Lattice Results on the Meson Electric Form Factor

W. Wilcox and R. M. Woloshyn
TRIUMF, University of British Columbia, Vancouver, British Columbia V6T 2A3, Canada
(Received 12 February 1985)

A calculation is outlined and results presented for the electric form factor, measured at two values of the momentum, of the pseudo-Goldstone meson within the staggered formulation of lattice fermions.

PACS numbers: 13.40.Fn, 11.15.Ha, 12.35.Eq

As numerical simulations of quantum chromodynamics (QCD) on the lattice improve it is necessary to devise calculations which provide more detailed tests of the theory. One important area for study is hadron internal structure.[1-3] Electromagnetic properties can provide clean and experimentally accessible information for this purpose. In this Letter we discuss the lattice calculation of the vector-current–hadron vertex function and present our first results for the pseudoscalar meson electric form factor.

The staggered scheme for putting fermions on the lattice is used.[4,5] The fermion action in terms of single-component fermion fields $\overline{\chi}$, χ and gauge-field matrices U_μ takes the form (color indices suppressed)

$$S_{\rm F}=\tfrac{1}{2}\sum_{x,u}\alpha_\mu(x)[\overline{\chi}(x)U_\mu(x)\chi(x+a_\mu)-\overline{\chi}(x+a_\mu)U_\mu^\dagger(x)\chi(x)]+ma\sum_x\overline{\chi}(x)\chi(x)\ , \tag{1}$$

where a is the lattice spacing and a_μ is the unit vector in the μ direction ($\mu=1,\ldots,4$). The quantity $\alpha_\mu(x)=(-1)^{\xi_\mu}$, where $\xi_\mu=\sum_{\nu<\mu}x_\nu$. Local phase transformation of the fermion field, $\chi(x)\to e^{i\omega(x)}\chi(x)$, $\overline{\chi}(x)\to\overline{\chi}(x)e^{-i\omega(x)}$, yields a vector current

$$j_\mu(x)=-\delta S_{\rm F}(x)/\delta\Delta\omega(x) \tag{2a}$$

$$=-\tfrac{1}{2}i\alpha_\mu(x)[\overline{\chi}(x)U_\mu(x)\chi(x+a_\mu)+\overline{\chi}(x+a_\mu)U_\mu^\dagger(x)\chi(x)]\ , \tag{2b}$$

where $\Delta\omega(x)=\omega(x+a_\mu)-\omega(x)$. This is conserved in the sense that the ensemble average $\langle\sum_\mu[j_\mu(x+a_\mu)-j_\mu(x)]\rangle=0$.

The interpretation of the fermion degrees of freedom in (1) is usually given in terms of flavored quark fields defined on hypercubes in the lattice. The hadron correlation functions, to be identified with continuum matrix elements, are constructed from interpolating fields made up from these *nonlocal* flavored quark fields. It has been shown that the two-point function, relevant for mass calculations, can be cast in a form that can be constructed using only *local* χ-field bilinear operators.[6,7] This is, of course, advantageous for numerical calculations. We have shown that our three-point function can also be calculated in a form that involves only local χ-field bilinear operators as interpolating fields. The derivation will be given elsewhere and here we only quote the final results used in our numerical study.

We want to construct current matrix elements for flavor nonsinglet meson states with nonzero electric charge. Specifically, we consider the pseudo-Goldstone meson associated with the exactly conserved (in the zero-mass limit) flavor nonsinglet axial current.[5] The usual flavor structure associated with the staggered fermions is not useful for constructing charged states since an "electric charge" defined *within* these flavors is not conserved.[8] The conserved vector current, (2b), if interpreted as nondynamical electric charge, assigns identical charges to all four staggered fermion flavors. We therefore introduce two sets of χ fields (labeled by u and d, with charges q^u and q^d, $q^u-q^d=1$) from which we construct charged-meson interpolating fields and the conserved electromagnetic currents $j^u(x)$ and $j^d(x)$, based on (2b).

