Transition to Oscillatory Convection in a ^{3}He–Superfluid-^{4}He Mixture

Yoshiteru Maeno, Hans Haucke, and John C. Wheatley
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
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The oscillatory instability of Rayleigh-Bénard convection is studied in a solution of 1.46% ^{3}He in superfluid ^{4}He for the Prandtl-number range $0.045 \leq \sigma \leq 0.15$. The observed behaviors of the oscillatory frequency and the onset condition support the theory of oscillatory convection for a classical, low-Prandtl-number, one-component fluid.

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Thermal convection has been one of the most extensively studied problems in the field of nonlinear dynamics. It has been recognized[1] that a cryogenic environment allows convection apparatus to have exceptionally high stability and precision of measurements. Because of the qualities of superfluidity, solutions of ^{3}He in superfluid ^{4}He have unique and wide ranges of important fluid parameters, not accessible to any other fluids studied. The measurements reported here suggest that such solutions constitute an outstanding physical system for the study of nonlinear dynamical phenomena.

Busse[2] has predicted that fluids with low Prandtl number σ (the ratio of the time scale for diffusion of heat to that of vorticity) are quite susceptible to an oscillatory convective instability. With ordinary one-component fluids, oscillatory convection has been studied either with a fixed σ[3,4] or in a narrow range of σ comparable to unity.[5] As a consequence, the very important σ dependences of the phenomena for low σ, which allows comparison with the Busse theory, have never been established. In previous work[6] we have shown that, at least in certain ranges of ^{3}He concentration and of temperature, solutions of ^{3}He in superfluid ^{4}He behave with respect to the transition to stationary convection like one-component fluids. In the present work we studied the oscillatory instability in a solution with ^{3}He molar concentration $x = 1.46\%$ in the temperature range from 0.70 to 1.05 K. We report the first observations of the σ dependences of both the oscillatory frequency and the onset condition over a wide range of σ much less than unity ($0.045 \leq \sigma \leq 0.15$), which, excepting effects likely due to the small cell aspect ratio, agree well with the predictions for classical, one-component fluids. Other qualities, not predicted by the Busse theory, such as the behaviors of the oscillatory amplitude and of heat conduction near the onset, are also presented.

In a solution of ^{3}He in superfluid ^{4}He, constancy of the partial chemical potential for ^{4}He requires that a temperature gradient be accompanied by a concentration gradient, such that the ^{3}He concentration is greatest where the temperature is lowest. Consequently, the cell must be heated from above to induce a convective instability. The Rayleigh number and the Prandtl number are defined,[6–8] in analogy to one-component systems, as

$$R \equiv \frac{g\beta d^3 \Delta T}{\nu_n \kappa}, \quad \sigma \equiv \frac{\nu_n}{\kappa}, \tag{1}$$

where g is the acceleration due to gravity, β is the effective thermal expansion coefficient, ΔT is the temperature difference across the fluid layer with thickness d, $\nu_n \equiv \eta/\rho_n$ is the ratio of the dynamic viscosity to normal fluid density, and κ is the effective thermal diffusivity. A stress parameter is defined as $\epsilon \equiv (R/R_c) - 1$, where R_c is the Rayleigh number at the onset of stationary convection. Convection in dilute solutions is known from both prior[6,9] and present work to occur in both high- and low-thermal-conductance states. The latter state, which is intrinsically noisy down to low values of ϵ and is believed to be the consequence of the superfluid motion, will be discussed elsewhere.[10] The present work is concerned with oscillatory convection in the high-conductance state.

The cell was constructed with top and bottom boundaries made of copper and side walls made of Vespel-22 graphite-loaded polyimide resin.[11] For all the data presented here, the cell geometry is rectangular with aspect ratios $\Gamma \equiv L/2d = 1.00$ and $\Gamma' \equiv W/2d = 0.70$, where L and W are the longer and shorter horizontal side lengths, respectively, and $d = 0.80$ cm is the cell height. The ratios of thermal conductivities of copper and of Vespel-22 to that of the fluid are estimated to be 1.5×10^3 and 1.3×10^{-2}, respectively, at 1 K. Thus, the top and bottom boundaries are highly conducting, while the side boundaries are thermally insulating. A small copper piece, called the "probe" and having 3.2% of the top boundary area directly in contact with fluid, was inserted into a hole in the center of the top plate and weakly thermally isolated from it by a Mylar film. The probe was fitted with a heater and one of the junctions of a thermocouple differential thermometer[12] (the probe TC); the other junction was in thermal contact with the top plate, and the leads were connected to a SQUID ammeter. The probe TC measures the temperature variations of the fluid below the probe with a rapid response time (15 ms at 1 K) and a very high sensitivity (10^{-7} K at 1 K with a 10-Hz low-pass filter).

A calibrated germanium resistance thermometer (GRT) was mounted in the top plate. The bottom plate temperature was controlled within several microkelvins at a value determined by another GRT, which was calibrated against the first one. In the measurements a constant heating rate $\dot{Q}$ was applied to the top plate. Both the temperature difference between the top and bottom plates, measured by the calibrated GRT, and the output of the probe TC, δT, were observed. The probe heater was *not* normally used.

