Bifurcation and Dynamical Symmetry Breaking in a Renormalization-Group–Improved Field Theory

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We formulate the renormalization-group–improved theory for an $m_r \neq 0$ (massive) QCD Lagrangian. The Green's functions are parametrized in terms of two renormalization-group–invariant quantities, M_0 and Λ_c. The theory exhibits usual analyticity structure for every $M_0 > \Lambda_c e^{1/6}$. In the limit $M_0 = \Lambda_c e^{1/6}$, m_r becomes zero. The theory exhibits bifurcation in this limit, and chiral symmetry remains broken. $\langle \bar{\psi}\psi \rangle$ is calculated in this limit.

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In this paper, we report on the continuing study of the physical and mathematical properties of dynamical symmetry breaking for the vectorlike QCD theory. This study was initiated[1] in an attempt to apply the Nambu–Jona-Lasino[2] mechanism to the renormalizable QCD. Our study has been carried out within the context of a renormalization-group[3] (RG) improved field theory, with the $\overline{\text{MS}}$ scheme[4] used exclusively for regularization. In this scheme, we write the complete Lagrangian as

$$L_{\text{tot}} = L_{\text{tree}} + L_{\text{counter}}, \tag{1}$$

with

$$L_{\text{tree}} = -\bar{\psi}\gamma\cdot\partial\psi - m_r\bar{\psi}\psi + ig_r\bar{\psi}\gamma_\rho T^a\psi A^a_\rho(\mu^2/4\pi)^{\epsilon/4} + \ldots \tag{2}$$

and

$$L_{\text{counter}} = -(Z_2-1)\bar{\psi}\gamma\cdot\partial\psi - (Z_m Z_2 - 1)m_r\bar{\psi}\psi + i(Z_1-1)g_r\bar{\psi}\gamma_\rho T^a\psi A^a_\rho(\mu^2/4\pi)^{\epsilon/4} + \ldots, \tag{3}$$

where $n = 4-\epsilon$, the Z's represent the $1/(\epsilon)$ counterterms needed to define finite, renormalized n-point Green's functions, and μ is the 't Hooft renormalization scale. To one-loop RG accuracy, and in Landau gauge,[5] $Z_2 = 1$, while ($T^aT^a = C_f I$, where I is the unit matrix)

$$Z_m = (1 + b\lambda_r/\epsilon)^{-\alpha}, \quad \alpha = 6C_f/b. \tag{4}$$

In (4), we have written λ_r for $g_r^2/16\pi^2$, and it satisfies the one-loop RG equation

$$\mu\,\partial\lambda_r/\partial\mu = -b\lambda_r^2 \equiv \beta_\lambda. \tag{5}$$

To the same one-loop RG accuracy, m_r satisfies the RG equation

$$\mu\,\partial m_r/\partial\mu = -6C_f\lambda_r m_r \equiv \beta_m. \tag{6}$$

All the n-point Green's functions calculated with the Lagrangian (1) now yield finite answers that depend on p^2, λ_r, m_r, and μ^2. Consider the chiral-flip (X.f.) part of the quark two-point function. It may be written in the form, accurate to one-loop RG,

$$\Gamma_{\text{X.f.}} = -im_r(\lambda_r/\lambda_p)^{-\alpha} \equiv -iM_p, \tag{7}$$

and will be in accord with renormalization as long as

$$\mu\frac{d}{d\mu}\lambda_p \equiv \left(\mu\frac{\partial}{\partial\mu} + \beta_\lambda\frac{\partial}{\partial\lambda_r} + \beta_m\frac{\partial}{\partial m_r}\right)\lambda_p = 0. \tag{8}$$

λ_p may be obtained iteratively from the equation ($x^{(n)} = 1/\lambda_p$ in the nth iteration, with $x^{(0)} = 1/\lambda_r$)

$$x^{(n+1)} = f(x^{(n)}, p^2, m_r^2, \mu^2, \lambda_r), \tag{9}$$

where

$$f(x,p^2,m_r^2,\mu^2,\lambda_r) = \lambda_r^{-1} + \tfrac{1}{2}b\int_0^1 dz \ln\{[m_r^2(\lambda_r x)^{-2\alpha} + Zp^2]/\mu^2 e^{1/3}\}. \tag{10}$$

The usual MS-renormalized perturbative series for the two-point function,[6] to one-loop RG accuracy, can be generated systematically by this iteration, with λ_p being the $n \to \infty$ limit of $x(n)$.

Since λ_p is a RG invariant, Eq. (9) and, in turn, Eq. (7) may be rewritten entirely in terms of RG invariants. Apart from the QCD invariant cutoff, Λ_c, defined to one-loop RG accuracy by the equation

$$\lambda_r^{-1} + \tfrac{1}{2}b\ln(\Lambda_c^2/\mu^2) = 0, \tag{11}$$

we also introduce the independent parameter, M_0, defined by

$$m_r = M_0[1 + \tfrac{1}{2} b \lambda_r \ln(M_0^2/\mu^2 e^{1/3})]^{+\alpha}, \tag{12}$$

which by (11) may be rewritten as

$$m_r = M_0[\tfrac{1}{2}\lambda_r b \ln(M_0^2/\Lambda_c^2 e^{1/3}]^{\alpha}, \quad M_0 \geq \Lambda_c e^{1/6}. \tag{13}$$

In order that m_r be real, M_0 has to be greater than or equal to the critical value listed in Eq. (13).

