigma)+\frac{i}{2}\sum_{n\neq0}\frac{1}{n}[\tilde{\alpha}_n^i e^{-2in(\tau+\sigma)}], \qquad (1)$$

where

$$[\alpha_n^i, \alpha_m^j] = [\tilde{\alpha}_n^i, \tilde{\alpha}_m^j] = \delta^{ij}\delta_{m+n,0}n,$$

$$[\tilde{\alpha}_n^i, \alpha_m^j] = 0, \quad [x^i, p^j] = i\delta^{ij},$$

whereas the fermionic coordinates are

$$S^a(\tau-\sigma) = \sum_{n=-\infty}^{+\infty} S_n^a e^{-2in(\tau-\sigma)},$$

where

$$\gamma^+ S_n = hS_n = 0, \quad \{S_m^a \bar{S}_n^b\} = (\gamma^+ h)^{ab}\delta_{m+n,0} \tag{2}$$

$[h = \frac{1}{2}(1 \pm \gamma_{11})]$, with a similar expression for the left movers $\tilde{S}^a(\tau+\sigma)$.

The states of the first quantized, type-II, closed strings are direct products of the Fock space states of the right and left movers. In addition, the basic one-particle operators of the theory (e.g., the super Poincaré generators) are direct sums of operators in the right- and left-moving sectors separately, and the vertex operators that describe the splitting and joining of the strings are direct products of left and right vertices.[2,9] Therefore one can, in principle, construct a consistent string theory in which the left- and right-handed components are treated asymmetrically—as long as each sector is internally consistent. We therefore construct the heterotic string to consist of fermionic-string right movers: eight transverse coordinates X^i ($i=1,\ldots,8$), eight Majorana-Weyl fermionic coordinates S^a; and of bosonic string left movers: eight transverse coordinates $\tilde{X}^i$ and sixteen internal coordinates $\tilde{X}^I$, $I=1,\ldots,16$.

$$\tilde{X}^I(\tau+\sigma) = x^I + p^I(\tau+\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^I e^{-2in(\tau+\sigma)}, \tag{3}$$

where

$$[\tilde{\alpha}_n^I, \tilde{\alpha}_m^J] = n\delta_{m+n,0}\delta^{IJ}, \quad [x^I, p^J] = \tfrac{1}{2}i\delta^{IJ}.$$

(The factor of $\frac{1}{2}$, familiar in light-cone quantization, arises since X^I is a function of $\tau+\sigma$ alone.)

If X^I is to satisfy the periodic boundary conditions of the closed string, either $p^I=0$ or else x^I parametrizes some compact space T. The simplest (and at present the only known consistent) choice for T is a sixteen-dimensional torus. Compactification of a conventional string coordinate on a nonsimply connected manifold introduces several new features.[10] There now exist classically stable topological configurations where a string coordinate, $X(\sigma) = x + 2\alpha' p\tau + 2NR\sigma + \ldots$, winds N times around the manifold as σ runs from 0 to π. In addition, of course, the momenta, p^I, are quantized in units of the inverse radius, $1/R$, of the manifold.

In order to achieve a consistent string theory involving only left-moving coordinates X^I to cancel anomalies and to preserve the geometrical structure of string interactions, we are forced to compactify on a special torus. T must be a "maximal" torus—a product of circles of equal radii, $R=(\alpha')^{1/2}=1/\sqrt{2}$, on which points are identified according to

$$X^I \equiv X^I + \sqrt{2}\pi R \sum_{i=1} e_i^I n_i, \tag{4}$$

where e_i^I are sixteen basis vectors [normalized so that $(e_i^I)^2=2$] which generate an integer, even, self-dual lattice. This means that $g_{ij} = \sum_{I=1}^{16} e_i^I e_j^I$ is integer valued and $\det g = 1$. In this case the allowed values of the momenta are

$$p^I = \sum_{i=1}^{16} n_i e_i^I \quad (n_i = \text{integer}), \tag{5}$$

and must equal the winding numbers N^I. [Note that the minimal value of $(p^I)^2$ is two.] There exist only two lattices of this type. One is Γ_{16} and coincides with the lattice of weights of $\mathrm{Spin}(32)/Z_2$, the other is a direct product, $\Gamma_8 \times \Gamma_8'$, where Γ_8 is the lattice of weights of E_8.

