Magnetoresistance Fluctuations in Mesoscopic Wires and Rings

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A simulation of the magnetoresistance of very small wires shows sample-specific, aperiodic structure due to the non-self-averaging nature of quasi-one-dimensional conductors in this size range. This explains earlier failures to observe flux-periodic magnetoresistance in very small rings. Analysis of the temperature and size dependence of the effect gives good agreement with experiment. A prediction is made for the experimental conditions in which oscillations periodic with flux hc/e will be observed.

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Physicists are accustomed to dealing with two classes of quantum-mechanical systems: (1) microscopic systems in which the energy levels are discrete and their spacing is typically much greater than kT (at low temperatures); (2) macroscopic systems in which the level spacing is always much less than kT. However, recently it has become possible to study experimentally solid-state systems in the intermediate "mesoscopic" size regime. In these systems the energy-level spacing is only a few orders of magnitude smaller than kT at low temperatures. I argue that quite generally one should see novel quantum-mechanical fluctuation phenomena in the transport coefficients of such systems. These *time-independent* sample-specific fluctuations are a direct consequence of the differing microscopic configurations of two mesoscopic systems with the same average physical properties. The fluctuations will be large if the states of the mesoscopic system are localized, as in a semiconductor, and small if the states are extended, as in a metal. Large fluctuations of this type have already been observed experimentally in one-dimensional semiconductor devices, where they appear as reproducible, noiselike structures in the conductance as a function of electron density.[1] Similar but much weaker structure has been seen very recently in the magnetoresistance of small metal wires and rings.[2,3]

In this paper I present a theory of the origin of this structure based on a detailed numerical simulation that agrees well with these experiments; the experiments should therefore be seen as the first observation of mesoscopic fluctuations in a metallic system. I also predict the experimental conditions necessary for observation of *periodic* magnetoresistance oscillations in normal-metal rings, despite the presence of these fluctuations. It must also be emphasized here that while there has been a great deal of theoretical work on resistance fluctuations in strongly localized, purely one-dimensional systems, surprisingly, there has been no theoretical work at all on fluctuations in weakly disordered wires where the transport is diffusive.

The experimental impetus for studying very small rings and wires was the observation by Sharvin and Sharvin of very weak periodic oscillations in the magnetoresistance of normal-metal cylinders,[4] with the periodicity of the superconducting flux quantum $hc/2e$. Working on much smaller systems (rings of 3000-Å diameter and 500-Å width) Webb, Washburn, Umbach, and Laibowitz[2] and Blonder[3] observed sample-specific, aperiodic, reproducible structure in the magnetoresistance which was two orders of magnitude larger than the Sharvin and Sharvin effect and which persisted without decrease in amplitude out to 8 T. The same structure was also seen in wires (Fig. 1), showing that, unlike either the Sharvin and Sharvin effect or the usual Bohm-Aharonov effect (which has flux period hc/e), it had nothing to do with the system's topology. The structure grew slowly with decreasing temperature roughly as $T^{-1/2}$, and had an amplitude of (0.01–0.1)% compared to the background resistance. An asymmetry in the structure was observed when the magnetic field direction was reversed.

In order to understand the experiments, I study a model first treated by Lee and Fisher[5,6] which consists of a nearest-neighbor tight-binding model on an infinite two-dimensional strip containing a finite disordered region of N sites in length and M sites in width. The Hamiltonian in the site representation is

$$H=\sum_{n,m}\{\epsilon_{n,m}|n,m\rangle\langle m,n|+[V^{x}_{n,n+1}(m)|n+1,m\rangle\langle m,n|+\text{c.c.}]+[V^{y}_{m,m+1}(n)|n,m+1\rangle\langle m,n|+\text{c.c.}]\},\quad(1)$$

where the site energies $\epsilon_{n,m}$ are random, and uniformly distributed between $\pm W/2$ in the disordered region,[7] and the hopping matrix elements are set equal to unity in the ordered region. Hard-wall boundary conditions are imposed in the transverse direction (y direction). A uniform magnetic field, B, normal to the wire, can be introduced into the disordered region by setting $V^{y}_{n,m}=\exp(2\pi i n\Phi_1/\Phi_0)$, where $\Phi_1=Ba^2$ is the flux through a unit cell of the lattice, and $\Phi_0=hc/e$ is the one-electron flux quantum. This model has been shown to give a negative *average* magnetoresistance in agreement with the theory of weak localization.[5] The key point here is that *ensemble averag-*

