Plasma Distribution Function in a Superthermal Radiation Field

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A plasma which is immersed in superthermal radiation suffers velocity-space diffusion which is enhanced by the photon-induced Coulomb-field fluctuations. This enhanced diffusion universally produces a power-law distribution $(E/E_0)^{-\kappa}$ at energy E larger than a critical energy E_0 where the transition energy E_0 and the power κ are inversely proportional to the photon-field intensity.

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When the Coulomb mean free path is smaller than the plasma size, the plasma is in thermal equilibrium with the electrostatic Coulomb field. The velocity-space diffusion coefficient then becomes proportional to the friction coefficient, a consequence of the fluctuation-dissipation theory,[1] and the equilibrium distribution becomes Maxwellian where the temperature is given by the proportionality constant. However, most plasmas we encounter both in the laboratory and in nature are not black bodies, and hence are not in equilibrium with photons. Although photons do not contribute directly to the velocity-space transport in a nonrelativistic plasma, superthermal radiation can induce fluctuations in the Coulomb field which produces enhanced velocity-space diffusion while the friction coefficient is not affected by photons. Consequently the enhanced diffusion coefficient is expected to be no longer proportional to the friction coefficient. In this Letter, we show that the proportionality constant is then given by the square of the test-particle velocity in the high-energy regime. This leads to a multiplicative stochastic process[2] in the velocity-space diffusion, and a power-law distribution function originates at the high-energy tail. The overall distribution function resembles the κ distribution[3] which is often used to fit the particle distribution function observed in space plasmas.[4]

We consider the Fokker-Planck equation to describe the evolution of the distribution function in the Coulomb field[5]:

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial \mathbf{v}}\cdot\left[\frac{1}{2}\mathbf{D}(\mathbf{v})\cdot\frac{\partial f}{\partial \mathbf{v}}-\mathbf{v}\gamma(\mathbf{v})f\right], \qquad (1)$$

where the diffusion tensor is given by

$$\mathbf{D}=D_{\parallel}\mathbf{I}+D_{\perp}(\mathbf{I}-\mathbf{v}\mathbf{v}/v^2).$$

The isotropic stationary solution of Eq. (1) is given by

$$f_0(v)\,dv=A\exp\left[\int v\frac{2\gamma(v)}{D_{\parallel}(v)}\,dv\right]4\pi v^2\,dv, \qquad (2)$$

where A is the normalization constant, and the longitudinal diffusion coefficient $D_{\parallel}$ is given by[5] $(\omega=\mathbf{k}\cdot\mathbf{v})$

$$D_{\parallel}(v)=\frac{q^2}{8\pi^3M^2v^2}\int d^3k\,\frac{\omega^2}{k^2}\langle E_l^2\rangle_{\mathbf{k},\omega}, \qquad (3)$$

where q, M, and v are the charge, mass, and the speed of the incident test particle, ω and $\mathbf{k}$ are the frequency and the wave vector of the Coulomb field E_l, and $\langle E_l^2\rangle$ is the spectral density. The frictional coefficient γ arises from the polarized Coulomb field due to the test particle and is given by the imaginary part of the longitudinal plasma dielectric function $\epsilon(\omega,\mathbf{k})$,

$$\gamma(v)=\frac{q^2}{8\pi^3\epsilon_0Mv^2}\,\mathrm{Im}\int d^3k\,\frac{\omega}{k^2}\frac{1}{\epsilon(\omega,\mathbf{k})}. \qquad (4)$$

When the plasma is in equilibrium with the Coulomb field, the fluctuation-dissipation theory[1] gives

$$\langle E_l^2\rangle_{\mathbf{k},\omega}=-\frac{2}{\epsilon_0}\frac{T}{\omega}\,\mathrm{Im}\frac{1}{\epsilon(\omega,\mathbf{k})}, \qquad (5)$$

where T is the plasma temperature in energy units. Hence, from Eq. (3),

$$D_{\parallel}^{\mathrm{eq}}(v)=-2T\gamma(v), \qquad (6)$$

and from Eq. (2) the stationary distribution becomes Maxwellian.

