Anderson Localization and the Theory of Dirty Superconductors

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We study the effect of Anderson localization in dirty superconductors. The scale dependence of the diffusion in the vicinity of the mobility edge results in a strong renormalization of the zero-temperature coherence length. This implies the breakdown of the Ginzburg criterion close to the metal-insulator transition and thus the importance of fluctuations in this regime. The upper critical field is calculated and possible experiments are also discussed.

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The interplay between strong disorder and superconductivity has attracted considerable attention in recent years. On the experimental side, measurements of H_{c2}, the upper critical field,[1-5] and of the superconducting transition[1,2,5-7] show considerable discrepancies with the "classical" Ginzburg-Landau-Abrikosov-Gorkov (GLAG) theory[8] and the H_{c2} calculation of Werthamer-Helfand-Hohenberg (WHH).[8] On the theoretical side, two main approaches have been employed; both are based on the anomalous renormalization of the Coulomb interactions due to the disorder. Fukuyama, Ebisawa, and Maekawa[9] have calculated the effects of disorder on the degradation of T_c and on H_{c2} by applying an expansion in $1/k_F l$, valid in the weakly localized regime only. Anderson, Muttalib, and Ramakrishnan[10] and later Coffey, Muttalib, and Levin[11] have studied the renormalization of the Coulomb pseudopotential and its effect on T_c and H_{c2}.

In this Letter we extend the GLAG theory of dirty superconductors to incorporate the scale dependence of the vanishingly small diffusion constant close to the mobility edge. Using standard BCS theory[12] without Coulomb interactions and the scaling theory of the Anderson transition,[13] we show that the zero-temperature coherence length is strongly affected by the disorder. This leads to the striking conclusion that in very disordered superconductors *fluctuation effects are very important.* This can result in a substantial reduction of the transition temperature in contrast to the Anderson theorem.[14] We then study the dependence of H_{c2} on temperature. We find that the WHH theory with $H_{c2}\propto(1/k_F l)(T_0/T_F)$ is replaced by $H_{c2}\propto(\xi_l k_F/g_c)(T_0/T_F)$ as the disorder becomes strong and saturates to the value $H_{c2}\propto(1/g_c)(T_0/T_F)^{2/3}$ close to the mobility edge. T_0 is the BCS transition temperature, T_F is the Fermi temperature, and g_c[13] is one-half of the dimensionless critical conductance of the material. ξ_l is the localization length which diverges at the Anderson transition.

Our main conclusion is that for systems close to the Anderson transition, the GLAG functional has to be modified to incorporate the effects of the scale dependence of the diffusion. We express the GLAG functional (we use the units $k_B=\hbar=1$)

$$F(\Delta)=(\rho/2\pi)\int d^dx\, \Delta^*(\mathbf{x})\{f(\nabla/i-e\mathbf{A})+\ln(T/T_0)\}\Delta(\mathbf{x})+U\int d^dx\, |\Delta|^4 \tag{1}$$

in terms of a function f which depends on the temperature via the temperature length l_T, the wave number q, and the localization length (ρ is the density of states). Its asymptotic behavior (in $d=3$) is given by

$$f(q,l_T,\xi_{\rm loc})=\psi(\tfrac{1}{2}+g_c l_T^3 q^2/L)-\psi(\tfrac{1}{2})+\tfrac{1}{2}\ln[1+g_c(qL)^2],\quad l_T>>L, \tag{2a}$$

$$f(q,l_T,\xi_l)=4.6g_c(l_T q)^2,\quad l_T<<L. \tag{2b}$$

We use the notation $L=\min(\xi_l,1/q)$. $l_T=(\rho T)^{-1/d}$ plays the role of a thermal length in the vicinity of the Anderson localization fixed point.[16,17]

The long-wavelength behavior of f is governed by a new renormalized coherence length

$$\tilde{\xi}_0=2.22g_c^{1/2}l_T(l_T/\xi_l)^{1/2},\quad l_T>>\xi_l, \tag{3a}$$

$$\tilde{\xi}_0=2.15g_c^{1/2}l_T,\quad l_T<<\xi_l. \tag{3b}$$

Comparison with the GLAG result $\xi_0^2\simeq v_F l/T$ reveals that for weak disorder, ξ_0 decreases as the square root of the mean free path. When l reaches the Yoffe-Regel limit ($k_F l\sim1$), localization sets in. A new length, the localization length, emerges and the replacement[13] $l\to(k_F^2\xi_l)^{-1}$ gives Eq. (3). As the disorder increases, the localization length diverges.

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When it crosses the thermal length, then l_T becomes the only relevant length in the problem. Replacing ξ_l by l_T in Eq. (3a), we arrive at Eq. (3b).