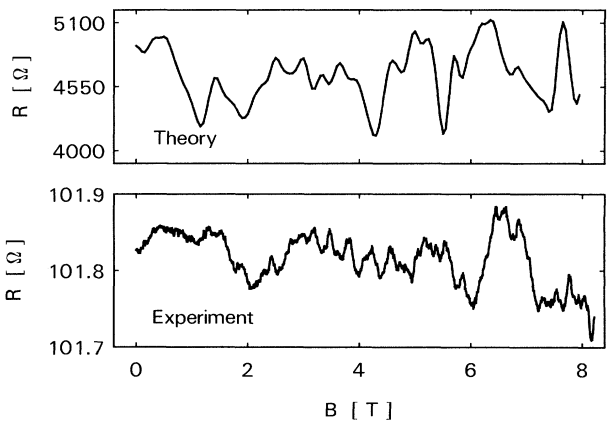

FIG. 1. Comparison of magnetoresistance structure from simulation and experiment. The experimental system is a Au-Pd wire of length 7900 Å and width 500 Å (reprinted with permission of Webb *et al.*, Ref. 2). The simulated wire is 400 by 40 sites, with $W=0.6$ and $E=0.2$.

ing is not appropriate if one is to model a system in this mesoscopic size range, since it misses the interesting sample-specific fluctuations.

The propagating eigenstates of the model in the ordered regions are just plane waves labeled by a continuous momentum index k_x and a discrete transverse momentum index (or channel number) k_y. The dispersion relation is

$$E_j = 2\cos(k_x a) + 2\cos[k_y(j)\,a],$$

where $k_y(j)=\pi j/(M+1)$, $j=1,\ldots,M$; the number of real solutions k_x for fixed energy gives the number of channels M_c at that energy. The conductance of the wire is calculated by first computing the transmission and reflection matrices, t and r, across the disordered region and then evaluating a multichannel Landauer formula due to Azbel[8] and Büttiker *et al.*[9]:

$$\frac{G}{2e^2/h}=\frac{\left(\sum_{i=1}^{M_c} T_i\right)\left(\sum_{j=1}^{M_c} v_j^{-1}\right)}{\sum_{k=1}^{M_c}(1+R_k-T_k)\,v_k^{-1}}, \tag{2}$$

where $T_i=\sum_{j=1}^{M_c}|t_{ij}|^2$ and $R_i=\sum_{j=1}^{M_c}|r_{ij}|^2$, and v_i is the channel velocity, $\partial E_i/\partial k_x$. I emphasize that the basic features of the magnetoresistance structure that are described below are independent of the details of formula (2), and could have been obtained by analyzing the square of the transmission matrix. A comparison of results for different multichannel Landauer formulas will be given elsewhere.[10]

The S matrix needed for the evaluation of Eq. (2) is obtained by use of a recursion Green's-function technique.[5,11] To calculate the conductance from (2), given the Green's function, one needs the relations[6,10]

$$|r_{ij}|^2=|i(v_i v_j)^{1/2}G_{ij}^{+}(0,0)-\delta_{ij}|^2, \tag{3a}$$

$$|t_{ij}|^2=v_i v_j|G_{ij}^{+}(0,N+1)|^2, \tag{3b}$$

where $G_{ij}^{+}(n,n')$ is the retarded Green's function with x coordinates n and n' and channel indices i and j. As in the experiment, wires with length about ten times their width were considered, and of sizes up to 500 by 50 sites.

