Scales of Deconfinement?

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We investigate the possibility of scales of deconfinement in SU(N) lattice gauge theories with infinitely heavy sources in different representations. We employ Monte Carlo methods for $N=4$, factorization for $N=\infty$ and strong-coupling mean-field techniques for even $N\leqslant 16$. No scales of deconfinement are found.

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Evidence has been presented for the existence of scales of chiral-symmetry breaking; at least in the quenched approximation different representations ($\mathbf{r}$) of massless quarks have $\langle \Psi_\mathbf{r} \Psi_\mathbf{r}\rangle \neq 0$ below different temperatures $T_c^\mathbf{r}$.[1] Here we investigate the possibility that the deconfinement transition[2] occurs at different temperatures $T_d^\mathbf{r}$ for different representations of infinitely massive quarks.[3]

The order parameter for the confinement of static sources of representation $\mathbf{r}$ is the trace of the Wilson line matrix in the $\mathbf{r}$th representation, $\chi_\mathbf{r}(W)$. We consider infinitely massive sources because only in that case is $\langle \chi_\mathbf{r}(W)\rangle$ (whose magnitude is $e^{-F_\mathbf{r}/T}$, where $F_\mathbf{r}$ is the change in free energy due to adding a representation-$\mathbf{r}$ quark to the system and T is the temperature) a true order parameter: $\langle \chi_\mathbf{r}(W)\rangle = 0$ ($\neq 0$) when the sources are confined (unconfined).

How could scales of deconfinement come about? For reasons which will become clear, let us first consider SU(4) gauge theory. With the notation of Table I, $\chi_\mathbf{f}(W) = \mathrm{tr}\, W$ and $\chi_\mathbf{g}(W) = \frac{1}{2}[(\mathrm{tr}\, W)^2 + \mathrm{tr}\, W^2]$ are both protected from getting an expectation value at low temperatures by a $Z(4)$ [center of SU(4)] global symmetry of the gauge theory which takes $W \to e^{\pi i n/2} W$. But $\chi_\mathbf{g}(W)$, unlike $\chi_\mathbf{f}(W)$, is invariant under the $Z(2)$ subgroup $W \to -W$. Thus $\mathbf{g}$ can deconfine at a lower temperture than $\mathbf{f}$ ($T_d^\mathbf{g} < T_d^\mathbf{f}$) if the $Z(4)$ symmetry first breaks down to $Z(2)$ (at $T_d^\mathbf{g}$) before breaking completely (at $T_d^\mathbf{f}$). Figure 1(a) shows a Monte Carlo configuration of Wilson lines at $T < T_d^\mathbf{g}$ where the $Z(4)$ symmetry is unbroken, Fig. 1(b) shows a configuration at $T > T_d^\mathbf{f}$ with completely broken $Z(4)$ symmetry, and Fig. 1(c) is a *hypothetical* configuration at $T_d^\mathbf{g} \leqslant T \leqslant T_d^\mathbf{f}$ in which a Z(2) symmetry remains with $\langle \chi_\mathbf{g}\rangle \neq 0$, $\langle \chi_\mathbf{f}\rangle = 0$. Is this latter phase likely to occur? A loose transcription of the phase $\langle \chi_\mathbf{g}\rangle \neq 0$, $\langle \chi_\mathbf{f}\rangle = 0$ into the language of effective-spin-theory might be $\langle S^2\rangle \neq 0$, $\langle S\rangle = 0$, with $S \in Z(4)$. Such a phase has been observed in studies of nearest-neighbor $Z(4)$ spin systems in $d=2$ dimensions[4]; we know of no searches for it with $d > 2$. From a statistical mechanics viewpoint, whether or not a $Z(2)$ phase occurs depends on the interactions of the spins or Wilson lines. From a physics point of view the existence of a phase with $\langle \chi_\mathbf{g}\rangle \neq 0$, $\langle \chi_\mathbf{f}\rangle = 0$ might require a reexamination of the belief that plasmas always yield Debye screening of all charges[5]—a belief based primarily on studies of Abelian theories.

