# Squeezing in a Rydberg Atom Maser

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We show theoretically that a Rydberg atom maser produces during its evolution a squeezed field with sub-Poissonian statistics. We give a semiclassical interpretation of this effect.

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Squeezed states of the electromagnetic field are characterized by fluctuations of one quadrature field smaller than the vacuum fluctuations. They could have interesting applications for quantum noise reduction.[1] These states are theoretically produced in a wide variety of nonlinear atom-field processes,[2] but have not yet been observed.

It has been shown recently that a good tool for the study of quantum radiative properties is a sample of Rydberg atoms inside a high-finesse resonant millimeter-wave cavity[3]: It is possible to prepare experimentally a system very close to the ideal Dicke model[4] ($N$ two-level atoms symmetrically coupled to a unique field mode). Very good agreement between observed and predicted radiative properties has been obtained.[5]

The aim of this Letter is to show theoretically that a Rydberg atom maser can produce during its evolution a large field with significant squeezing. We think that this new proposal should be seriously considered as a means for experimental observation of squeezed states.

An interesting additional feature of this system is its semiclassical nature which allows an astonishingly simple interpretation of squeezing in terms of Bloch vector evolution.

*Description of the physical system.*—The Rydberg atom system will be described as an angular momentum $J=N/2$ coupled to a unique field mode[3] ($J_z$ is the operator associated with the atomic energy). We will restrict ourselves to the case of a strictly resonant interaction and neglect cavity-mode damping and atomic relaxation.

The ordinary superfluorescence problem, which has been widely discussed,[6] corresponds to a completely excited atomic system and an empty field mode as the initial state. There is no phase information in this initial state and the two dispersions $\Delta a_1$ and $\Delta a_2$ of the two field quadratures [$a_1 = a + a^\dagger$, $a_2 = i(a - a^\dagger)$; $a$, an annihilation operator] thus remain equal, so that squeezing ($\Delta a_1 < 1$) is forbidden by the Heisenberg inequality, $\Delta a_1 \Delta a_2 \geqslant 1$.

We will instead consider two types of initial states containing some phase information. In the first one ("spontaneous superradiance" problem) all of the atoms are prepared in the same coherent superposition of their two states (Bloch coherent state of the angular momentum[7]) and the field is empty. In the second one ("triggered superradiance"), the atoms are totally excited and the field is prepared in a Glauber coherent state.[8]

*Semiclassical predictions.*—When the number of atoms is large enough, the Dicke problem can be approximated by the semiclassical "Bloch pendulum",[3] which obeys the equations

$$\dot{\theta} = -\beta; \quad \dot{\beta} = \sin\theta. \tag{1}$$

The angular position $\theta$ of the pendulum is related to the atomic angular momentum [$J_z \leftrightarrow (N/2)\cos\theta$] whereas the angular velocity is connected with the field variable ($a \leftrightarrow \sqrt{N}\beta$). The Bloch pendulum can be considered as the one-dimensional problem associated with the energy

$$\epsilon = \beta^2/2 + (1 - \cos\theta). \tag{2}$$

The two types of initial states correpsond respectively to $\epsilon < \epsilon_c$ (spontaneous superradiance) and $\epsilon > \epsilon_c$ (triggered superradiance) with $\epsilon_c = 2$.

Quantum fluctuations of the atomic and field variables are accounted for by position and velocity fluctuations in the initial state of the pendulum, which can be shown to be equal to

$$\Delta\theta_0 = \Delta\beta_0 = 1/\sqrt{N}. \tag{3}$$

We show in Fig. 1 the temporal evolution of the distribution in $\{\theta, \beta\}$ phase space, calculated from Eqs. (1) for different initial conditions. It is clearly apparent that squeezing, i.e., a reduction of the dispersion $\Delta\beta^2$, occurs in this evolution especially around $\theta = 0$ (minimum of the atomic energy). The value of $\Delta\beta^2$ corresponding precisely to $\theta = 0$ can be deduced from energy conservation. A straightforward calculation gives a reduction factor of $1 - \epsilon/\epsilon_c$ for $\epsilon < \epsilon_c$ and $1 - \epsilon_c/\epsilon$ for $\epsilon > \epsilon_c$. Large amounts of squeezing are thus predicted in both cases when $\epsilon$ gets close to the critical energy $\epsilon_c$.

We have determined the minimum value of $\Delta\beta^2$, which is reached before $\theta = 0$ for $\epsilon < \epsilon_c$ and after for $\epsilon > \epsilon_c$, and plotted the corresponding squeezing $S = N\Delta\beta^2$ as solid lines in Fig. 2 [Fig. 2(a) is for $\epsilon < \epsilon_c$, 2(b) for $\epsilon > \epsilon_c$].

One must emphasize that the area of the distribution is conserved (Liouville theorem). It follows that the angular dispersion $\Delta\theta^2$ is increased near $\theta = 0$.