The results of a typical calculation are compared in Fig. 1 with the experimental results from a Au-Pd wire of length 7900 Å. One sees that the simulation gives aperiodic structure that is qualitatively similar to that found experimentally, including its persistence to high fields without decrease in amplitude. The electrons in the simulation are spinless, and thus spin-orbit scattering and spin-flip scattering from magnetic impurities, which are present in the experiment, appear not to be responsible for any of the qualitative features of the structure. This indicates that the existence of such structure is universal in metal wires (in this mesoscopic size range).

Although the simulated wire is substantially more disordered than the experimental wires, it *is* in the relevant regime of length scales: The sample length is much longer than the elastic mean free path, but shorter than the localization length. The structure found in the simulation is, however, 100 to 1000 times larger than the experimental structure. This difference will be discussed in detail below.

A careful study of the magnetoresistance over a very small field range reveals a crucial feature: No structure is seen until the change in the flux through the area of the wire is roughly of order two or three times Φ_0, and the density of structure as a function of field is proportional to the area of the wire.[10] This agrees well with the observed density in the experiment. Equally importantly, it suggests that the structure arises from an interference effect between electronic trajectories in the wire, since the change in the phase of the electronic wave functions due to the field scales as the sample area.

The calculation shown in Fig. 1 is done for a fixed electronic energy corresponding to zero temperature, whereas the experiment shows a $T^{-1/2}$ growth of the structure with decreasing temperature. This is *not* the usual one-dimensional weak-localization temperature dependence, because the sample lengths are known to be shorter than the inelastic scattering lengths at the relevant temperatures. To understand the experimental temperature dependence, one needs to examine the variation of the structure in field as a function of energy. In particular, there should be some correlation length in energy, E_c, over which the structure in field remains the same, and outside of which one obtains structure essentially uncorrelated with that at the ini-

tial energy. At finite temperature the incident electron distribution is smeared out over a range kT around E_F; therefore the conductance should be, roughly speaking, the sum of kT/E_c uncorrelated patterns, and should be reduced in amplitude by $(E_c/kT)^{1/2}$ compared to the zero-temperature structure. This explains the observed increase of the structure proportional to $T^{-1/2}$ as temperature decreases, as long as kT is much greater than E_c. What then is this correlation length, E_c?

A sensible hypothesis is that the energy correlation length is at least as large as the average energy-level spacing of the sample in isolation. This hypothesis is substantiated by studying the structure as a function of energy (Fig. 2). When one changes the electron energy by less than an energy-level spacing a correlated pattern is obtained, but when it is changed by several level spacings an uncorrelated pattern is found. Studying smaller wires reveals that changing the energy by the same absolute amount (which is now less than the level spacing) does not produce an uncorrelated pattern. Thus the relevant energy scale does appear to be at least as large as the level spacing. However, there are physically reasonable arguments that the scale may be even larger than the level spacing,[12] and because of limitations on system size, it is hard to decide this question by simulation alone. At any rate, by averaging over more and more energies separated by more than the numerically determined correlation length, one approximately simulates the conductance as a function of temperature. The structure amplitude is measured by the standard deviation ΔG around G_{av}, the *field* average of G for a field sweep of a fixed

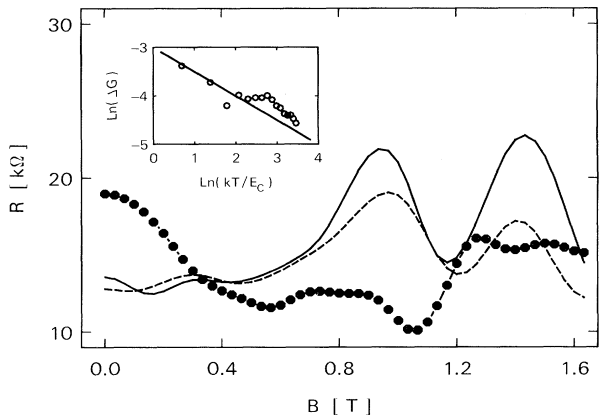

FIG. 2. Magnetoresistance structure at three different energies for a 200×20-site wire with energy-level spacing 0.002. The solid line is $E=0.200$, and the dashed line $E=0.201$; these curves show correlated structure. The dotted curve is $E=0.210$, which is poorly correlated. The inset plots ΔG, the structure amplitude, vs temperature in units of E_c calculated as described in the text. The data are consistent with the predicted $T^{-1/2}$ dependence (solid line).

number of flux quanta through the sample area. The results (inset, Fig. 2) are consistent with the $T^{-1/2}$ law predicted above, as are the experimental results.