If $Z(4)$ does break to $Z(2)$ for SU(4) gauge theory it would be expected that all representations with 2(mod4) blocks would deconfine (because of invariance under $W \to -W$) while representation with 1(mod2) blocks would remain confined until the remaining symmetry breaks at $T_d^\mathbf{f}$. [The remaining representations with 0(mod4) blocks are never confined because they can always be screened by gluons.] Similarly, for any group, the possible deconfining scenarios can be easily enumerated with the scale at which any representation deconfines determined by its N-ality. In particular, SU(2) and SU(3) would not be expected to have deconfinement scales.

We performed Monte Carlo simulations[6] of SU(4) in 2+1 and 3+1 dimensions to look for evidence of separate deconfining transitions for the $\mathbf{f}$, $\mathbf{g}$, and $\mathbf{h}$ representations. {$\chi_\mathbf{h}(W) = \frac{1}{2}[(\mathrm{tr}\, W)^2 - \mathrm{tr}\, W^2]$ is invariant under $W \to -W$ and thus would be expected to go nonzero when $\chi_\mathbf{g}$ does.} No such evidence was found. For example, Fig. 2(a) shows the behavior of $\langle \chi_\mathbf{f}\rangle$ and $\langle \chi_\mathbf{h}\rangle$ as functions of the coupling β (increasing β corresponds to raising the temperature) on a 2×25^2 lattice; there appears to be no distinction between the two represtentations. The picture for a 5×10^3 lattice [Fig. 2(b)] looks very similar; in addition, Figs. 1(a) and 1(b) show the distribution of Wilson line variables for one configuration before and after the transition. There is no indication of a $Z(2)$-

TABLE I. Designation of representations.