The three-point function which we calculate is (with color indices suppressed)

$$A(\mathbf{p},\mathbf{q};t_z,t_x)=\langle 0|\sum_{\mathbf{z}}e^{-i\mathbf{p}\cdot\mathbf{z}}(-1)^{\mathbf{z}}\overline{\chi}^d(\mathbf{z},t_z)\chi^u(\mathbf{z},t_z)\sum_{\mathbf{x}}e^{i\mathbf{q}\cdot\mathbf{x}}\rho(\mathbf{x},t_x)\overline{\chi}^u(0)\chi^d(0)|0\rangle\ , \tag{3}$$

where $\rho(\mathbf{x},t_x)=iq^u j_4^u(x)+iq^d j_4^d(x)$ and $(-1)^{\mathbf{z}}$ means $(-1)^{z_1+z_2+z_3}$. We also need the two-point function[9]

$$G(\mathbf{p};t_z)=\langle 0|\sum_{\mathbf{z}}e^{-i\mathbf{p}\cdot\mathbf{z}}(-1)^{\mathbf{z}}\overline{\chi}^d(\mathbf{z},t_z)\chi^u(\mathbf{z},t_z)\overline{\chi}^u(0)\chi^d(0)|0\rangle\ . \tag{4}$$

For $t_x, t_z-t_x>>1$, we have

$$A(\mathbf{p},\mathbf{q};t_z,t_x)\to\left\{Z(p)Z(p')\frac{(1+e^{E_pa})}{(1+e^{-E_pa})}\frac{(1+e^{-E_{p'}a})}{(1+e^{E_{p'}a})}\right\}^{1/2}e^{-E_{p'}t_x}e^{-E_p(t_z-t_x)}\langle\pi^+(\mathbf{p})|\rho(0)|\pi^+(\mathbf{p}')\rangle\ , \tag{5}$$

where

$$\langle \pi^+(\mathbf{p})|\rho(0)|\pi^+(\mathbf{p}')\rangle = \frac{(E_p+E_{p'})}{2(E_pE_{p'})^{1/2}}F(q), \quad (6)$$

with $E_p = \{\mathbf{p}^2+M^2\}^{1/2}$ and $\mathbf{p}'=\mathbf{p}-\mathbf{q}$. We use $|\pi^+(\mathbf{p})\rangle$ to label the pseudoscalar-meson ground state with unit charge and degenerate (light) valence quarks. The quantity $F(q)$ is the electric form factor and the quantity $Z(p)$ is related to the two-point function by

$$G(\mathbf{p};t_z) \underset{t_z \gg 1}{\longrightarrow} Z(p)\exp(-E_p t_z). \quad (7)$$

Numerical calculations were done in a model for QCD with use of only SU(2) color to save on computing time. The lattice size was $10^2\times 20\times 16$ with the current-carrying momentum in the 3 direction. Thirty-two gauge-field configurations were prepared by a heat-bath Monte Carlo technique[10] in quenched approximation and the Wilson gauge-field action[11] at $\beta=2.3$. The gauge fields were constructed on a $10^3\times 16$ lattice and then doubled in the 3 direction.

Quark propagators were calculated by the conjugate-gradient algorithm.[12] Antiperiodic boundary conditions were used on fermion fields in the spatial directions. However, the fermion coupling was put equal to zero across the time boundary of the lattice. This is similar, but not identical, to the boundary condition used by Bernard *et al.*[13] The advantage of this choice is that we see simple exponential falloff of the correlation function over a large time interval. In calculation of the form factor the problem of nonvacuum contamination can be corrected by the taking of a geometric mean of correlation functions calculated for appropriate momenta.