One immediate consequence of this result is a dramatic increase in the size of the critical region where superconducting fluctuations govern the critical behavior. The classical Ginzburg criterion states that mean-field theory breaks down when $(T-T_0)/T_0 \sim (T_0\rho\xi_0^3)^{-2}$.[18] This is a very small number ($\lesssim 10^{-10}$ for dirty superconductors and even less for type I) since ξ_0 is very large. Close to the mobility edge, the Ginzburg criterion results in $(T-T_0)/T_0 \simeq g_c^{-3}\xi_l^3(\rho T_0)$ for $l_{T_0} >> \xi_l$. As ξ_l grows and $l_{T_0} << \xi_l$, $(T-T_0)/T_0 \simeq g_c^{-3}$. Since g_c is a number of order unity, close to the Anderson transition the superconducting transition could be very similar to the λ transition in liquid helium.[19]

When the Ginzburg parameter is of the order unity, the reduction in T_c is of the order of T_c itself and cannot be calculated by perturbation theory. This provides an explanation to the strong degradation of T_c in a highly disordered system. (Localization decreases T_c via fluctuations but leaves the mean-field value of the transition temperature[14] unchanged.)

Turning to magnetic properties, we note that Eqs. (2) and (3) predict a dependence of H_{c2} on T and T_0 different from the WHH theory. The upper critical field is calculated from the eigenvalue equation

$$f(l_H^{-1}) + \ln(T/T_0) = 0. \tag{4}$$

$l_H^2 = c/eH$ is the magnetic length and we took into account that a small magnetic field introduces another scale in the localization transition.[20] Our mean-field calculation is valid outside the critical region down to $T=0$. There are two different regimes depending on the value of l_{T_0} (the length determined by the transition temperature) and the localization length.

For $l_{T_0} >> \xi_l$

$$H_{c2}(T=0) = 0.14\frac{\xi_l\rho T_0}{g_c}\frac{c}{e} \quad \text{and} \quad -T_0\frac{dH_{c2}}{dT}\bigg|_{T_0} = 0.2\frac{\xi_l\rho T_0}{g_c}\frac{c}{e}. \tag{5}$$

Hence the disorder enhances the value of H_{c2} at zero temperature beyond its classical value $H_{c2} = (c/e)1.74\,T_0/v_F l$.

As we approach the mobility edge and $l_{T_0} << \xi_l$, H_{c2} saturates to the value

$$H_{c2}(T=0) = 0.27\left(\frac{T_0\rho}{g_c(1+g_c)^{1/2}}\right)^{2/3}\frac{c}{e} \quad \text{and} \quad -T\frac{dH_{c2}}{dT}\bigg|_{T_0} = 0.215\frac{(T_0\rho)^{2/3}}{g_c}\frac{c}{e}. \tag{6}$$

Near T_0, $H_{c2}(T)$ has a linear dependence on T. Thus, depending on the value of g_c, $R = H_{c2}^{(0)}/(T\,dH_{c2}/dT)_{T_0}$ will determine the nature of the crossover between the small-q regime ($T \simeq T_0$) and $T \simeq 0$. For $g_c \gtrsim 1$, R is larger than 1 and thus we predict for this class of materials a kink in the H_{c2} vs T curve.[21]

For small enough g_c (still of the order unity), R becomes less than 1, implying a negative second derivative all the way from T_0 to $T=0$. The experimental work on dirty superconductors near the mobility edge concentrates mainly on materials where our theory does not necessarily apply. These are homogeneous superconductors or if adopting the approach of Anderson, Muttalib, and Ramakrishnan,[10] also A-15's. In both cases either the morphology can enhance the Coulomb interaction or dimensionality effects[22] will dominate.

The most recent work on homogeneous dirty superconductors at this regime is of Hebard and Paalanen.[2] They investigated the behavior of H_{c2} upon temperature as well as the degradation of T_c for In/InO_x films. Substantial reduction of T_c was observed for high-resistivity films. Also the behavior of H_{c2} versus temperature was not of WHH form as they found a kink in H_{c2} for $T > T_c$. In trying to fit their data by our theory, we get very good agreement for the low-temperature behavior of H_{c2} including the kink. Close to T_c, the experiment shows an effective slope greater than the theoretical one. This, together with the degradation of T_c, was explained by those authors using a pair-breaking parameter. We argue that both effects can be understood qualitatively by noting the change in the critical behavior near T_c due to fluctuations. Moreover, since the thickness of the sample is finite (~ 100 Å) a crossover to a two-dimensional critical behavior is expected near T_c which by itself can decrease T_c[7] and cause an effective higher slope. In general, those features of degradation of T_c, the kink in the H_{c2} curve, and anomalously high $H_{c2}(0)$ observed for all dirty superconductors are at least qualitatively explained by our theory.