Representation r	Corresponding Young Tableau
f	□ (fundamental)
g	□□
h	□ □

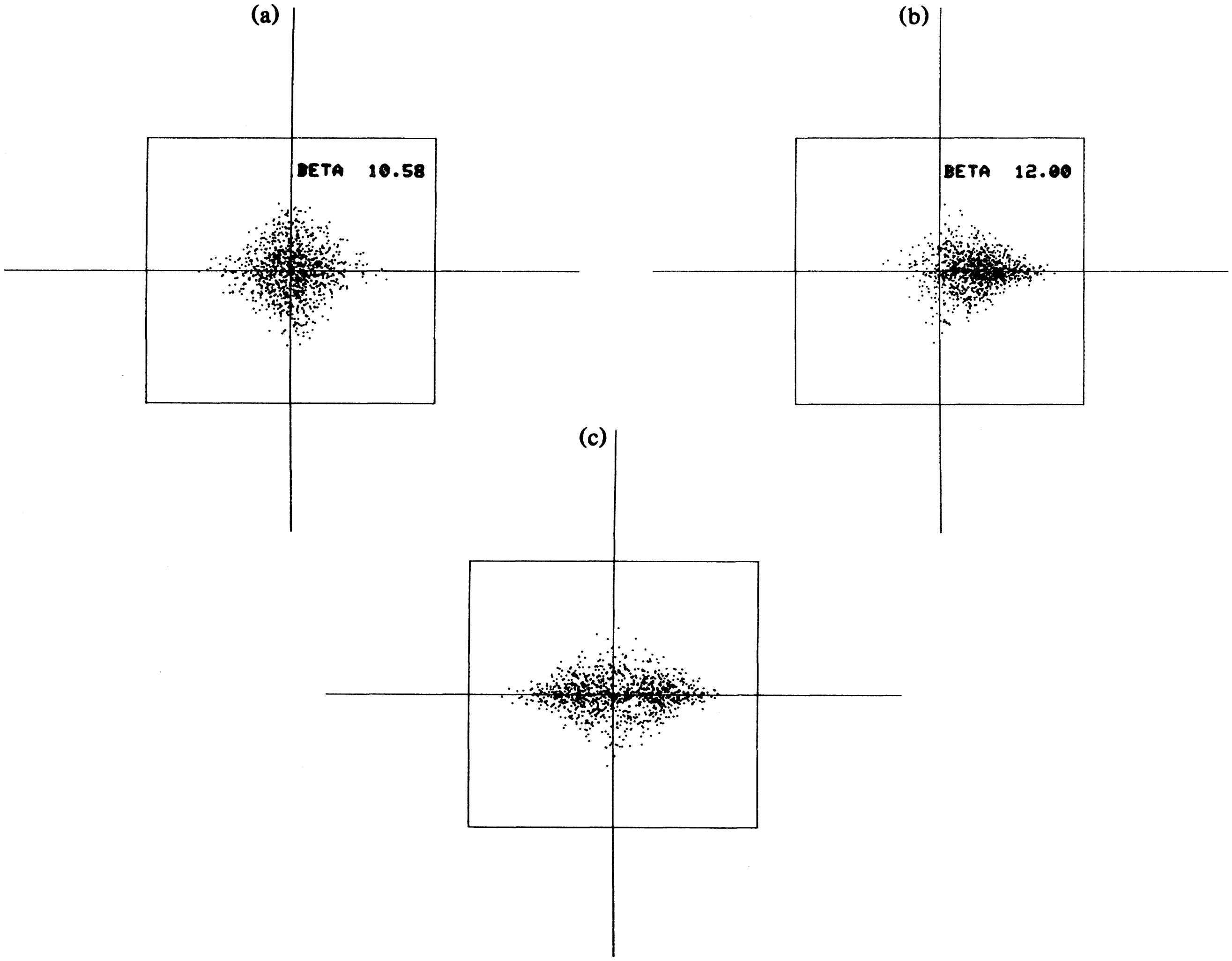

FIG. 1. (a) SU(4) Monte Carlo plot of $\mathrm{Im\,tr}\,W$ vs $\mathrm{Re\,tr}\,W$ for a particular configuration just below T_d^g. The global $Z(4)$ symmetry is unbroken (up to finite-size effects). (b) Same as (a), but at $T > T_d^f$, i.e., with the $Z(4)$ symmetry completely broken. (c) Hypothetical SU(4) plot of $\mathrm{Im\,tr}\,W$ vs $\mathrm{Re\,tr}\,W$ at $T_d^g \leq T \leq T_d^f$, i.e., with $Z(4)$ broken to $Z(2)$ $(W \rightarrow -W)$.

symmetric phase such as that shown in Fig. 1(c).

Any representation **r**, with r blocks, satisfies

$$\chi_{\mathbf{r}}(W) = \sum\nolimits_n C^n_{i_1,\ldots,i_{r-nN}} W_{i_1} W_{i_2} \cdots W_{i_{r-nN}}$$

[plus a constant if $r \equiv 0(\mathrm{mod}N)$] where W_i are the eigenvalues of W.

As N [of SU(N)] approaches infinity, we can use the factorization property[7]

$$\langle AB \rangle = \langle A \rangle \langle B \rangle [1 + O(1/N^2)],$$

which implies that

$$\langle \chi_{\mathbf{r}}(W) \rangle = \sum\nolimits_n C^n_{i_1,\ldots,i_{r-nN}} \langle W_{i_1} \rangle W_{i_2} \rangle \ldots \langle W_{i_{r-nN}} \rangle + O(1/N^2).$$

As long as any (nontrivial) remnant of the $Z(N)$ symmetry remains, the eigenvalues of W satisfy $\langle W_i \rangle = e^{2\pi i n/N} \langle W_i \rangle$, $n \neq 0$, and thus are prevented from getting an expectation value. This implies that $\langle \chi_{\mathbf{r}}(W) \rangle$ is forced to remain zero, up to terms $O(1/N^2)$, until the $Z(N)$ symmetry is completely broken. Thus it appears that scales of deconfinement do not occur in the limit $N \rightarrow \infty$.