 

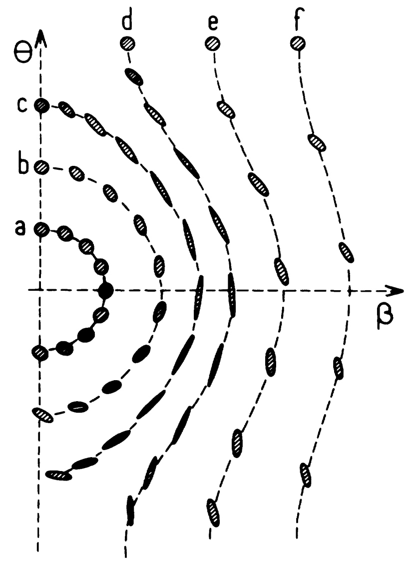


FIG. 1. Classical evolution of the distribution in the $\{\theta, \beta\}$ phase space (same scale on the two axes). The circles labeled *a*, *b*, *c*, *d*, *e*, and *f* represent six different initial states: *a*, *b*, and *c* correspond to spontaneous superradiance ($\beta_0=0$) with $\theta_0=\pi/4$, $\pi/2$, and $3\pi/4$, respectively; and *d*, *e*, and *f* to triggered superradiance ($\theta_0=\pi$) with $\beta_0=1$, 2, and 3, respectively. The ellipses represent the displacement and deformation of the distribution at equally spaced times (the time interval is $T/16$, where $T$ is the period of small oscillations).

*Quantum calculations.*—We have numerically solved the Schrödinger equation of the Dicke problem and computed the field variances. The solid squares in Fig. 2 represent the optimal squeezing ($S=\Delta a_1^2$) obtained around the time of the first minimum of the atomic energy, as a function of the initial conditions for $N=10$ and $N=100$ atoms. As expected, the quantum results are closer to the semiclassical predictions for $N=100$ than for $N=10$. But the discrepancy increases rapidly when the energy $\epsilon$ gets close to $\epsilon_c$. As a result, there is a limit on the squeezing which can be obtained for a given number $N$ of atoms. When $N$ increases, this limit becomes lower and corresponds to an energy $\epsilon$ closer to $\epsilon_c$.

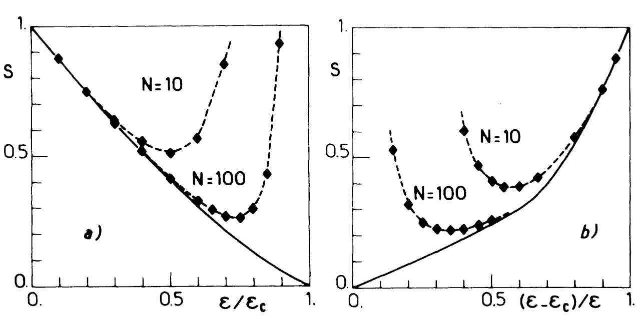


FIG. 2. Minimum value of the variance $\Delta a_1^2$ as a function of the initial conditions. Semiclassical predictions are indicated by solid lines, and quantum results for $N=10$ and $N=100$ by solid squares. $\epsilon$ is the energy characterizing the initial state and $\epsilon_c$ is the critical energy. (a) $\epsilon<\epsilon_c$; (b) $\epsilon>\epsilon_c$.

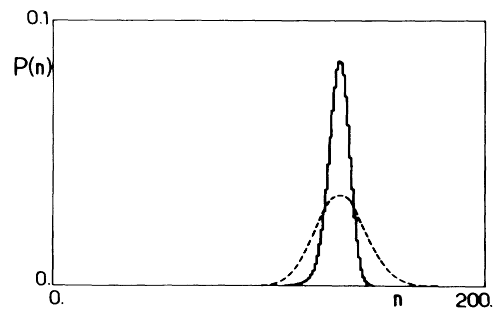


FIG. 3. Sub-Poissonian distribution $P(n)$ (solid line) obtained for $N=170$, $\epsilon/\epsilon_c=0.8$, compared to a Poisson distribution (dashed line).

As the computation time increases roughly as $N^3$, we have restricted our calculations to $N \leq 170$. The best quantum result which we have obtained is $S=0.18$ for $N=170$ and $\epsilon/\epsilon_c=0.8$.

Let us stress that squeezing is associated in this case with sub-Poissonian statistics. In the semiclassical approach, we predict

$$\Delta n^2/\bar{n}=S<1. \tag{4}$$

The quantum results for this parameter are quite similar to those obtained for $S$ although the discrepancies with the semiclassical predictions are slightly different. As an example, we have plotted in Fig. 3 (solid line) the distribution $P(n)$ of the number of photons in the mode for $N=170$ and $\epsilon/\epsilon_c=0.8$. This distribution is narrower (by a factor larger than 2 since $\Delta n^2/\bar{n}=0.18$) than a Poisson distribution (dashed line in Fig. 3).

In conclusion, we have shown that a Rydberg atom maser produces during its evolution a large amount of squeezing in a field containing a large number of photons. We have been able to give a semiclassical interpretation of this squeezing in terms of Bloch pendulum evolution.

Getting close to this model in a realistic experiment seems possible. The first stage of such an experiment, i.e., preparation of the squeezed field, can be achieved by well-known techniques.[5] One can indeed prepare a large number of Rydberg atoms inside a high-finesse resonant cavity and interrupt the atom-field interaction by Stark detuning when the optimum squeezing is reached. The second stage, i.e., detection of the field statistical properties, is more difficult. Detection by other Rydberg atoms[5] could be applied directly to the investigation of sub-Poissonian statistics but observation of squeezing would require field detection by heterodyne mixing.[1] The low noise and high quantum efficiency required seem attainable by the best presently built mixers.[9]

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