In the presence of a superthermal radiation field, however, the fluctuation in the Coulomb field is enhanced as a result of the induced scattering by photons. This leads to an enhancement of the diffusion coefficient.

$$D_{\parallel}(v)=D_{\parallel}^{\rm eq}(v)+D_{\parallel}^{\rm NL}(v), \quad (7)$$

where $D_{\parallel}^{\rm eq}$ is the diffusion coefficient due to the equilibrium field given by Eq. (5) and $D_{\parallel}^{\rm NL}$ is the diffusion coefficient due to the photon-induced longitudinal field, $E_l^{\rm NL}$,

$$D_{\parallel}^{\rm NL}(v)=\frac{q^2}{8\pi^3M^2v^2}\int d^3k\,\frac{\omega^2}{k^2}\langle(E_l^{\rm NL})^2\rangle_{\mathbf{k},\omega}. \quad (8)$$

The induced longitudinal field is given by the induced density perturbation of electrons,

$$E_l^{\rm NL}(\omega,\mathbf{k})=\frac{ien_0}{\epsilon_0\epsilon(\omega,\mathbf{k})k}\int f^{\rm NL}(\omega,\mathbf{k},\mathbf{v})\,d^3v, \quad (9)$$

where $f^{\rm NL}$ is the induced velocity distribution function of the electrons. If we write the electric field of the photons as

$$\mathbf{E}(t)=\mathbf{E}_0(\omega_0)e^{-i\omega_0 t}+\text{c.c.}, \quad (10)$$

$f^{\rm NL}$ is easily obtained from the Vlasov equation:

$$f^{\rm NL}(\omega,\mathbf{k},\mathbf{v})=\frac{(e/m)^2}{(\mathbf{k}\cdot\mathbf{v}-\omega)}[(\hat{\mathbf{k}}\cdot\mathbf{E}_0(\omega_0)\hat{\mathbf{k}}\cdot\mathbf{E}_l(\omega-\omega_0,\mathbf{k})]\hat{\mathbf{k}}\cdot\frac{\partial}{\partial\mathbf{v}}\left(\frac{\mathbf{k}\cdot\mathbf{v}-\omega}{\mathbf{k}\cdot\mathbf{v}-\omega+\omega_0}\hat{\mathbf{k}}\cdot\frac{\partial f_0}{\partial\mathbf{v}}\right)+(\omega_0\to-\omega_0), \quad (11)$$

where $\mathbf{E}_l(\omega-\omega_0,\mathbf{k})$ is the equilibrium longitudinal electric field whose spectral density is given by Eq. (5) and $\hat{\mathbf{k}}$ $(=\mathbf{k}/k)$ is the unit vector in the $\mathbf{k}$ direction. Substituting Eqs. (9) and (11) into Eq. (8), we obtain the photon-induced diffusion coefficient

$$D_{\parallel}^{\rm NL}(v)=\frac{q^2|r_0|^2T}{12\pi^3M^2v^2\epsilon_0}\int d^3k\frac{\omega^2\,\mathrm{Im}\,\epsilon_l(\omega-\omega_0,\mathbf{k})\,|\chi_e(\omega,\mathbf{k})-\chi_e(\omega-\omega_0,\mathbf{k})|^2}{(\omega-\omega_0)\,|\epsilon(\omega,\mathbf{k})|^2|\epsilon(\omega-\omega_0,\mathbf{k})|^2}+(\omega_0\to-\omega_0), \quad (12)$$

where

$$|r_0|^2=|eE_0/m\omega_0^2|^2=3\langle|e\hat{\mathbf{k}}\cdot\mathbf{E}_0/m\omega_0^2|^2\rangle \quad (13)$$

is the square of the direction-averaged excursion distance of electrons in the photon electric field, and χ_e is the electron susceptibility.

Let us evaluate $D_{\parallel}^{\rm NL}(v)$ first for induced low-frequency fluctuations such that $\omega-\omega_0\simeq 0$. The integration (12) can be evaluated by use of $d^3k=2\pi\times k_\perp\,dk_\perp\,dk_\parallel=(2\pi/v)k_\perp\,dk_\perp\,d\omega$, $\epsilon(\omega,\mathbf{k})\simeq\epsilon(\omega_0,\mathbf{k})$, $|\chi_e(\omega,\mathbf{k})-\chi_e(\omega-\omega_0,\mathbf{k})|^2\simeq k_{\rm D}^4/k^4$, and the sum rule

$$\frac{1}{\pi}\int_{-\infty}^{\infty}d\omega'\frac{1}{\omega'-\omega}\frac{\mathrm{Im}\,\epsilon(\omega',\mathbf{k})}{|\epsilon(\omega',\mathbf{k})|^2}=1-\mathrm{Re}\frac{1}{\epsilon(\omega,\mathbf{k})},$$

to give

$$D_{\parallel}^{\rm NL}\simeq\frac{|r_0|^2k_{\rm D}^2q^2T}{6\pi M^2v\epsilon_0}\frac{k_{\rm D}^4}{k_{\rm D}^2+\omega_0^2/v^2}\frac{1}{|\epsilon(\omega_0,0)|^2}, \quad (14)$$

where $k_{\rm D}$ $(=\omega_{pe}/v_{Te})$ is the Debye wave number. We note that when the radiation field has a frequency spectrum close to the plasma frequency, $\epsilon(\omega_0,0)\simeq 0$, and thus the photon-induced diffusion is enhanced. The contribution from induced high-frequency fluctuations $(\omega-\omega_0\sim\omega_{pe})$ can be evaluated in a similar way. The result gives $D_{\parallel}^{\rm NL}$ which is proportional to v^{-3} and can be shown to be much smaller than the contribution from the low-frequency fluctuation given in Eq. (14).