To complete the construction of the heterotic string we express $X^{\pm} = (1/\sqrt{2})(X^0 \pm X^9)$ in terms of the physical degrees of freedom, as usual in light-cone gauge,[11] $X^+ = x^+ + p^+\tau$ and

$$X^-(\sigma,\tau) = x^- + p^-\tau + \frac{i}{2}\sum\frac{1}{n}(\alpha_n^- e^{-2in(\tau-\sigma)} + \tilde{\alpha}_n^- e^{2in(\tau+\sigma)}). \tag{6}$$

In the heterotic string α_n^- is constructed as in the fermionic string

$$\alpha_n^- = \frac{1}{p^+}\sum_m \alpha_m^i \alpha_{n-m}^i + \frac{1}{2p^+}\sum_m \left(m - \frac{n}{2}\right) :\bar{S}_{n-m}\gamma^- S_m: \tag{7}$$

whereas $\tilde{\alpha}_n^-$ is constructed as in the bosonic string

$$\tilde{\alpha}_n^- = \frac{1}{p^+}\sum_m (\tilde{\alpha}_m^i \tilde{\alpha}_{n-m}^i + \tilde{\alpha}_m^I \tilde{\alpha}_{n-m}^I). \tag{8}$$

The mass operator is

$$\tfrac{1}{2}\alpha'(\mathrm{mass})^2 = N + (\tilde{N} - 1) + \tfrac{1}{2}\sum_I (p^I)^2, \tag{9}$$

where $N(\tilde{N})$ are the number operators for the right (left) movers (i.e., $N = p^+\alpha_0^-$, $\tilde{N} = p^+ :\tilde{\alpha}_0^- :$). The subtraction of 1 from $\tilde{N}$ arises from the normal ordering of the bosonic number operator $\tilde{N}$; it is also required in order to maintain Lorentz invariance. Finally, we must constrain the physical states so that

$$N = \tilde{N} - 1 + \tfrac{1}{2}\sum_I (p^I)^2. \tag{10}$$

This implies that the unitary operator $U(\Delta) = \exp 2i\Delta[N - \tilde{N} + 1 - \frac{1}{2}\sum_I (p^I)^2]$, which shifts σ (in X^i, $\tilde{X}^i$, $\tilde{X}^I$) by Δ, is the identity operator on physical states, thereby ensuring that there is no distinguished point on the closed string.

The physical states of the heterotic string are direct products of Fock space states ($|\ \rangle_R \times |\ \rangle_L$) of the right-moving fermionic string and the left-moving bosonic string, subject to the constraint (10). The right-handed ground state is annihilated by α_n^i, S_n^i ($n > 0$), and N, and forms an irreducible representation of the zero mode oscillators S_0^a, containing eight transverse bosonic states $|i\rangle_R$ and eight fermionic states $|a\rangle_R$. The left-handed ground state, annihilated by $\tilde{\alpha}_n^i$, $\tilde{\alpha}_n^I$ ($n > 0$), and $\tilde{N}$, with zero p^I is removed from the physical Hilbert space by the constraint. Therefore the heterotic string is free of tachyons! The lowest mass states are direct products of $|i\rangle_R$ or $|a\rangle_R$ with $\tilde{\alpha}_{-1}^i|0\rangle_L$, $\tilde{\alpha}_{-1}^I|0\rangle_L$, or $|p^I\rangle_L$ [with $(p^I)^2 = 2$] and are all massless. The states $|i \text{ or } a\rangle_R \times \tilde{\alpha}_{-1}^j|0\rangle_L$ form the irreducible $N=1$, $D=10$ supergravity multiplet. The states $|i \text{ or } a\rangle_R \times \alpha_{-1}^I|0\rangle_L$ and $|i \text{ or } a\rangle_R \times |p^I\rangle_L$ [with $(p^I)^2 = 2$] form an irreducible $N=1$, $D=10$, super Yang-Mills multiplet of G [either SO(32) or $E_8 \times E_8$]—the Lie group whose maximal torus coincides with T. These consist of sixteen neutral vector mesons (plus their supersymmetric partners) with $p^I = 0$, which are the ordinary Kaluza-Klein gauge bosons arising from the $U(1)^{16}$ isometry of T. The additional 480 charged vectors with $(p^I)^2 = 2$, which complete the adjoint representation of G, are special to closed-string theory. Given the work of Frenkel and Kac, who have used string vertices to construct representations of Kac-Moody-Lie algebras, it is not surprising that the standard geometrical string interactions will yield a heterotic string whose states form representations of G with G-invariant interactions (note that massive excited states can have arbitrarily large values of the charges p^I, corresponding to arbitrary representations of G).