In the region $4 < N < \infty$ we contented ourselves with an exploratory search using crude strong-coupling, mean-

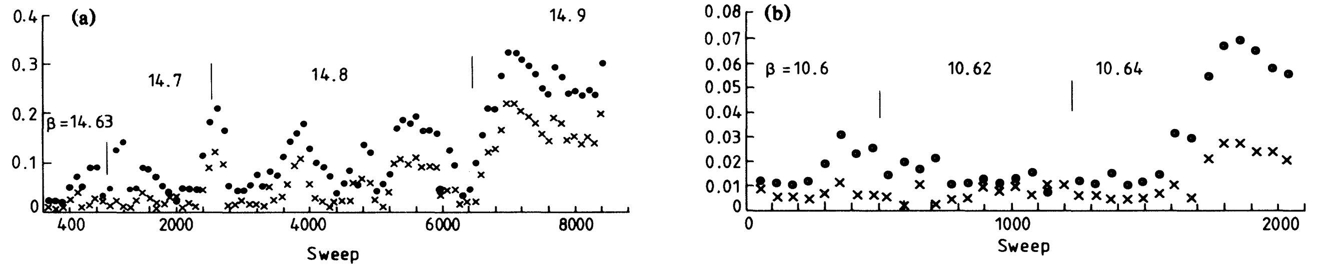

FIG. 2. (a) SU(4) Monte Carlo data on a 2×25^2 lattice showing $\chi_f/4$ (dots) and $\chi_h/6$ (crosses) vs sweep number for increasing values of the coupling, β. (b) Same as (a), but on a 5×10^3 lattice.

field lattice techniques. Our philosophy was that if any evidence of scales was found a more thorough investigation would follow whereas a null result would not, of course, be conclusive. In a region of coupling-constant space (with no "bulk" or large-N phase transition) the effective action in terms of Wilson lines can be approximated by[8]

$$S[W]=\sum_{\mathbf{x},\hat{i}}\ln\left[\sum_{\mathbf{r}}\left(\frac{\tilde{\beta}_{\mathbf{r}}}{d_{\mathbf{r}}}\right)^{N_\tau}\chi_{\mathbf{r}}(W(\mathbf{x}))\chi_{\mathbf{r}}^*(W(\mathbf{x}+\hat{i}))\right], \tag{1}$$

where

$$\tilde{\beta}_{\mathbf{r}}=\int dU\,\chi_{\mathbf{r}}^*(U)\exp[(\beta/2N)\mathrm{tr}(U+U^\dagger)],$$

$d_{\mathbf{r}}$ is the dimension of $\mathbf{r}$, and N_τ is the number of links in the temporal direction. Applying the standard mean-field technique[8,9] involves a saddle-point approximation of the partition function

$$Z=\int[dW][dK]\exp\{S[W]-\mathrm{Re}\,\mathrm{tr}[K^\dagger W]+V[K]\}, \tag{2}$$

with $e^{V[K]}\equiv\int[dU]\exp\{\mathrm{Re}\,\mathrm{tr}[K^\dagger U]\}$. The usual mean-field Ansatz is $W=w1$, $K=k1$. We use $W=w_1 1+iw_2\begin{pmatrix}-1&0\\0&1\end{pmatrix}_{N\times N}, K=k_1 1+ik_2\begin{pmatrix}-1&0\\0&1\end{pmatrix}_{N\times N}$ for even N. A solution with $w_2\neq 0$, $w_1=0$ at some coupling β would indicate the possibility of a $\langle\chi_g\rangle\neq 0$, $\langle\chi_f\rangle=0$ phase with $Z(N)$ broken down to $Z(2)$ ($W\to -W$ which leaves the eigenvalues of W unchanged when $w_1=0$). $S[W]$ was expanded about $\beta=0$ to order β^4 with $N_\tau=2$, $4\leq$ even $N\leq 16$, and the saddle points were found by numerical methods. The result was that the saddle point which maximized the integrand of (2) had $w_2=0$ before and after w_1 went nonzero. The values of β at which w_1 went nonzero are compatiable with results obtained with the $W=w1$, $K=k1$ Ansatz[8] as would be expected. All the phase transitions were first order.

We discussed how scales of deconfinement might occur in gauge theories. We search for such scales using Monte Carlo methods for SU(4), factorization for SU(∞), and crude strong-coupling mean-field techniques for SU(N), $4\leq$ even $N\leq 16$. No scales were found, yet it may be that some other action or group gives rise to scales or even that some of the other exotic phases found in spin systems[4] have their counterparts in corresponding finite-temperature gauge theories.

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[7]We thank M. B. Halpern for reminding us of the use of factorization in relating different $\chi_r(W)$ at $N=\infty$.

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