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Dynamics-Independent Null Experiment for Testing Time-Reversal Invariance

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It is shown that it is impossible to construct, in any reaction in atomic, nuclear, or particle physics, a null experiment that would unambiguously test the validity of time-reversal invariance independently of dynamical assumptions.

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The testing of the validity of symmetry laws and, specifically, of time-reversal invariance has remained a center of attention in nuclear and particle physics. In the study of particle reactions, there are three conceivable ways to construct such tests. The first kind involves the measurement of any observable for a particular reaction (differential cross section, some polarization quantity, spin correlations, etc.) and its comparison with the corresponding observable for the time-reversed reaction. This method encounters considerable experimental difficulties, since the two reactions in question may be drastically different in terms of instrumentation, possibly carried out on completely different kinds of accelerators, and hence the relative normalization of the two measurements is very difficult to determine with more than a moderate degree of accuracy. Such tests, therefore, can at present be carried out at all but the lowest energies to an accuracy of 5%, and even at the lowest energies, for nuclear reactions,[1] only to an accuracy of 0.5%. A proposal for molecular beams[2] claims 0.1% or so. Even then, problems and uncertainties might arise, as shown by a specific instance in the past history of this approach.

The second method involves a reaction which is "self-conjugate" under time reversal, that is, a reaction which under time reversal goes into the same reaction. For such instances time-reversal constraints result in relationships between different observables.[3] For example, in pp elastic scattering, there is such a relationship between the simple polarization (P) and the simple asymmetry (A).

In such instances we at least deal with only one reaction, but even so, the techniques of the two experiments yielding the two measurements with a relationship between them may be quite different; thus problems of normalization still exist. Therefore such tests are also limited in accuracy, though perhaps not as much as those of the first type. The rough order of magnitude of 1% might be ascribed to the present-day accuracy of such tests.

The third type of test one could conceive of is a null experiment, that is, some observable which must vanish when time-reversal invariance holds. Precedents for such null experiments to test conservation laws exist, for example, in the case of parity conservation, where the component of the simple polarization in the reaction plane must be exactly zero, completely independent of dynamics, when parity conservation holds.[4] This fact has been used in one form or another to explore the tiny admixtures of weak-interaction effects in strong interaction phenomena. Experimentally, the very attractive feature of such tests is that a null experiment can be performed to a very high degree of accuracy, such as one part in 10^5, 10^6, or perhaps even

10^7, vastly superior to the accuracy levels indicated for the first two types of tests.

Such null experiments have not been carried out on particle reactions in connection with time-reversal invariance, and hence attention would naturally be focused on the formulation of experiments of this type for the testing of time-reversal invariance or measurement of tiny time-reversal-noninvariant effects.

The aim of the present note is to show that the above described objective is impossible. In other words, we will show that it is impossible, in any particle reaction, to construct a null experiment in a dynamics-independent way that would test time-reversal invariance by itself, i.e., to find an experimental observable the vanishing of which would allow us to conclude unambiguously that time-reversal invariance holds independently of dynamical assumptions.

It is important to emphasize the phrase *"dynamics independent."* If one is allowed to make some assumptions about dynamics, even if very mild ones (e.g. "no final-state interaction", or "a Hamiltonian of a certain form," or "interaction of a certain tensorial type," etc.), it may be possible to circumvent the above theorem. Indeed if for example the interaction is electromagnetic, the form of which we believe we know, we may very well find experimental quantities that must vanish. If, however, we want to be completely free of dynamical assumptions, and argue only on the basis of the structure of the reaction matrix as determined by general conservation laws, the above theorem holds.

The demonstration of the validity of this theorem will be carried out in the so-called optimal formalism[5] of polarization phenomena. This is the class of formalisms in which the relationship between observables and the bilinear combinations of amplitudes ("bicoms") is the simplest. The result, however, should be independent of the particular formalism we use.

We then start with an arbitrary reaction with two particles in and two particles out, which is self-conjugate under time reversal, that is, with elastic scattering of some sort. The spins of the particles in the reaction can be arbitrary.