When $\dot{Q}$ is increased from zero, the first instability is the onset of stationary convection. It is detected by a change in the thermal conductance $K = \dot{Q}/\Delta T$. Here and below, $\dot{Q}$ and the time-averaged temperature difference ΔT have been corrected for the side-wall conductance and for the Kapitza boundary resistance between copper and liquid helium. The Nusselt number is defined as $N_{\mathrm{Nu}} = K/K_c$; the stress parameter may be written as $\epsilon = (\Delta T/\Delta T_c) - 1$. Here K_c and ΔT_c are the conductance and temperature difference at the onset of convection, both of which depend on the mean cell temperature T_m through the temperature dependence of the fluid parameters. As $\Delta T/T$ is not always small and the bottom plate temperature is held fixed, the variation in the effective K_c and ΔT_c with ΔT may not be negligible. The values of K_c and ΔT_c that we used to evaluate any N_{Nu} or ϵ are those valid at T_m for the corresponding ΔT. The critical Rayleigh number obtained from Eq. (1), using ΔT_c and the fluid parameters evaluated at the mean temperature of the cell, is $R_c = 1930$ ($\pm 7\%$) over the full temperature range. This essentially constant value of R_c strongly suggests that deviations from one-component behavior are not significant. As in the previous work,[6] this value is noticeably lower than $R_c = 2430$, which is the expected value in both classical fluids[13] and ^{3}He-^{4}He superfluid solutions.[8] This discrepancy, though not understood, is not excessive when compared with current work for one-component fluids.[14]

At higher ϵ, the stationary convective state becomes time dependent, as indicated with highest stability and lowest noise by regular oscillations in the probe-TC output. Figure 1(a) shows the rms amplitude of the time-dependent component of the temperature measured by the probe TC plotted against ϵ for $T_m = 0.716$ K and for zero probe-heater power. The rms amplitude is *linearly* dependent on ϵ close to the onset. We fitted a straight horizontal line to the data points without oscillations [indicated by triangles in Fig. 1(a)], and a straight line to those with oscillations (circles). The intersection of these two lines was taken to define the onset point, ϵ_0. To demonstrate that the oscillations are not caused by the small horizontal temperature gradient near the probe, measurements were made with the probe heater providing 3.7% of the total heat into the top plate. A very similar transition to the oscillatory state was observed with ϵ_0 shifted downward by 0.3%.

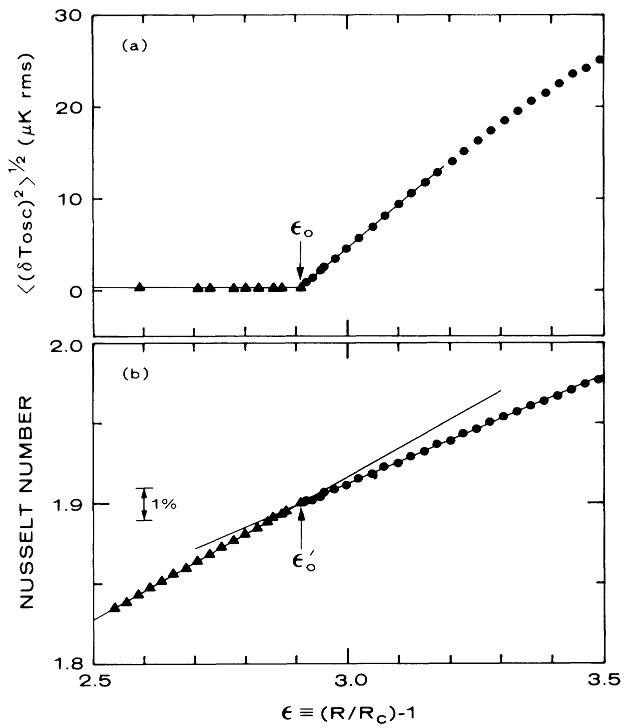

FIG. 1. (a) The rms amplitude of the time-dependent component of the probe-thermocouple output. (b) The time-averaged Nusselt number. In both figures $T_m = 0.716$ K and $\Delta T = 27.8$ mK at the onset of oscillations; triangles indicate the data without oscillations and circles indicate the data with oscillations.

The time-averaged N_{Nu} near ϵ_0 is plotted in Fig. 1(b). The points below and above the onset are fitted with straight lines to determine the intersection point, ϵ_0'. The onset parameter varies from $\epsilon_0 = 2.91$ at 0.716 K to 9.56 at 1.054 K, but for a given temperature ϵ_0 and ϵ_0' are in good agreement with each other, with an average difference of 0.2%. A change in the slope of N_{Nu} vs ϵ indicates that the total heat transport is suppressed by the oscillations, the reduction being linearly proportional to $\epsilon - \epsilon_0$ near ϵ_0; this is true in the entire range of σ that we have investigated. Our observation is in contradiction to previous work with mercury ($\sigma = 0.036$),[3] in which N_{Nu} is enhanced by the oscillations.

