Outward Effective Mass of Quark and Baryon Magnetic Moments

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(Received 5 March 1985)

The valence quark a is postulated to respond, when probed by a soft photon, with the outward effective mass defined by the relation $m_a^B = M_B - m_b - m_c$, where M_B is the physical mass of baryon B, and m_b and m_c are the effective masses of the spectator valence quarks b and c in the usual sense. The observed characteristics of baryon magnetic moments are shown to be well described in the naive scheme of the additive quark model if the new notion of the outward effective mass of quarks is incorporated.

PACS numbers: 13.40.Fn, 12.35.Ht, 14.20.–c, 14.80.Dq

Recent progress in the accurate measurements[1–8] of baryon magnetic moments has confirmed the existence of considerable discrepancies between the experimental results and the theoretical predictions based on the additive quark model with SU(6) spin-flavor wave functions.[9,10] Various attempts have been made to improve the situation by including effects such as symmetry breaking,[11,12] configuration mixing,[13] relativistic corrections,[14] and meson-cloud contributions.[15] Until now, however, no attempt has succeeded in finding a convincing mechanism to fill the gap between theory and experiment. In this communication we point out that if we incoporate the new concept of an effective mass with which a quark is probed from the outside of a baryon by means of a soft photon, then the additive quark model is renovated so as to explain well the observed characteristics of baryon magnetic moments.

The additive quark model describes the magnetic moment of baryon B by a vector addition of the moments $\mu^B(q)$ of valence quarks q ($q = u,d,s$) which are assumed to be free spinning particles. Namely, the magnetic moment $\mu(B)$ of the baryon B is given by the expectation value

$$\mu(B) = \langle B\,\tfrac{1}{2}, 56 | \textstyle\sum_q \mu^B(q)\sigma_z^q | B\,\tfrac{1}{2}, 56\rangle, \qquad (1)$$

where the state vector $|B\,\frac{1}{2}, 56\rangle$ belongs to the 56-plet representation of the SU(6) spin-flavor symmetry, and $\boldsymbol{\sigma}^q$ is the Pauli spin matrix for quark q. The assumption that the quarks have the intrinsic magnetic moments $\mu^B(q) = \mu(q)$ irrespective of baryonic species enables us to express eight observable baryon magnetic moments in terms of three parameters: $\mu(u)$, $\mu(d)$, and $\mu(s)$. In the simplest description, in which quarks are supposed to be free Dirac particles without anomalous magnetic moments, $\mu(q)$ is postulated to be

$$\mu(q) = e_q/2m_q, \qquad (2)$$

where e_q and m_q are the electric charge and the effective mass parameter for quark q. Comparison of this naive theory with the latest experimental results made in Table I shows that, though qualitatively in the right direction, the fit gives a very large χ^2. Another decisive defect of this theory is found in the relation of the mass parameters of u and d quarks. All analyses along this line lead, without exception, to the inequality $m_u > m_d$ which contradicts the relation $m_u < m_d$ derived from the investigation of electromagnetic mass differences among hadron isomultiplets.[16] This contradiction requires reconsideration of the physical meaning of the mass parameter m_q in Eq. (2).

Notice that the measurement of baryon magnetic moments involves photons carrying small momentum transfer as probes. In other words, the baryon magnetic moments are softly seen by photons with long wavelength. Therefore, if the postulates of the additive quark model in Eqs. (1) and (2) are valid, the parameter m_q must be interpreted as the effective mass of the quark q inside the baryon as observed from the outside by the soft photon. Since quarks are deeply (and perhaps eternally) confined inside the baryon and the soft photon can see a coherent internal structure of the baryon, the mass parameter appearing in the expression of magnetic moments must reflect the effect of confinement. In contrast, the electromagnetic contributions to the baryon mass M_B arise mainly from the virtual exchange of somewhat hard photons with momentum of the order of the strong-interaction mass scale, and the hard photons created and annihilated inside the baryon naturally see quarks as individual particles. In this way we are led to the standpoint that the effective mass of the quark observed from the outside of the baryon by the soft photon should be distinguished from the effective mass of the ordinary sense with which quarks are assumed to interact with each other inside the baryon. Let us call the former, which depends closely on the properties of the baryon B in which quarks are confined, the outward effective mass m_q^B of the probed quark q, and the latter, the inward effective mass—or simply effective mass—m_q of quark q. Although the inward effective mass m_q of the quark q in baryon B may also depend on the characteristics of B, this dependence is assumed hereafter to be so weak as to be negligible, and the values of m_q ($q = u,d,s$) are postulated approximately to be common to all baryon species.