Let us start with the reaction being constrained only by Lorentz invariance, that is, let us consider the case when we do not know whether other symmetry constraints (e.g., parity conservation) hold or not. The arbitrary observable in an arbitrary reaction (i.e., involving particles with arbitrary values of spins) can then be written[5] in terms of the amplitudes as

$$\begin{aligned}\mathscr{L}(uvH_p,UVH_P;\xi\omega H_q,\Xi\Omega H_Q)\\ =\tfrac{1}{2}KZ_1Z_2H_{wW}[&D(\xi,u;\Xi,U)D^*(\omega,v;\Omega,V)+wD(\omega,v;\Xi,U)D^*(\xi,u;\Omega,V)\\ &+pD(\xi,v;\Xi,U)D^*(\omega,u;\Omega,V)+pwD(\omega,u;\Xi,U)D^*(\xi,v;\Omega,V)\\ &+PD(\xi,u;\Xi,V)D^*(\omega,v;\Omega,U)+PwD(\omega,v;\Xi,V)D^*(\xi,u;\Omega,U)\\ &+pPD(\xi,v;\Xi,V)D^*(\omega,u;\Omega,U)+pPwD(\omega,u;\Xi,V)D^*(\xi,v;\Omega,U)], \qquad (1)\end{aligned}$$

where H_p is the real (imaginary) part for $p=+1$ (-1), $Z_1=1+pq-p+q$, $Z_2=1+PQ-P+Q$, $w=pq$, $W=PQ$, and $K=1$ unless $w=W=-1$, in which case $K=-1$. The rest of the notation is as follows: $D(\xi,u;\Xi,U)$ is the amplitude for the reaction $A+B\rightarrow C+D$, and the four indices u, U, ξ, and Ξ denote the spin projections, possibly chosen differently for each particle. The observables for this reaction are denoted by $\mathscr{L}(uvH_p,UVH_P;\xi\omega H_q,\Xi\Omega H_Q)$.

We will now demonstrate the nonexistence of null experiments for time-reversal invariance in four different cases: (a) Lorentz invariance only; (b) Lorentz invariance plus parity conservation; (c) Lorentz invariance plus identical-particle constraints; and (d) Lorentz invariance, parity conservation, and identical-particle constraints. The method of proof will be the same in all four cases. It consists of the following steps: (1) We exhibit the constraints among the amplitudes. (2) We substitute these constraints into Eq. (1). (3) We write down Eq. (1) for the time-reversal reaction. (4) We demand that the observable which is the result of step (2) be the negative of the observable which is the result of step (3), and show that this demand cannot be satisfied.

Since the proof involves the comparison of a reaction with its time-reversed partner, we clearly need to consider only observables in which $u=\xi$, $v=\omega$, $U=\Xi$, and $V=\Omega$. In such observables $p=q$, $P=Q$, $w=W=+1$, $Z_1=Z_2=2$, $K=1$, and H_{wW} is the real part. Furthermore, the constraints of time-reversal invariance in the transversity frame (which we will use for our proof) are[6]

$$D(c,a;d,b)=D^t(a,c;b,d). \qquad (2)$$

Inserting these constraints into Eq. (1) we get for the particular set of observables we need to consider

$$\mathscr{L}(uvH_p, UVH_P; uvH_p, UVH_P) = 4\,\mathrm{Re}[D(uu,UU)D^*(vv,VV) + D(vv,UU)D^*(uu,VV) + 2pD(uv,UU)D^*(vu,VV) + 2PD(uu,UV)D^*(vv,VU) + 2pPD(uv,UV)D^*(vu,VU)]. \quad (3)$$

Correspondingly, the expression for the time-reversed reaction, that is, for the reaction $C+D \rightarrow A+B$, is

$$\mathscr{L}(uvH_p, UVH_P; uvH_p, UVH_P) = 4\,\mathrm{Re}[D(uu,UU)D^*(vv,VV) + D(vv,UU)D^*(uu,VV) + 2pD(uv,UU)D^*(vu,VV) + 2PD(uu,UV)D^*(vv,VU) + 2pPD(uv,UV)D^*(vu,VU)]. \quad (4)$$

We are now ready to complete the proof in the four cases mentioned above.