Next, the effect of system size on the amplitude of the structure was examined by studying $\delta G=\Delta G/G_{av}$ as a function of the number of transverse channels, M_c, for a fixed ratio of the wire's length to its width. Surprisingly, the structure amplitude decreases quite slowly, the best power-law fit is $M_c^{-0.3}$, and the data are not inconsistent with the hypothesis that the amplitude becomes independent of channel number for $M_c \gg 1$. This is an important result, since a naive argument which assumes that the t_{ij} are uncorrelated and have random phases suggests that the amplitude of the fluctuations to background should scale as M_c^{-1} and this would predict an amplitude two orders of magnitude smaller than is seen experimentally. This puzzling discovery requires further study.[13] An extrapolation based on this result, those of Figs. 1 and 2, and the hypothesis concerning the magnitude of E_c mentioned below predicts resistance fluctuations in the experimental system of about 1% at 10 mK, somewhat larger than the experimental observation. However, the simulation clearly shows that the amplitude of the fluctuations to background increases with increasing disorder and this may account for the remaining discrepancy.

Finally, the simulation does show an asymmetry when the field is reversed, a result which is consistent with the form of Eq. (2).[14] However, this feature of the structure is the only one which is sensitive to differences in the various proposed multichannel Landauer formulas[9,14] and therefore a careful study of the asymmetry will be presented elsewhere.[10]

The physical origin of the aperiodic structure may be understood as follows. Consider a one-dimensional ring connected to leads with a flux Φ through the hole. There is an interference term in the total transmission coefficient whose sign depends on the phases of the transmission amplitude through each arm of the ring. The magnetic field changes the relative phase of the contribution from each arm of the ring by an amount $2\pi\Phi/\Phi_0$; thus it gives magnetoresistance oscillations which are periodic in Φ_0.[15] For a real ring with finite width, and with a field penetrating the metal as well as the hole, the electrons traversing the inner and outer perimeters acquire different phases from the field because the trajectories enclose different amounts of flux. Thus, variation of the field causes the phase-dependent contributions to the resistance to vary incoherently, and does not give rise to one periodicity in field. The same interference occurs between the electronic trajectories in a *wire* in a magnetic field. If the wire were macroscopic in its dimensions, these incoherent phase-dependent terms would average to zero (although they would not do so for a macroscopic

ring, where it is known that $\Phi_0/2$ oscillations survive ensemble averaging[9,16]). However, in this mesoscopic size range, if inelastic scattering is absent, these phase-dependent contributions do not self-average, and their variation gives the magnetoresistance fluctuations described above. On the basis of this picture, in which the fluctuations arise from the non-self-averaging character of the wires, I predict that one should observe the fundamental Φ_0 periodic oscillations in normal-metal rings *if* the ratio of the area of the hole to that of the wire is much greater than 1. But it is crucial to study them over a small enough field range that *the change in the flux through the wire is much less than* Φ_0 in order to avoid the aperiodic fluctuations. In a ring of 2-μm diameter and 500-Å linewidth about 10 to 50 oscillations periodic in Φ_0 should be observable when the field is varied over a range of 200 G. When the field is varied over a larger range the Fourier-transformed magnetoresistance should have strongly bimodal form, with most of its weight centered around a high frequency corresponding to flux Φ_0 through the hole, and low frequencies corresponding to fluxes of order Φ_0 through the wire.

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Note added.—Since the submission of this paper for publication, experimental results by Webb *et al.*[17] have shown convincingly the existence of magnetoresistance oscillations with flux period hc/e in metal rings of larger diameter than those studied previously.

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