We can now write Eq. (7) directly as the implicit equation ($M_c^2 = M_0^2 e$)

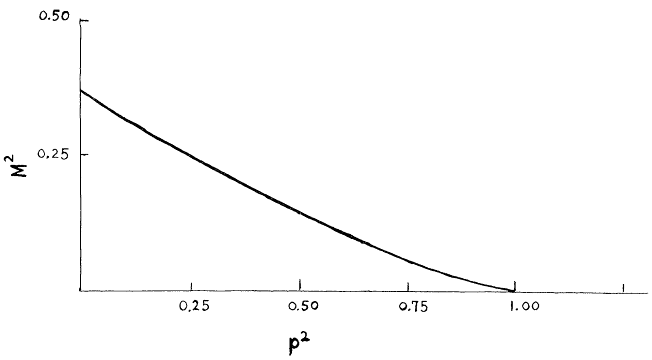

FIG. 1. M_p as a function of p^2 in the range $0 \leq p^2 \leq \bar{M}_c^2$, in units of $\bar{M}_c^2$.

$$M_p = M_0 \left\{ 1 + \frac{\lambda_0 b}{2} \ln \left[\frac{M_p^2}{M_c^2} \left(1 + \frac{p^2}{M_p^2} \right)^{M_p^2/p^2 + 1} \right] \right\}^{-\alpha}, \tag{14}$$

$$\lambda_c^{-1} = \tfrac{1}{2} b \log(M_c^2/\Lambda_c^2 e^{1/3}) \geq 0, \tag{15}$$

where in Eq. (14) we have further performed the integration called for in (10).

For every λ_0 in the range

$$0 \leq \lambda_0 < \infty,$$

M_p is an analytic function of p^2 in the cut p^2 plane, with the cut along the negative p^2 (timelike) axis. The location of the branch point depends on the parameter λ_0, being given by

$$p^2(\text{branch point}) = -M_0^2 e[1 - (2/b\lambda_0)(1 - e^{-1/2\alpha}) + O(\lambda_c^{-2})]. \tag{16}$$

At the branch point, $M_p^2 + p^2$ vanishes, so that the propagator actually has a pole at the branch point.

The high-energy behavior of M_p may easily be obtained from Eq. (14), and is given by

$$M_p \underset{p^2 \to \infty}{\longrightarrow} M_0[\tfrac{1}{2}\lambda_c b \ln(p^2/M_c^2)]^{-\alpha}. \tag{17}$$

Behavior (17) is well known from earlier studies of renormalization-group theory. However, at the critical limit when M_0 equals $\Lambda_c e^{1/6}$, λ_0 becomes infinite and the right-hand side of (17) vanishes.

In fact, at the critical limit, bifurcation sets in.[7] For $p^2 \geq \bar{M}_c^2$ ($= \Lambda_c^2 e^{4/3}$, the critical mass squared), M_p tends to the chiral symmetric phase, i.e., M_p vanishes. For $p^2 < \bar{M}_c^2$, M_p opts for the chiral-*broken* phase, i.e., M_p is not equal to zero (see Fig. 1).

In the complex p^2 plane only the region inside as well as along the boundary of the "fig leaf" will have M_p not equal to zero. This domain may be deduced from Eq. (14) by setting of the argument of the logarithm equal to one. As before, at the timelike point $p^2 = -\bar{M}_c^2$, $M_p^2 + p^2$ vanishes. The propagator, at the bifurcation limit, is analytic inside the domain, but has a singularity at the timelike cusp node shown in Fig. 2.

With this M_p, we are finally in the position to calculate $\langle \bar{\psi}\psi \rangle$.[8] A careful analysis of the renormalization properties shows that, for all λ_0,

$$\langle \bar{\psi}_\alpha \psi_\alpha \rangle = \lim_{x \to y} -\langle T[\psi_\alpha(x) \bar{\psi}_\alpha(y)] \rangle \tag{18}$$

$$= -4N_c \int \frac{dp^2}{16\pi^2} \frac{p^2 M_p}{p^2 + M_p^2}. \tag{19}$$

In (19) we have already continued to the Euclidean p^2 and integrated over the angles. For λ_0 finite, the integral produces infinities, a situation which complicates the renormalization of $\langle \bar{\psi}\psi \rangle$. This will be discussed elsewhere. In the critical limit, however, $\lambda_0 \to \infty$, the integral is finite, and $\langle \bar{\psi}\psi \rangle$ does not need renormalization. Indeed, in the bifurcation limit, the vacuum expectation value, apart from N_c (the dimensionality of the fermion representation), is universal. The integral can be performed numerically, with the result that

$$\langle \bar{\psi}\psi \rangle = -0.0398 N_c \Lambda_c^3. \tag{20}$$

To compare with current-algebra determinations, it is necessary to multiply Eq. (20) by $M (= \Lambda_c e^{1/6})$ to find

$$M \langle \bar{\psi}\psi \rangle = -0.0470 N_c \Lambda_c^4. \tag{21}$$

An accepted value of 0.15 GeV for Λ_c yields a value of 9.24×10^{-5}, which is remarkably close to the current-

