Isospin Dependence of s-wave Pion Absorption in the $A = 3$ Nuclei

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On the basis of microscopic models for s-wave pion absorption on nucleon pairs the contribution of $T=1$ pairs compared to $T=0$ pairs should be strongly suppressed. A direct consequence, not previously noted, is that the $1s$ absorption widths in the pionic atoms ^{3}H and ^{3}He should be equal, aside from renormalization due to the different π^--atomic wave-function length scales. The ^{3}H width, which is not directly observable, is thus predicted to be 2.25 ± 0.55 eV. It is shown that this value is consistent with observed branching ratios for radiative capture in the mirror pair.

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Reliable calculation of medium-energy (100–250 MeV) pion-absorption cross sections in complex nuclei has recently become possible because of the work of Lee and Ohta.[1] In their model, pions interacting in p waves relative to nucleon pairs form $\frac{3}{2},\frac{3}{2}$ isobars which undergo pionless decay through interactions with the nuclear medium. At lower energies where the nucleus becomes somewhat more transparent to pions, the absorption of pions in s waves relative to nucleon pairs becomes significant. The spin-isospin dependence of this interaction is the subject of this Letter.

At each of the major "meson factories"[2–4] complete kinematical determinations of post-pion-absorption nuclear states with ^{3}He targets have been carried out. The analysis of the low-energy portions of these current experiments is expected to constrain tightly the parameters in any particular s-wave pion absorption model. For example, the Backenstoss[3] experiment with stopped π^- reveals that those final states from absorption on spin-singlet, $T=1$, s-state nucleon pairs are reduced by at least an order of magnitude compared to corresponding states on spin-triplet, $T=0$ pairs. However, in this paper we restrict ourselves to a comparison of $1s$ absorption widths in the mirror pair ^{3}He and ^{3}H and point out the implications for the two-nucleon absorption model parameters.

A pionic-atom wave function contains mainly low-momentum components and so the momentum-energy transfer arguments in favor of two-nucleon absorption are particularly valid. In the lowest order the pion-nucleus scattering matrix is

$$T = (-4\pi/\epsilon_\pi)\sum_i [b_0 + b_1 \mathbf{t}\cdot\boldsymbol{\tau}_i]\delta(\mathbf{r}-\mathbf{r}_i), \tag{1}$$

where the isoscalar and isovector scattering lengths are[5] $b_0 = (-0.001 \pm 0.007)\mu^{-1}$ and $b_1 = (-0.090 \pm 0.006)\mu^{-1}$, respectively. In the two-nucleon absorption model, the pion scatters off shell, giving about half its rest energy to the struck nucleon, and propagates through the nucleus until it is absorbed on a second nucleon through the interaction $G/4\pi\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\tau}\cdot\boldsymbol{\phi}(\mathbf{x})$. Summing over the possible permutations of the nucleons, we can write the two-nucleon effective s-wave absorption operator as[6,7]

$$H_{ab} = \psi_\pi(0)\{G_0(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\nabla(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)\cdot\mathbf{a}' + (\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\nabla[2G_1(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)\cdot(\mathbf{a}'\times\mathbf{t}) - G_0(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)\cdot\mathbf{a}']\}f(\mathbf{x}), \tag{2}$$

where $f(\mathbf{x})$ is the static pion propagator between nucleons 1 and 2, $\psi_\pi(0)$ is the $1s$ pion wave function evaluated at the origin, and $G_i = Gb_i$. The annihilation operators a'^q are related to the standard annihilation operators a^q by

$$\boldsymbol{\tau}\cdot\mathbf{a}' = \sum_q (-1) q_\tau q_{a'} - q = \tau^0 a^0 + \tau^{+1} a^{+1} + \tau^{-1} a^{-1},$$

and have been introduced so that all operators in Eq. (2) have the same rotation-group properties. Note that the first term, proportional to G_0, is nonzero only when acting on a $^1S_0$ nucleon pair state while the second is nonzero only for $^3S_1$ pair states. Since the isoscalar pion-nucleon scattering length b_0 is at least an order of magnitude smaller than the isovector scattering length b_1 (in the soft-pion limit[8] $b_0/b_1 = 0$), we have $G_0/G_1 \ll 1$ and absorption takes place essentially only on $^3S_1$ pairs. To the extent that ^{3}He and ^{3}H contain the same number of deuteron-like np pairs the absorption rates in the two nuclei, aside from the renormalization of $\psi_\pi(0)$, should be the same.