TABLE I. Comparison of theoretical results with observed baryon magnetic moments (in nuclear magnetons).

$\mu(B)$	Naive quark model	Present attempt: Formula	Present attempt: Numerical result	Experiment	(Ref.)
$\mu(p)$ [a]	2.793	$\frac{4}{3}\mu^p(u)-\frac{1}{3}\mu^p(d)$	2.793	2.793	(Ref. 1)
$\mu(n)$ [a]	−1.913	$\frac{4}{3}\mu^n(d)-\frac{1}{3}\mu^n(u)$	−1.913	−1.913	(Ref. 1)
$\mu(\Lambda)$	−0.607	$\mu^{\Lambda}(s)$	−0.612	-0.6138 ± 0.0047	(Ref. 2)
$\mu(\Sigma^+)$	2.671	$\frac{4}{3}\mu^{\Sigma^+}(u)-\frac{1}{3}\mu^{\Sigma^+}(s)$	2.058	2.33 ± 0.13	(Ref. 3)
$\mu(\Sigma^-)$	−1.093	$\frac{4}{3}\mu^{\Sigma^+}(d)-\frac{1}{3}\mu^{\Sigma^-}(s)$	−0.793	-0.89 ± 0.14	(Ref. 4)
$\mu(\Xi^0)$	−1.427	$\frac{4}{3}\mu^{\Xi^0}(s)-\frac{1}{3}\mu^{\Xi^0}(u)$	−1.253	-1.253 ± 0.014	(Ref. 7)
$\mu(\Xi^-)$	−0.486	$\frac{4}{3}\mu^{\Xi^-}(s)-\frac{1}{3}\mu^{\Xi^-}(d)$	−0.501	$-0.69\pm0.04\pm0.02$	(Ref. 8)
$\mu(\Sigma\Lambda)$	−1.630	$\frac{1}{3}\sqrt{3}\mu^{\Sigma\Lambda}(d)-\frac{1}{3}\sqrt{3}\mu^{\Sigma\Lambda}(u)$	−1.384	$-1.82^{+0.25}_{-0.18}$	(Ref. 1)
m_u (MeV)	337.8		291.0		
m_d (MeV)	321.9		313.7		
m_s (MeV)	515.0		456.4		
$\chi^2(\Lambda,\Xi^0)$	156		0.12		
χ^2	178		17.8		

[a] In both fits the nucleon magnetic moments were used as inputs.

The outward effective mass m_q^B must be determined as the measure of inertia of the probed quark q that is deeply confined inside the baryon B carrying the physical mass M_B as a whole. Without entering into the maze of the yet-unsolved problem of confinement, we take here an empirical and heuristic viewpoint. The valence quarks q_a, q_b, and q_c of the baryon B are assumed to have, respectively, the effective masses m_a, m_b, and m_c. Even though such a portion of the confining energy that can be assigned to the individual valence quarks is presumed to be distributed already to the inward effective mass of the quarks, the physical baryon mass M_B differs in general from the sum of the effective masses $m_B=m_a+m_b+m_c$. The difference M_B-m_B represents the indivisible and irreducible part of the confining energy. Here we postulate *a posteriori* that the soft photon perceives, at the long-wavelength limit, the valence quark q confined in baryon B as if it has outwardly and effectively the mass

$$m_q^B=m_q+(M_B-m_B). \tag{3}$$

Namely, the valence quark q inside baryon B is assumed to behave, when observed softly from outside, as a free pointlike entity with the effective mass of inertia m_q^B which differs from the inward effective mass m_q by the irreducible amount of the confining energy M_B-m_B.