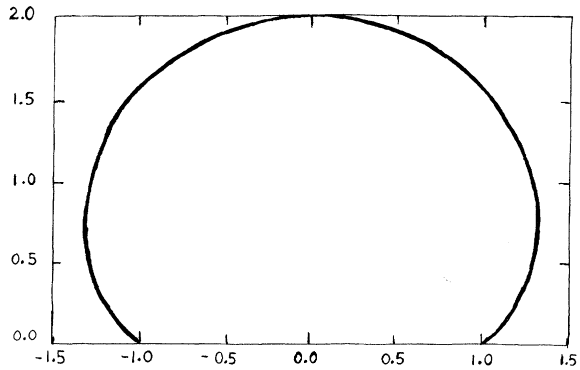

FIG. 2. Shown here is the upper half of the "fig leaf" domain. The timelike region is along the negative real axis. Both axes are in units of $\overline{M}_c^2$. The lower half of the domain is obtained by reflection about the real axis.

algebra estimate of 8.8×10^{-5} for the up quark.[9]

Finally, we make a few concluding remarks:

(i) The analysis given here is a generalization of our earlier work on the RG analysis of the Nambu–Jona-Lasinio mechanism for QCD. We have carried out the discussion here to one-loop RG accuracy only for simplicity. The generalization to two-loop accuracy will be discussed elsewhere.

(ii) We emphasize once again the universal character of the bifurcation limit. The function shown in Figs. 1 and 2 does not depend on the Clebsch-Gordan coefficients of the group; it only depends on the spin-1 nature of the gluon that has been exchanged. This analysis can therefore be applied to the class of hypercolor theories.

(iii) An open question for the moment is how the temperature can affect the bifurcation limit.

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[1]L. N. Chang and N. P. Chang, Phys. Rev. D **29**, 312 (1984); N. P. Chang and D. X. Li, Phys. Rev. D **30**, 790 (1984).

[2]Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961), and **124**, 246 (1961).

[3]M. Gell-Mann and F. E. Low, Phys. Rev. **95**, 1300 (1954); E. C. G. Stückelberg and A. Peterman, Helv. Phys. Acta **26**, 499 (1953); C. G. Callan, Phys. Rev. D **2**, 1541 (1970); K. Symanzik, Commun. Math. Phys. **18**, 227 (1970).

[4]G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972); G. 't Hooft, Nucl. Phys. **B61**, 455 (1973).

[5]In this paper, we choose only to work in the Landau gauge where wave-function renormalization effects vanish. In Chang and Li, Ref. 1, the full gauge dependence of the $\overline{\mathrm{MS}}$-perturbative series has been discussed and the gauge independence of the dynamically generated mass has been demonstrated.

[6]For an exposition on the $\overline{\mathrm{MS}}$ renormalization procedure, see J. C. Collins, *Renormalization* (Cambridge Univ. Press, Cambridge, England, 1984). The actual $\overline{\mathrm{MS}}$-perturbative series for the fermion two-point function has been worked out to two loops in Ref. 1.

[7]For a clear exposition on bifurcation theory, see Chow Shui-Nee, *Bifurcation Theory and Dynamical Systems,* Lecture Notes Series No. 22, prepared by C. J. Chapman (Mathematics Department, National University of Singapore, Singapore, 1984).

[8]We thank D. X. Li for discussions during our earlier attempt at the calculation of $\langle\bar{\psi}\psi\rangle$.

[9]For reviews on current algebra see, e.g., S. L. Adler and R. F. Dashen, *Current Algebra and Application to Particle Physics* (Benjamin, New York, 1968); H. Pagels, Phys. Rep. **16**, 221 (1975). For the latest work and earlier references on QCD calculation of $\langle\bar{\psi}\psi\rangle$, see V. Elias and M. D. Scadron, Phys. Rev. D **30**, 647 (1984); S. Narison, CERN Report No. CERN-TH 3990/84 (unpublished); S. L. Adler, and A C. Davis, Nucl. Phys. **B244**, 469 (1984); P. Castorina and S. Y. Pi, Phys. Rev. D **31**, 411 (1985); A. Le Yaouanc, L. Oliver, S. Ono, O. Pene, and J. C. Raynal, Phys. Rev. Lett. **54**, 506 (1985); and Y. P. Kuang and Y. P. Yi, Phys. Energias Fortis Phys. Nucl. **6**, 488 (1978).