Palmer *et al.* Respond: Three approximations were employed in our paper[1] to proceed from the discrete level model to the asymptotic expression for $q(t)$: (a) replacement of the sum by an integral, (b) approximation of

$$\sum_{k}^{n} k^{-p} \text{ by } \zeta(p) - n^{1-p}/(p-1),$$

and (c) evaluation of the integral by saddle-point methods. Each of these can be improved by standard techniques. The corrections are insignificant in the long time limit but, as Zwanzig points out,[2] quite important at short and intermediate times.

We regard the discrete level model as a framework for incorporating constraints carefully, not as an appropriate model for real systems. There is neither experimental evidence nor theoretical reason to expect a discrete set of relaxation times in real systems. The continuum limit of the discrete model, in which the sum is replaced by the integral, is expected to be closer to real systems, and forms our major thesis. In this case approximations (a) and (b) are irrelevant, as is the comparison of the discrete model with the asymptotic form. Approximation (c) may be controlled, but it is more relevant to note that the integral itself may be evaluated and compared to the $\exp(-t/\tau)^{\beta}$ form. We find a reasonable fit over many decades of time, although the best-fit values of β and τ vary somewhat with the domain chosen. As an example, we show in Fig. 1 the region in which $0.99 > q(t) > 0.01$ for the case $\lambda = \mu_0 = 2$ chosen by Zwanzig. We agree that this is the region relevant to real-time experiments [frequency-space experiments depend more on the asymptotic tail, where our asymptotic evaluation best approximates $q(t)$]. The integral (circles) is well fitted by $q(t) = \exp[-(t/\tau)^{\beta}]$ with $\beta = 0.563$ (solid line). In contrast, the asymptotic form with $\beta = 0.419$ only fits well for $t/\tau_0 > 1$; the broken line shows this form with the Gaussian term of the saddle-point integral included. Finally, the dotted line shows the discrete sum model evaluated by Zwanzig. It is not well approximated by the integral or by the asymptotic form. The discrepancy is removed at large t if $\mu_n = \mu_0/(n+1)$ is used in place of $\mu_n = \mu_0/n$, but remains at small t.

The main points are the following: (1) The discrete model was used only as a framework to generate hierarchical constraints. The integral for $q(t)$ corresponds to physical situations. (2) Experiments are fitted to stretched exponentials over many decades of intermediate times. Our numerical evaluation of the integral for $q(t)$ is well fitted over five decades of intermediate times with $\beta = 0.563$. Therefore, Zwanzig's tests of comparing the discrete sum to an asymptotic form are not meaningful. (3) The asymptotic form of $q(t)$ gives an expression for β valid only for very long times for which the stretched exponential form is more and more exact. The analytic expression for β shows how it is related to the constraints present in the system and reveals qualitative trends in the effective β obtained by fitting $q(t)$ at short and intermediate times.

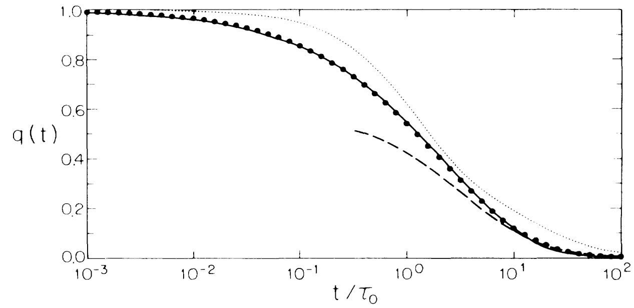

FIG. 1. The relaxation function $q(t)$ plotted against a logarithmic time scale (five decades) for the case $\lambda = \mu_0 = 2$. The lines and symbols are explained in the text.

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