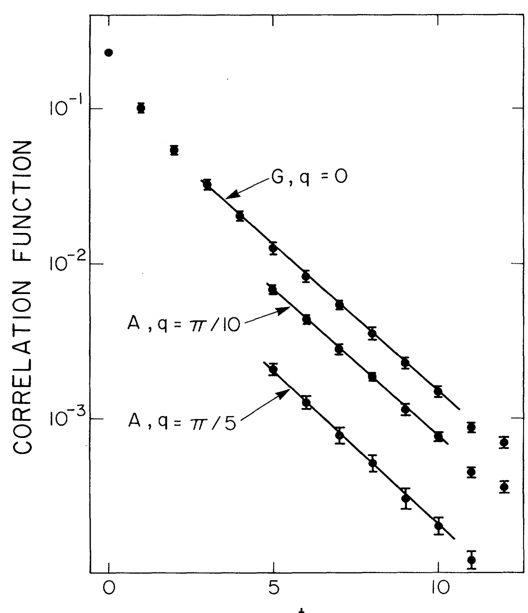

FIG. 1. Plot of the two-point function $G(0;t_z)$ and the three-point function $A(0,\mathbf{q},t_z,t_z)$ (for $q=\pi/10$ and $q=\pi/5$) as functions of t_z. The solid lines are single-exponential fits.

The three-point function was calculated as the derivative of a two-point function with the charge operator acting as a source.[14,15] This means that in the two-point function one of the quark propagators is calculated not with the action S_F of Eq. (1) but with

$$S_F^{(\alpha)} = S_F - \alpha \sum_{\mathbf{x}} e^{i\mathbf{q}\cdot\mathbf{x}}\rho(\mathbf{x},t_x). \quad (8)$$

The derivative with respect to α (at $\alpha=0$) gives the three-point function. This derivative is obtained numerically by calculation of two-point functions at $\alpha=0$ and $\alpha=0.05$. A check of this procedure at $q=0$, where the three- and two-point functions are related,[9] indicates that our derivative is good to within a few percent.

Propagators, with and without the source, were calculated for three different spatial starting points in each of the thirty-two gauge-field configurations. All calculations were done for the quark mass parameter $ma=0.025$. The data summed over all configurations are shown in Fig. 1 for the two-point function $G(0;t_z)$ and the three-point function $A(0,\mathbf{q};t_z,t_x)$. The three-point function was calculated at two values of momentum, for $q=\pi/10$, the lowest nonzero value on our lattice, and for $q=\pi/5$. The charge operator is located at $t_x=4$. This actually means that it involves lattice points at time steps four and five. From the two-point function we infer that the pseudoscalar meson mass $Ma=0.44\pm0.01$. As expected from (5) the three-point function falls with the sample slope as $G(0;t_z)$.

The form factor is extracted from a combination of two- and three-point functions at large time separa-

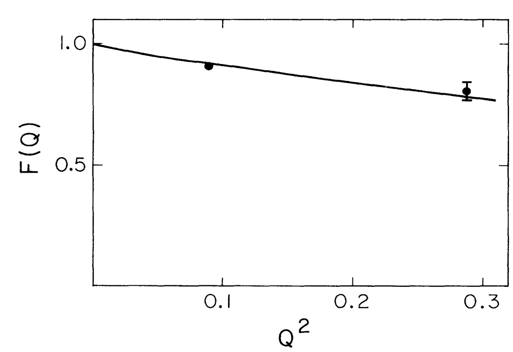

FIG. 2. Plot of the electric form factor F vs the Minkowskian four-momentum transfer squared (in lattice units). The solid line is a monopole form factor $(1+Q^2a^2/\lambda^2)^{-1}$ with $\lambda^2=1.05$.

tions:

$$\left\{\frac{A(0,\mathbf{q};t_z,t_x)A(\mathbf{q},\mathbf{q};t_z,t_x)}{G(0;t_z)G(\mathbf{q};t_z)}\right\}^{1/2} \rightarrow \frac{(E_q+M)}{2(E_qM)^{1/2}}F(q). \quad (9)$$

This not only simplifies the calculation but also corrects for nonvacuum contamination in the Z factors of (5) and (7). The results for F, plotted as a function of Minkowskian four-momentum transfer squared $Q^2=\mathbf{q}^2-(E_q-M)^2$, are shown in Fig. 2. These results were obtained by an averaging of (9) over time steps t_z numbers five to nine. The errors are statistical only and were calculated by combination of the uncertainties of the two- and three-point functions *including* the covariances between these quantities.[16] The solid line in Fig. 2 is a monopole form factor[17] $(1+Q^2a^2/\lambda^2)^{-1}$ with $\lambda^2=1.05$. The conversion to physical units depends on the calculation of the lattice spacing. Using the value $a\simeq 0.16$ fm obtained by Gutbrod and Montvay,[18] we infer an rms charge radius from the monopole form factor of about 0.38 fm. The older value $a\simeq 0.23$ fm based on the evaluation of the Creutz ratio[10] would give 0.55 fm.