The absorption width for a particular isospin state of the pion+nucleus is defined as

$$\Gamma_{ab}^{1s}(T) = 2\pi\sum_n \delta(E_n - E_0 - m_\pi)[\langle\psi_{1/2}|\times\langle 1|]_T^t \sum_{i<j} H_{ab}^\dagger(ij)|\psi_n\rangle\langle\psi_n|P_T\sum_{k<l} H_{ab}(kl)[|\psi_{1/2}\rangle\times|1\rangle]_T^t. \tag{3}$$

The isospin projection operator $P_T = \sum_r |\eta_T^t([f]r)\rangle\langle\eta_T([f]r)|$ is a sum over all basis states of the three-nucleon representation of SU(2) appropriate to a particular T value. In terms of these isospin widths, the widths for the mirror pair can be expressed as

$$\Gamma^{1s}_{ab}(^3\mathrm{He}) = \tfrac{2}{3}\Gamma^{1s}_{ab}(T=\tfrac{1}{2}) + \tfrac{1}{3}\Gamma^{1s}_{ab}(T=\tfrac{3}{2}), \quad \Gamma^{1s}_{ab}(^3\mathrm{H}) = \Gamma^{1s}_{ab}(T=\tfrac{3}{2}).$$

Approximating the three-nucleon ground-state wave function by its completely symmetric part and neglecting any spin-isospin dependence of the nuclear Green's function we find that the difference between the two isospin widths is proportional to G_0^2:

$$\Gamma^{1s}_{ab}(T=\tfrac{1}{2}) - \Gamma^{1s}_{ab}(T=\tfrac{3}{2}) = 2\pi G_0^2\,\{24[\langle \mathrm{direct}\rangle + \langle \mathrm{exchange}\rangle]\}. \tag{4}$$

This is much smaller than contributions from those states of lower space symmetry which have been neglected. The direct and exchange terms are integrals involving the three-body Green's function, the pion propagator, and the ground-state space function

$$\begin{aligned}\langle \mathrm{direct}\rangle &= \psi_\pi^2(0)\langle \Phi(1'2'3')\nabla'_{12}\cdot f(\mathbf{x}'_{12})[-(i/\pi)\mathrm{Im}G(\mathbf{x}'_i;\mathbf{x}_i)]\nabla_{12}f(\mathbf{x}_{12})|\Phi(123)\rangle,\\ \langle \mathrm{exchange}\rangle &= \psi_\pi^2(0)\langle \Phi(1'2'3')\nabla'_{12}\cdot f(\mathbf{x}'_{12})[-(i/\pi)\mathrm{Im}G(\mathbf{x}'_1;\mathbf{x}_1)\nabla_{13}f(\mathbf{x}_{13})|\Phi(123)\rangle.\end{aligned} \tag{5}$$

Clearly, equality of the two isospin widths implies equality of the two physical $1s$ widths. Unfortunately, a direct experimental comparison of the two widths is not possible. While the ^{3}He total width[9] (absorption+charge exchange+radiative capture) and the individual branching ratios[10] have been measured, the effect of absorption on K x-ray widths is too small to measure in ^{3}H. The ^{3}H absorption width must be obtained indirectly by combination of measured values of the branching ratios for absorption and radiative capture[11] with theoretical estimates of the total radiative capture rate, $\Gamma^{1s}_{ab}(^3\mathrm{H})$ = (percent absorbed)/(percent radiatively captured) $\times\,\Lambda^{1s}_{\pi\gamma}$(calculated). For π^- on ^{3}He, Phillips and Roig[12] have calculated partial radiative capture rates for transitions to one-, two-, and three-body nuclear states. Unfortunately, their rate for transitions to the three-body final state which could be used to estimate $^3\mathrm{H}(\pi^-,\gamma)3n$ is less than half of the measured rate[10] for $^3\mathrm{He}(\pi^-,\gamma)p2n$ and may be an unreliable number. For that reason, we have used a closure-plus-correction method to estimate the total radiative capture rates in ^{3}H and ^{3}He. The unrenormalized $1s$ ^{3}H absorption width is shown, within broad experimental error, to be the same as the $1s$ ^{3}He width.

The impulse approximation applied to radiative pion capture has been very successful[13] and leads to expressions for transition rates formally the same as those for muon capture. Bernabeu,[14] who was interested in calculating total muon-capture rates for muonic atoms, noted that the simple closure approximation to the sum over final states leads to rates which are rapidly varying functions of the average neutrino energy. If the average energy is thought of as the starting point for a Taylor series expansion, then the closure result is the lowest-order term and the sum over the additional terms must lead to a result independent of the neutrino (or in our case photon) energy. He found that by including correction terms through the double commutator of the muon-capture transition operator with the nuclear Hamiltonian he could obtain a theoretical rate which went through a maximum; the stationary value of the rate around the maximum was then taken as the best estimate of the total rate.