(a) Lorentz invariance only.—Comparison of Eqs. (3) and (4) shows that each term in Eq. (3) is equal in magnitude and sign to the corresponding term in Eq. (4). Thus the two observables are not the negative of each other, as we would have to have if we have an observable vanishing under time-reversal invariance and thus forming a null experiment.

We have not included, in the above, observables in which the outgoing particle indices are not equal to the incoming particle indices. For such observables, the time-reversed reaction would involve a different observable, so that the criterion used above [Eq. (3) = $\pm$Eq. (4)] could not apply. The way for such an observable to vanish would be for the eight individual terms in Eq. (1) to cancel one another. We see, however, that, in particular, the first term cannot ever be canceled by any of the other terms. This conclusion holds even if the values of some or all of the indices are taken to be equal to each other. This can be easily established by an inspection of Eq. (1).

(b) Lorentz invariance together with parity conservation.—(1) The constraints of parity conservation on the amplitudes in the transversity frame are[6]

$$D(c,a;d,b) = \eta_{\mathrm{int}}(-1)^{a+b+c+d+2(S_A+S_D)} D^P(c,a;d,b). \quad (5)$$

We see, therefore, that about half of the amplitudes vanish identically, while half of them remain unchanged. (2),(3) The above constraints mean that in Eqs. (3) and (4) some (or all) of five terms vanish. The others, however, will continue to be equal to the corresponding terms in the other equation, as they did in the case of Lorentz invariance alone, and hence the proof given for Lorentz invariance alone applies here also.

(c) Lorentz invariance together with identical-particle constraints.—(1) Here we consider the special reaction $A+A \rightarrow C+C$ and impose the constraints stemming from the identical nature of the two initial and two final particles. These constraints on the amplitude are[6]

$$D(c,a;d,b) = (-1)^{2(a+c)+2(S_A+S_C)} D^I(d,b;c,a). \quad (6)$$

(2) We now substitute these constraints into Eqs. (3) and (4). We continue to get pairwise equality between the corresponding terms and hence we continue to fail to obtain an observable that vanishes as a result of time-reversal invariance and thus could serve as a null experiment.

(d) Lorentz invariance together with parity conservation and identical-particle constraints.—Compared to (c) above, the only difference here is that some of the amplitudes vanish and hence some of the five terms (or all of them) also vanish because of parity conservation. The remaining terms, however, will continue to equal the corresponding terms in the other equation both in magnitude and in sign; hence we still fail to have an observable which vanishes as a result of time-reversal invariance and thus could serve as a null experiment for testing the symmetry. This completes the proof.

It was necessary to repeat the proof in each of the above four cases because the more information we have on the validity of various conservation laws other than time-reversal invariance the more constrained the M matrix becomes. Hence, it would have been conceivable that an experiment becomes a null experiment when we know that, e.g., parity is conserved but is not a null experiment when that extra knowledge is not available.

The above result depends crucially on the experimental observables being *bilinear* functions of the amplitudes. There is one unique relationship that circumvents this type of dependence, namely, the optical theorem (based on probability conservation) in which something bilinear in amplitudes is related to something linear in amplitudes, although only in a special way (namely, utilization of the imaginary part of the forward reaction amplitude). This circumstance has been used by Stodolsky[7] to propose a test of time-reversal invariance which involves asymmetry measurements in *total* cross sections which could possibly be carried out to a quite high degree of accuracy.[8] Unfortunately, the quantity in Ref. 7 vanishes *either* when parity conservation holds *or* when time-reversal invariance holds, and hence this null experiment cannot be interpreted unambiguously as a test of time-reversal invariance. It is, in this respect, similar to the neutron's electric dipole moment, and hence is of no interest in the present context.

The significance of the result proven above is that it precludes us from obtaining, in the foreseeable future, a general, sensitive, and dynamics-independent test of time-reversal invariance in any atomic, nuclear, or particle reaction. It would seem, therefore, that the only way to increase the sensitivity of our time-reversal invariance tests in general types of reactions is to increase the accuracy of the first two types of time-reversal tests mentioned at the opening of this note. With their precision standing at the present at 0.1% or worse, it will be many years from now, if ever, that we can use them for the type of sensitivity in time-reversal invariance tests that we expect to provide something of importance.

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