Lorentz invariance in $D=10$ is easily established since the generators, $J^{\mu\nu} \equiv \int_0^\pi d\sigma[\bar{X}^\mu P^\nu - \bar{X}^\nu P^\mu] + K^{\mu\nu}$ [where $\bar{X}^\mu = X^\mu + \tilde{X}^\mu$, $P^\mu = (d/d\tau)\bar{X}^\mu$, $K^{ij} = i/8 \times \sum \bar{S}_{-n}\gamma^{ij-} S_n$, etc.] act separately on the right and left movers. Since the right movers (left movers) are those of the fermionic (bosonic) string in its critical dimension of 10(26), the would-be anomalies cancel. In addition the hybrid string contains one supersymmetry, generated by

$$Q^a = i(p^+)^{1/2}(\gamma + S_0)^a + 2i\frac{1}{(p^+)^{1/2}}\sum(\gamma_i S_{-n})^a \alpha_n^i, \tag{11}$$

which acts on right movers alone, and satisfies (as in the fermionic string)

$$\{Q^a, Q^b\} = -2(n\gamma^\mu P)^{ab}. \tag{12}$$

The interactions of the heterotic string, which correspond geometrically to the splitting and joining of closed strings, can be constructed as direct products of the vertex operators for the fermionic and bosonic strings. We have constructed the vertices for the emission of massless closed strings and have shown that they are Lorentz and G invariant.[12] We have also examined the one-loop diagrams and have seen that these are consistent with unitarity (the restriction to self-dual lattices comes from this requirement) and are finite.[12] While we have not calculated the string hexagon diagrams, the equivalence of the heterotic low-energy field theory to the anomaly-free $D=10$ $N=1$ theories convinces us that they will be anomaly free. Thus we have established the existence of two new consistent closed-string theories, which naturally lead, by a string Kaluza-Klein mechanism, to the gauge symmetries of SO(32) or $E_8 \times E_8$. These theories differ in essential ways from open-string gauge theories (for example the gauge coupling $g^2 = \kappa^2/\alpha'$ as opposed to $g^2 = \kappa\alpha'$ for open strings).

Fermionization of two-dimensional field theories often simplifies their structure, as in the case of the nonlinear sigma model with a Wess-Zumino term.[13] We have also constructed a version of the heterotic string in which the internal coordinates are fermions [in this case, $E_8 \times E_8$ is realized on its SO(16) $\otimes$ SO(16) subgroup]. Details of this construction will be presented in Ref. 12.

Additional possibilities for use of the above mechanism to generate new string theories are severely limited. Attempts to compactify the $D=26$, bosonic string to $D=10$ on a sixteen-dimensional torus[14] are doomed, since they would produce a gauge group of $G \times G$, would not contain $D=10$ fermions or supersymmetry, and would have tachyons. Compactification of the type-II closed fermionic string will produce no new gauge symmetries associated with winding strings.

Finally, we note that the heterotic $E_8 \times E_8$ string is perhaps the most promising candidate for a unified field theory. One can easily contemplate physically interesting compactifications of this theory to four dimensions, including the possibility that the $E_8 \to E_8'$ symmetry is unbroken, thereby implying the existence of a "shadow world" consisting of E_8' matter which interacts with us (E_8 matter) only gravitationally.[12]

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