Following Bernabeu's notation, the reduced rate for radiative π^- capture is defined as

$$\Lambda^{1s}(a\to b) = K(k_{ab})\phi_\pi^2(Z)\langle R_{1s}^2\rangle Z\Lambda_r^{1s}(a\to b), \tag{6}$$

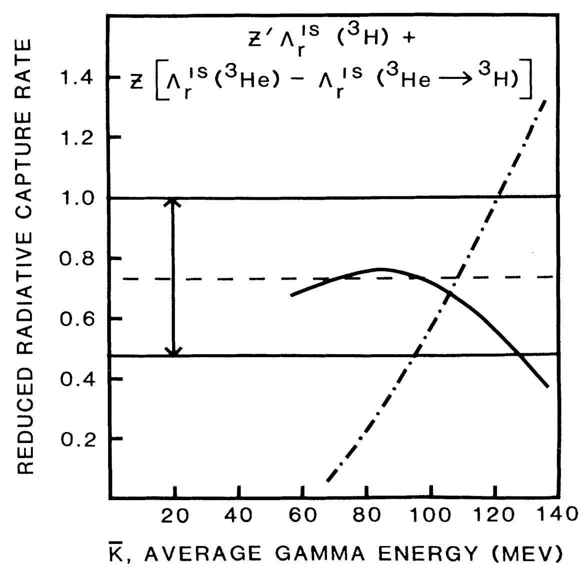

FIG. 1. The sum of the reduced rates for radiative pion capture to continuum states of the three-body nuclei calculated from Eq. (8). The solid curve shows the full calculation and the dot-dashed curve the closure approximation from Eq. (7), but with the ground-state to ground-state transition removed. The dashed horizontal line is the same quantity extracted from experimental data with the assumption that $x=0$ in Eq. (10). An experimental error of about 30% has been assigned, due primarily to large experimental errors in the total ^{3}He width and the radiative-capture branching ratio in ^{3}H.

where

$$K(k)=4\pi\frac{(1+k/M)(1+m_\pi/M)2A^2}{(1+k/M_A)m_\pi^2},$$

$$Z\Lambda_r(a\to b)$$
$$=\frac{k_{ab}}{\langle R_{1s}^2\rangle m_\pi}\times\frac{1}{4}\sum_{\lambda,S_a,S_b}|\langle b|\sum_j O_{j\lambda}(\mathbf{k}_{ab})|a\rangle|^2,$$

and

$$O_\lambda(\mathbf{k})=\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\epsilon}}_\lambda^*\tau^-R_{1s}(r)e^{-i\mathbf{k}\cdot\mathbf{r}},$$

where $\Lambda^{1s}(a\to b)$ is the partial radiative width, k_{ab} is the photon energy, A is the strength of the s-wave photoproduction amplitude,[13] and $R_{1s}(r)$ is the ratio of the radial wave function, modified by strong interaction and finite size effects, to the $1s$ Coulomb point function evaluated at the origin, $\phi_\pi(Z)$. The total reduced width for radiative capture is the sum of all the partial widths. For the ^{3}H, ^{3}He mirror pair the closure estimate for the total reduced widths is[15]

$$Z\Lambda_r(Z,\bar{k})=(Z\bar{k}/m_\pi)[1-\beta(Z)F_{\rm rel}(\bar{k})], \tag{7}$$

where $\beta(1)=1$, $\beta(2)=\frac{1}{2}$. The strong k dependence is evident. $F_{\rm rel}(k)$ is the form factor for an np pair in the ground state. The method of finding corrections to closure elucidated by Bernabeu can be rigorously applied only to isoscalar nuclei. We can sum the reduced rates over both members of the isospin doublet to restore symmetry in which case no error is made in introducing the double commutator. The sum of the total reduced widths for radiative capture less the transition $^3\text{He}+\pi^-\to{}^3\text{H}+\gamma$ takes the form

$$\Lambda_r^{1s}(^3\text{H})+2[\Lambda_r^{1s}(^3\text{He})-\Lambda_r^{1s}(^3\text{He}\to{}^3\text{H})]=[1+\omega d/d\bar{k}][\Lambda_r^{1s}(1,\bar{k})+2\Lambda_r^{1s}(2,\bar{k})]$$
$$-\frac{d}{d\bar{k}}\left\{\frac{\bar{k}}{m_\pi}\times\frac{1}{2\times4}\sum_{t,m,\lambda}\frac{1}{4}\langle m,t|[[O_\lambda^\dagger,H],O_\lambda]+[O_\lambda^\dagger,[H,O_\lambda]]|m,t\rangle\right\}-\frac{k}{m_\pi}F_m^2(k). \tag{8}$$

Here $\bar{\omega}$ is the excitation energy of the nucleus corresponding to the expansion photon energy $\bar{k}$, H is the nuclear Hamiltonian, and $F_m(k)$ is the magnetic form factor[16] for ^{3}He evaluated at the photon energy corresponding to the $^3\text{He}\to{}^3\text{H}$ transition.