Figures 1(a) and 1(b) contain data points taken with both increasing and decreasing ϵ. No measurable hysteresis was observed around ϵ_0. A critical slowing-down effect was also observed both below and above ϵ_0.

The oscillatory frequency f was very stable and was measured with a precision of about 0.02%. In Fig. 2

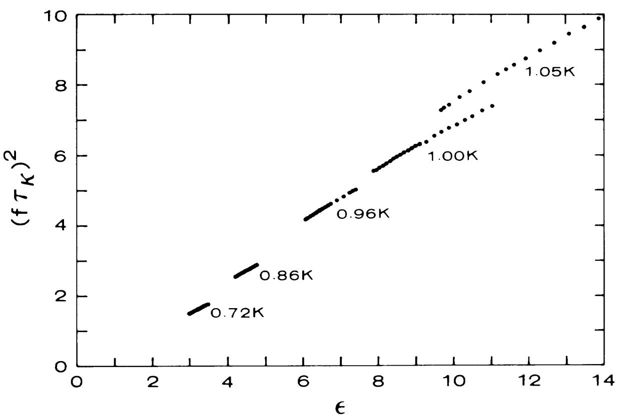

FIG. 2. The square of the oscillatory frequency, normalized by the vertical thermal diffusion time τ_κ, vs $\epsilon \equiv (R/R_c) - 1$.

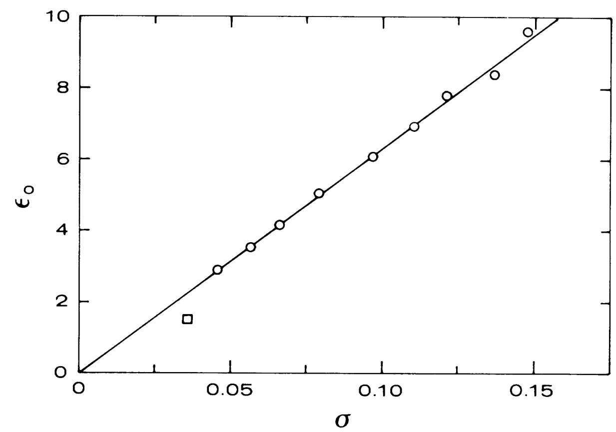

FIG. 3. The stress parameter at the onset of oscillations plotted against the Prandtl number σ. Also included (square) is the result in Ref. 3 for mercury in a rectangular cell with $\Gamma = 1.00$ and $\Gamma' = 0.50$.

the square of f normalized by the vertical thermal diffusion time[15] $\tau_\kappa = 4d^2/(\pi^2\kappa)$ is plotted against ϵ for various temperatures. At 0.72 K, $\tau_\kappa = 2.0$ s and 1.05 K, $\tau_\kappa = 13$ s. Least-squares fits with straight lines to the data at nine different temperatures and with $\epsilon - \epsilon_0$ less than 1.0 give zero-frequency intercepts at $\epsilon/\epsilon_0 = 0.04 \pm 0.01$. The period of the convective-roll oscillation is expected to be closely related to the turnover time of the rolls. Thus, the square of the frequency would vary as ϵ as long as the convective velocity varies as $\epsilon^{1/2}$.[16] The slope of $(f\tau_\kappa)^2$ vs ϵ decreases from 0.70 at 1.05 K to 0.54 at 0.72 K. The change is too large to be accounted for by a lack of precision in the experiment or by errors in the thermal diffusivity values used. Hence the magnitude of the convective velocity is not strictly scaled by κ.

According to the predictions[17] that assume rigid horizontal boundaries but no side boundaries, the oscillatory frequency should increase closely as $\epsilon^{1/2}$ and scale nearly with τ_κ, in agreement with our observation. For a given value of ϵ, $(f\tau_\kappa)^2$ is expected to decrease by about 30% as σ is decreased from 0.15 to 0.045 (Fig. 12 of Ref. 17). We have observed this tendency as seen in the decrease in slope at lower temperatures (and hence lower σ) in Fig. 2.

In Fig. 3 the onset parameter, ϵ_0, is plotted versus the Prandtl number. A least-squares fit to a line, $\epsilon_0 = A + B\sigma$, gives $A = -0.06 \pm 0.10$ and $B = 63.9 \pm 1.0$. Also included in the figure is the result of Fauve and Libchaber[3] using mercury with $\sigma = 0.036$ in a rectangular cell with $\Gamma = 1.00$ and $\Gamma' = 0.50$. The onset parameter is predicted to vary as $\epsilon_0 \propto \sigma^\alpha$ with α very close to 1.0 for $\sigma < 0.1$ (Fig. 11 of Ref. 17). This σ dependence of ϵ_0 agrees with our observation. The values of ϵ_0 observed are an order of magnitude greater than the predicted ones, suggesting that the side walls severely suppress the instability. We have no simple heuristic interpretation of the observation that ϵ_0 increases linearly with σ for small σ.

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