Apparently, the outward effective mass of the probed quark may be defined as the difference between the physical baryon mass and the sum of the inward effective masses of the spectator valence quarks. It is worthwhile to emphasize that the confining energy M_B-m_B is assigned to the probed quark only, and that other spectator quarks are assumed to carry constantly the inward effective mass. As is obvious from the identities

$$\begin{aligned} M_B &= m_a^B+m_b+m_c \\ &= m_a+m_b^B+m_c \\ &= m_a+m_b+m_c^B, \end{aligned} \tag{4}$$

there is no risk of double or triple counting of confining energy.

In place of Eq. (2), the magnetic moment of valence quark q inside baryon B is given by the relation

$$\mu^B(q)=e_q/2m_q^B, \tag{5}$$

in terms of the outward effective mass. The baryon magnetic moments $\mu(B)$ are calculated from the addition formula (1) with these quark magnetic moments. By adjustment of the values of inward effective masses of quarks just like the case of the naive quark model, the best fits of the baryon magnetic moments to the experimental results are found as given in Table I. In the fit the experimental data of $\mu(p)$ and $\mu(n)$ are used as input and their contributions are not added to χ^2. It should be noticed that the values of $\mu(p)$, $\mu(n)$, and $\mu(\Lambda)$ are essentially determined by m_u and m_d, and that m_s can be chosen mainly to adjust to the precisely measured value of $\mu(\Xi^0)$. In consequence, the quark mass parameters are fixed as follows:

$$m_u = 291.0 \text{ MeV}, \quad m_d = 313.7 \text{ MeV},$$
$$m_s = 456.4 \text{ MeV}.$$

Although the difference $m_d - m_u$ is somewhat large, we get the desired inequality $m_u < m_d$. The χ^2 of 17.8 for five degrees of freedom which corresponds to a confidence level of about 0.3% is not satisfactory. However, this value should not be taken seriously, because the main contributions to χ^2 come from the moments of charged hyperons and the $\Sigma\Lambda$ transition moment whose experimental data are not yet decisive. As for the neutral hyperons Λ and Ξ^0, the situation is highly improved. While the χ^2 of two magnetic moments $\mu(\Lambda)$ and $\mu(\Xi^0)$ amounts of 156 in the naive quark model, it is reduced to 0.12 in the present scheme. In particular, in contrast to the prediction of the naive quark model on $\mu(\Xi^0)$, which differs from the observed value by more than twelve standard deviations, the present scheme can adjust the value of $\mu(\Xi^0)$ exactly to the latest experimental result. As emphasized by Bunce *et al.*,[6] "the simplest picture of baryon structure in terms of (u,d,s) quarks is incomplete" unless the problem of the $\mu(\Xi^0)$ discrepancy is resolved. Our intuitive approach has succeeded in finding one simple solution to this problem with the aid of the new concept of the outward effective mass of quarks without the introduction of any extra parameter other than those of effective quark masses.