If the test particle is an *electron*, at $v>v_{Te}$ Eq. (14) gives

$$D_{\parallel}^{{\rm NL}(e)}=\frac{k_{\rm D}^2|r_0|^2}{6\pi}\frac{v_{Te}^3}{v}\omega_{pe}\frac{k_{\rm D}^3}{n_0}\frac{1}{|\epsilon(\omega_0,0)|^2}, \quad (15)$$

where n_0 is the electron number density. This should be compared with the longitudinal diffusion coefficient of electrons due to the equilibrium Coulomb field[6] at $v<v_{Te}$,

$$D_{\parallel}^{{\rm eq}(e)}=\frac{1}{2\pi}\frac{v_{Te}^5}{v^3}\omega_{pe}\frac{k_{\rm D}^3}{n_0}\ln\Lambda, \quad (16)$$

where $\ln\Lambda$ is the Coulomb logarithm. We see that $D_{\parallel}^{{\rm NL}(e)}$ becomes larger than $D_{\parallel}^{{\rm eq}(e)}$ at

$$(v/v_{Te})^2>3|\epsilon(\omega_0,0)|^2\ln\Lambda/k_{\rm D}^2|r_0|^2.$$

We note that the friction term $\gamma(v)$ is not modified because the induced fluctuations by the photon field are not correlated with the polarized field by the test particle. Therefore, the stationary distribution function of the test particle in the presence of the photon field is given by

$$f_0(v)\,dv=A\exp\left(\int\frac{2\gamma v}{D_{\parallel}^{{\rm NL}(e)}+D_{\parallel}^{{\rm eq}(e)}}dv\right)4\pi v^2\,dv$$
$$=A\left(1+\frac{v^2}{2\kappa v_{Te}^2}\right)^{-\kappa}4\pi v^2\,dv, \quad (17)$$

where

$$\kappa=\frac{3\ln\Lambda}{2|r_0|^2k_{\rm D}^2}|\epsilon(\omega_0,0)|^2 \quad (18)$$

and the normalization $\int_0^\infty f_0(v)4\pi v^2\,dv=1$ gives

$$A=\frac{2\kappa-3}{4\sqrt{2}(\pi\kappa)^{3/2}v_{Te}^3}\frac{\Gamma(\kappa)}{\Gamma(\kappa-\frac{1}{2})}$$

and Γ is the gamma function. We see that the stationary distribution function is given by a power law $f(v) \simeq v^{-2\kappa}$ at large velocity. In terms of the energy distribution $f_0(E)$, at $E > E_0$,

$$f_0(E) \simeq (E/E_0)^{-\kappa}, \tag{19}$$

with the transition energy E_0 being given by

$$E_0/E_{Te} \equiv v_0^2/2v_{Te}^2 = \kappa, \tag{20}$$

where E_{Te} is the electron thermal energy mv_{Te}^2. The distribution function obtained in Eq. (17) resembles the κ distribution often used to fit the particle data in space plasmas.[3]

Let us now consider an *ion* test particle. If the test-particle speed is larger than the electron thermal speed, we see from Eq. (14) that the result is identical to the case of an electron test particle and the power-law distribution results. This may be important when the electron temperature is much smaller than the ion temperature. On the other hand, if the ion test-particle speed is less than the electron thermal speed (yet larger than the ion thermal speed), Eq. (14) gives

$$D_{\parallel}^{\mathrm{NL}(i)} \simeq \frac{|r_0|^2 k_{\mathrm{D}}^2}{6\pi} \frac{v}{v_{Te}} v_{Ti}^2 \omega_{pe} \frac{k_{\mathrm{D}}^3}{n_0} \left(\frac{m_e}{m_i} \right)^{1/2}. \tag{21}$$

Since $D_{\parallel}^{\mathrm{eq}(i)}$ in this velocity range is also proportional to v, Eq. (21) shows that the total diffusion coefficient $D_{\parallel}$ is slightly modified in magnitude and the resultant stationary distribution remains Maxwellian.

In conclusion, we have shown that in the presence of nonequilibrium photons, the stationary distribution function of electrons and ions obeys a power law at velocity much larger than the electron thermal speed. The ion distribution function in the velocity range between the electron and the ion thermal speeds remains Maxwellian, although this result does not rule out a possible production of a power law distribution for ions in this velocity range by low-frequency electromagnetic waves.

Since most plasmas we encounter in the laboratory, in space, and in astrophysics are not black bodies, the present result applies inherently to most plasmas and is believed to play a crucial role in modifying the plasma transport coefficients.

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