Universality among Scalar Spin Systems

In a recent Letter, Barma and Fisher[1] have discussed the critical behavior of the scalar Klauder and double-Gaussian models in two dimensions. These models are both parametrized by $0 \leq y \leq 1$, so that $y=0$ corresponds to the Gaussian model and $y=1$ to the spin-$\frac{1}{2}$ Ising model. In a series of earlier papers, Baker and Johnson[2] have investigated the extent of universality of critical exponents for the s^4 continuous-spin Ising model. Baker and Johnson[2] concluded that, for two- and three-dimensional models, there were five cases of critical exponents: (i) $\tilde{g}_0=0$; (ii) $\tilde{g}_0 \neq 0$, a single-peaked distribution; (iii) border model, $\tilde{A}_0=0$; (iv) a double-peaked distribution; (v) Ising model, $\tilde{g}_0=\infty$. The critical exponents [e.g., γ in magnetic susceptibility, $\chi \propto (T-T_c)^{-\gamma}$] between adjacent classes should differ from each other.

As a numerical example, using a ten-term high-temperature series, Baker and Johnson[2] found that $\gamma_b = 1.96 \pm 0.07$ for the border model, case (iii), which differs markedly from the exact value $\gamma = 1.75$ for the Ising model.[3] Barma and Fisher[1] report confirmation of this result, $\gamma = 1.95 \pm 0.05$, for appropriate values of y in both the Klauder and double-Gaussian models, using a 21-term series. Within numerical error, the border model indices so computed[2] provide a realization of one of the members of the Friedan-Qiu-Shenker[4] catalog of all possible two-dimensional systems possessing conformal invariance and unitarity. Namely, from $m=5$ in their list, we find $\gamma = 2$, $\nu = \frac{20}{19}$, and $\eta = \frac{1}{10}$.

Also, Barma and Fisher[1] in a two-variable analysis find a "significant minority" indicating a critical point near the border model. Nevertheless, they "do not believe that a new type of criticality" is implied, but suggest instead "incipient tricriticality." The Griffiths-Hurst-Sherman inequalities[5] hold for the border model, and so χ is a monotonically decreasing function of the square of the magnetic field, H, at fixed temperature. This feature precludes tricritical behavior and certain types of "incipient tricriticality," such as is manifested by the one-dimensional Blume-Capel model.[6]

A tricritical point here means that $\partial^2\chi/\partial H^2$ is anomalously small. Presumably "incipient tricriticality" might mean that it is small in some sense. Simple scaling ideas lead to the idea that the critical-point limit of $(\partial^2\chi/\partial H^2)[\chi^2\chi^{(\delta+1)/(\delta-1)}]^{-1}$ should be finite, if the relation $H \propto M^\delta$, where M is the magnetization, is to hold on the critical isotherm. The direct application of Fisher's inequalities[7] shows that, aside from a lattice-dependent constant factor, this quantity is rigorously greater than g, the corresponding renormalized coupling constant for $\lambda\phi^4$ boson field theory. If the view of Barma and Fisher[1] is adopted that the border model does not have a different type of criticality, then g should take on its supposedly universal value $g^* \simeq 14.5$, and $\partial^2\chi/\partial H^2$ is well bounded away from zero and so not small.

In regard to the suggestion[1] that the border-model fixed point is unstable and therefore small values of y must flow to the (also unstable) Gaussian fixed point, we point out that $y=0$ is a point of nonuniform approach (as we[2] have discussed for the s^4 model), and the limit $y \to 0^+$ corresponds to the strong-coupling limit of ϕ_2^4 field theory (a stable fixed point) and is quite distinct from $y \equiv 0$. Therefore, small y is our[2] class (ii) and not the Gaussian model.

In our opinion, the moments of the single-spin distribution function $\tilde{M}_n$ of the Blume-Capel[6] model at the tricritical point, which behave for $n \geq 1$ as $\tilde{M}_{2n+2}/\tilde{M}_{2n} = E$, E a constant, when compared to those of the border model, which go asymptotically as $\tilde{M}_{2n+2}/\tilde{M}_{2n} \propto \sqrt{n}$, are sufficiently dissimilar that they cannot be relied on to give any indication of "incipient tricriticality," contrary to the suggestion of Barma and Fisher.[1]

It is our opinion that the idea of "incipient tricriticality" does not explain our previous observations, and the border model indeed illustrates a real limitation on critical exponent universality.

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George A. Baker, Jr.
J. D. Johnson
Theoretical Division
Los Alamos National Laboratory
University of California
Los Alamos, New Mexico 87545

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