The kinetic-energy contribution to the double commutator is trivial and the potential terms have been reduced to integrals for local central, tensor, and spin-orbit potentials by Bernabeu and Cannata.[17] In order to obtain numerical results we have used the potentials and the two-parameter Gaussian ground-state wave function of Kanada, Kaneko, and Tang[18] which are consistent with the average three-body form factor and the Coulomb-energy difference between ^{3}H and ^{3}He. A sum of Gaussian functions is also consistent with our approximation for the relative form factor $F_{\rm rel}(k^2)=F_m(3k^2)$.

The theoretical estimate for the sum of the total reduced rates for radiative pion capture in ^{3}H and ^{3}He was calculated from Eq. (8) and is represented by the solid curve in Fig. 1. It attains a maximum value of 0.76 at about $k=87$ MeV. The closure result is shown as the dot-dashed curve.

If the $1s$ pion absorption width for ^{3}H is expressed in terms of that for ^{3}He through

$$\Gamma_{ab}^{1s}(^3\text{H})=\frac{\phi_\pi^2(1)}{\phi_\pi^2(2)\langle R_{1s}^2\rangle}(1+x)\Gamma_{ab}^{1s}(^3\text{He}), \tag{9}$$

where the parameter x characterizes the departure from equality of the widths, then the sum of the reduced radiative rates is proportional to the sum of the ratios of radiative capture widths to absorption widths plus a correction term depending on x:

$$\Lambda_r^{1s}(^3\text{H})+2[\Lambda_r^{1s}(^3\text{He})-\Lambda_r^{1s}(^3\text{He}\to{}^3\text{H})]$$
$$=\frac{\Gamma_{ab}^{1s}(^3\text{He})}{K(\bar{k})\phi_\pi^2(2)\langle R_{1s}^2\rangle}\left\{\frac{\Lambda^{1s}(^3\text{H})}{\Gamma_{ab}^{1s}(^3\text{H})}+\frac{[\Lambda^{1s}(^3\text{He})-\Lambda^{1s}(^3\text{He}\to{}^3\text{H})]}{\Gamma_{ab}^{1s}(^3\text{He})}\right\}+\frac{x}{1+x}\Lambda_r^{1s}(^3\text{H}). \tag{10}$$

The experimental values of the width ratios inside the braces were extracted from the branching ratios for radiative capture and absorption listed in Table VIII of Ref. 13. The ^{3}He tabulated values were corrected slightly to remove contributions from p-wave processes. A closure estimate of the $2p$ radiative capture rate was combined with Landua's value[19] for the total $2p$ level width of $0.7\pm0.2\times10^{-3}$ eV and a 0.1 weighting factor for p-state reactions to correct the branching ratios. The sum of the experimental width ratios was found to be 0.149 ± 0.027. Taking $\Gamma_{ab}^{1s}(^3\text{He})=19.1\pm4.8$ eV (68.3% of the measured total width[19] of the ^{3}He $1s$ level, 28 ± 7 eV) and $\langle R_{1s}^2\rangle=1.06$ (the result of fitting the negative-energy shift[9] of the $1s$ level with a complex square well[20]) and setting $x=0$ we find that the "experimental" sum of the total reduced rates for radiative capture comes out to be

$$\{\Lambda_r^{1s}(^3\text{H})+2[\Lambda_r^{1s}(^3\text{He})-\Lambda_r^{1s}(^3\text{He}\to{}^3\text{H})]\}_{x=0}=0.73\pm0.27, \tag{11}$$

in excellent agreement with the stable value of the theoretical result. The large errors quoted in Eq. (11) come chiefly from the 25% uncertainty in the ^{3}He absorption width.

In summary, we have argued that the spin and isospin dependence of the two-nucleon pion absorption operator predicts equality of the $1s$ pionic absorption widths for ^{3}H and ^{3}He if the pion-nucleon scattering lengths incorporated in the model are set at their free-particle values. In principle, the $1s$ absorption width for ^{3}H can be most accurately determined by combination of the experimental branching ratio for radiative capture of pions on ^{3}H of $(4.5 \pm 0.8)\%$[11] with a three-body calculation of the total radiative rate. The results of such an approach are reported in Table IX of Ref. 13 to be $\Gamma_{ab}^{1s}(^3\mathrm{H}) = 1.02 \pm 0.18$ eV. This value, based on the work of Phillips and Roig[12,21] is probably too low for reasons already discussed and disagrees with our prediction of 2.25 ± 0.55 eV obtained from a Coulombic renormalization of the ^{3}He width. The latter value is favored by the results of our sum-rule calculation of the radiative capture rates.

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