So far, the baryon confinement energy $M_B - m_B$ is assumed to be lumped totally into the outward effective mass of the probed quark. It is relevant here to examine a class of models in which the confinement energy is included fractionally in the outward mass as

$$m_q^B = m_q + \lambda(M_B - m_B) \tag{6}$$

with an extra parameter λ $(0 \leq \lambda \leq 1)$. By application of the same procedure as above, the best fits of calculated magnetic moments to the observed values are found for each fixed λ. For such fits the variations of $\chi^2(\Lambda, \Xi^0)$ for neutral hyperons and χ^2 for all hyperons are plotted against λ $(0.5 \leq \lambda \leq 1)$ in Fig. 1. As a result of the strong restriction from the nucleon magnetic moments used as input, it is impossible to satisfy $\chi^2(\Lambda, \Xi^0) < 10$ for $0 \leq \lambda \leq 0.49$. In the $\chi^2(\Lambda, \Xi^0)$ curve there are two minima, at $\lambda = 0.58$ with $\chi^2(\Lambda, \Xi^0) = 0.00004$ and $\chi^2 = 9.262$, and at $\lambda = 0.92$ with $\chi^2(\Lambda, \Xi^0) = 0.001$ and $\chi^2 = 16.43$. So far as the baryon magnetic moments are concerned, the choice of $\lambda = 0.58$ seems to be favorable since both $\chi^2(\Lambda, \Xi^0)$ and χ^2 take there the smallest values. In this case, however, the values of the inward quark masses turn out to be

$$m_u = 203.9 \text{ MeV}, \quad m_d = 319.6 \text{ MeV},$$
$$m_s = 395.5 \text{ MeV},$$

in which the mass difference $m_d - m_u$ is too large to be consistent with isospin symmetry. The minimum at $\lambda = 0.92$ occurs for the inward quark masses

$$m_u = 287.0 \text{ MeV}, \quad m_d = 314.1 \text{ MeV},$$
$$m_s = 450.9 \text{ MeV}.$$

For this choice the χ^2 of 16.43 for four degrees of freedom corresponds to a confidence level of 0.25%, and the u and d mass difference is not very large. With this analysis we find that to explain the characteristics of baryon magnetic moments without sharply violating the approximate symmetry of isospin, all or almost all—more than 92%—of the confinement energy should be lumped into the outward effective mass of quarks.

The essence of the present treatment is the assumption that the quark q probed by the soft photon responds, not with the inward effective mass m_q, but with the outward effective mass m_q^B. To get further experimental support for this working rule it seems relevant to analyze carefully the radiative decays of vector particles into pseudoscalar particles. In particular, accurate experiments on the radiative decay widths of charmed particles $D^{*+} \rightarrow D^+ + \gamma$, $D^{*0} \rightarrow D^0 + \gamma$, and $F^{*+} \rightarrow F^+ + \gamma$ are desired, since the mass differences between the triplet-spin and singlet-spin states are comparatively small. In this Letter a purely empirical viewpoint was taken, because it is at present a far-reaching problem to place the notion of the outward effective mass of a probed quark on a solid foundation

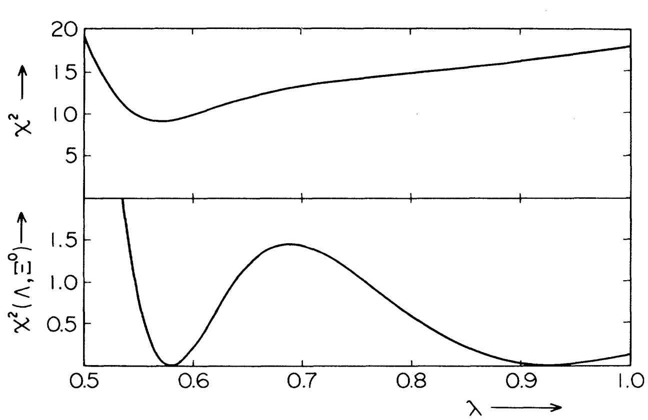

FIG. 1. The variation of χ^2 vs λ: $\chi^2(\Lambda, \Xi^0)$ for neutral hyperons and χ^2 for all hyperons.

of the basic theory of quark interaction, i.e., the $SU(3)_c$ gauge interaction, whose behavior at low energy and small momentum transfer is not yet clarified. The solution to this problem will yield an important clue to the question of what is the confinement mechanism, and consequently, what is the quark.

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