We next sketch the main points on the way to derivation of Eq. (2). Starting from the expression for the kernel of the GLAG functional in terms of the exact eigenstates of the electrons in the disordered metal, we follow de Gennes[8]:

$$f(q) = \frac{1}{g} - T\sum_n \int d^d(x-x')\exp[i\mathbf{q}\cdot(\mathbf{x}-\mathbf{x}')] \sum_{p_1,p_2} \frac{\langle \phi_{p_1}(\mathbf{x})^*\phi_{p_2}(\mathbf{x})\phi_{p_1}(\mathbf{x}')\phi_{p_2}^*(\mathbf{x}')\rangle}{(iU_n - \epsilon_{p_1})(-iU_n - \epsilon_{p_2})}. \tag{7}$$

Here G is the BCS coupling constant, $U_n=(2n+1)\pi T$ are the Matsubara frequencies, and the angular brackets denote the average over the impurities. $\phi_p(x)$ are the solutions of the Schrödinger equation in the presence of a random potential $V(\mathbf{x})$:

$$\{[\nabla/i-(e/c)\mathbf{A}]^2+V(\mathbf{x})\}\phi_p(\mathbf{x})=\epsilon_p\phi_p(\mathbf{x}). \tag{8}$$

In zero magnetic field, the wave functions are real and the term in Eq. (7) is an average over the square of the single-particle Green's function. It is well established[16,17] that

$$\int d^d(x-x')\exp[i\mathbf{q}\cdot(\mathbf{x}-\mathbf{x}')]\left\langle \mathbf{x}'\left|\frac{1}{iU_n-H_0}\right|\mathbf{x}'\right\rangle\left\langle\mathbf{x}'\left|\frac{1}{-iU_n-H_0}\right|\mathbf{x}\right\rangle=\frac{1}{D(\xi_l,q,|U_n)q^2+2|U_n|} \tag{9}$$

with a scale-dependent diffusion parameter. Close to the mobility edge, it exhibits universal (i.e., independent of the microscopic details) scaling behavior. $D(\xi_l,q,|U_n|)\sim g_c/L\rho$, where we define $L=\min\{l_U,\xi_l,q^{-1}\}$ as the shortest length in the problem $[l_U=(\rho U)^{-1/d}$ is the frequency length]. This result and a detailed analysis of the crossover behaviors in this problem can be found in Ref. 17.

In the presence of a magnetic field, we make the replacement $-i\nabla\rightarrow -i\nabla-(e/c)\mathbf{A}$. This introduces a phase factor in the particle-particle propagator which is valid since the magnetic length for fields less than H_{c2} is larger than the range of the kernel and cannot be much smaller than the localization length. The critical behavior of the second term in Eq. (7) near the orthogonal fixed point in a weak magnetic field can be derived using the techniques of Ref. 17. A weak magnetic field introduces a small mass proportional to the cyclotron resonance and therefore an additional scale, $l_H=(c/eH)^{1/2}$, emerges in Eq. (9). L is now defined as $L=\min\{l_U,\xi_l,q^{-1},l_H\}$.

To summarize, we presented a theory which combines the anomalous behavior of the diffusion propagator near the mobility edge together with the BCS treatment of the superconducting state. This framework[21] should be used to analyze experiments on materials that are close to the metal insulator transition[1-7] where k_Fl is no longer the relevant parameter.[22] In particular, they can explain the anomalously high H_{c2} found at low temperatures in several experiments[3,4] as discussed in Ref. 11.

The importance of fluctuations, as was shown above, can reveal a new kind of superconducting transition similar to the one found for liquid helium.[23] This can be found, e.g., in measurements of the specific heat for very dirty superconductors as a cusp should appear. For this purpose, granular materials are probably not good because of very high Coulomb interaction (via charging energy of the grains) that can wash out superconductivity. Materials for which the disorder is introduced on the atomic scale[2] (e.g., solid solutions of metal-insulator mixtures that can be produced by very fast quenching) are more plausible to exhibit this phenomenon.

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[18]The coefficient of the $|\Delta|^4$ term is not substantially modified by the disorder.

[19]To observe this last crossover, one may have to look for systems with relatively high transition temperature as the

disorder increases. These can be, e.g., granular systems. See also the conclusions.

[20]We note that the scaling theory of localization for weak fields can, in principle, be quite different from that of strong fields as we crossover from the orthogonal to the unitary fixed points for the localization problem.

[21]A more detailed analysis of the various crossovers of the function $f(q)$ [and hence $H_{c2}(T)$] will be given in a following publication.

[22]We note that for superconductivity near the percolation threshold [see, e.g., S. Alexander, Phys. Rev. B **27**, 1541 (1983), and S. Alexander and E. Halevi, J. Phys. (Paris) **44**, 805 (1983)] one expects percolation to dominate the transition in a certain regime of length scales. [This was observed experimentally; for a review see, e.g., G. Deutscher, A. Kapitulnik, and M. L. Rappaport, in *Percolation Structures and Processes,* edited by G. Deutscher, R. Zallen, and J. Adler (Hilger, London, 1983)]. Nevertheless, close enough to the mobility edge the percolation fixed point is unstable against the localization one, and our conclusions should be applicable in that regime. For granular materials also, the charging energy as well as Josephson coupling between grains are important to change the nature of the superconducting transition.

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