The results presented here are, in a sense, preliminary. A number of important systematic effects remain to be studied. These include the extrapolation in quark mass to physical values and the approach to the continuum. As far as finite lattice size effects are concerned, none were found in the detailed study of the meson spectrum in SU(2) color by Billoire *et al.*[7] This is consistent with our finding that the meson is a compact object.

In conclusion, we have shown that it is feasible to calculate directly an important physical observable, the electric form factor, for a meson on the lattice. Our results provide evidence that the quarks in a lattice meson are indeed localized in a compact object significantly smaller than the lattice volume.

We thank the Nuclear Research Centre, University of Alberta, for use of their VAX 11/780. This work is supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1]R. M. Woloshyn and W. Wilcox, TRIUMF Report No. TRI-PP-84-91 (to be published); W. Wilcox and R. M. Woloshyn, TRIUMF Report No. TRI-PP-84-90 (to be published).

[2]K. Ishikawa, G. Schierholz, H. Schneider, and M. Teper, Nucl. Phys. **B227**, 221 (1983).

[3]K. Barad, M. Ogilvie, and C. Rebbi, Phys. Lett. **143B**, 222 (1984).

[4]J. Kogut and L. Susskind, Phys. Rev. D **11**, 359 (1975); L. Susskind, Phys. Rev. D **16**, 3031 (1977); N. Kawamoto and J. Smit, Nucl. Phys. **B192**, 100 (1981).

[5]H. Kluberg-Stern, A. Morel, and O. Napoly, Nucl. Phys. **B220**, 447 (1983).

[6]H. Hamber and G. Parisi, Phys. Rev. D **27**, 208 (1983); J. P. Gilchrist, G. Schierholz, H. Schneider, and M. Teper, DESY Report No. DESY 84-021 (unpublished).

[7]A. Billoire, R. Lacaze, E. Marinari, and A. Morel, Centre d'Etudes Nucléaires de Saclay Report No. SPhT/84-92 (unpublished).

[8]Here we mean the electric charge as defined, for example, in Ref. 5.

[9]The two- and three-point functions satisfy an integrated Ward identity $A(0,0;t_z,t_x)=G(0;t_z)$ for $t_z>t_x$ which has also been noticed by the authors of Ref. 3.

[10]M. Creutz, Phys. Rev. D **21**, 2308 (1980).

[11]K. G. Wilson, Phys. Rev. D **10**, 2445 (1974).

[12]F. S. Beckman, in *Mathematical Methods for Digital Computers,* edited by A. Ralston and H. S. Wilf (Wiley, New York, 1960), p. 62.

[13]C. Bernard, T. Draper, K. Olynyk, and M. Rushton, Nucl. Phys. **B220**, 508 (1983).

[14]S. Gottlieb, P. B. Mackenzie, H. B. Thacker, and D. Weingartern, Phys. Lett. **134B**, 346 (1984).

[15]C. Bernard, in *Gauge Theory on a Lattice: 1984,* edited by C. Zachos, W. Celmaster, E. Kovacs, and D. Sivers (National Technical Information Service, Springfield, Va., 1984), p. 85.

[16]This is necessary since the fluctuations in the two- and three-point functions are correlated. See P. R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, New York, 1969) Chap. 4, for a discussion of error propagation.

[17]This is a reasonable phenomenological form at low-momentum transfer. See S. R. Amdendolia *et al.*, Phys. Lett. **146B**, 116 (1984).

[18]F. Gutbrod and I. Montvay, Phys. Lett. **128B**, 411 (1984).