Relativistic Impulse Approximation, Nuclear Currents, and the Spin-Difference Function

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(Received 14 November 1984)

Traditional nonrelativistic impulse-approximation treatments of p-nucleus scattering using a local nucleon-nucleon t matrix neglect nuclear currents which are intrinsic to relativistic approaches also employing a local t matrix. Inclusion of these current terms is essential to the understanding of the spin-difference function, $(D_{qk}+D_{kq})+i(P-Ay)$, as shown by comparison with $^{12}C(p,p')^{12}C^*(12.71\text{ MeV}, 1^+, T=0)$ spin-difference data at 150 MeV.

PACS numbers: 24.10.Cn, 25.40.Ep

The vastly improved quality and variety of medium-energy p-nucleus scattering observables,[1,2] especially the spin-transfer observables D_{ij}, has created pressure for refinements to theoretical treatments based on the impulse approximation. Recent developments have included more complete representations of the NN force, medium modifications,[3] and exchange processes.[4] Chief among the inelastic spin observables generating this pressure is the spin-difference function,

$$\Delta_s=(D_{qk}+D_{kq})+i(P-A_y), \tag{1}$$

which vanishes for elastic scattering and has been difficult to understand in standard nonrelativistic approaches.[1] In the nonrelativistic approach the spin-difference function vanishes in impulse approximation and a nonzero value is achieved only through explicit treatment of nonlocalities in the NN force. For example, Love and Comfort[5] have investigated the effects of nonlocalities arising from knock-on exchange and found only partial success with the 150-MeV data. In contrast, a relativistic impulse treatment possesses a structure automatically having those terms necessary for a nontrivial spin-difference function.[6]

In this work we investigate a contribution to the spin-difference function arising in impulse approximation from the nuclear convection-current operator. Familiar from electron scattering,[7] the nuclear current operator is neglected in the nonrelativistic impulse approximation. In the relativistic treatment it appears automatically and provides a nonzero spin-difference function even in plane-wave calculations. When distortions are added, the relativistic theory goes a long way toward describing the 150-MeV $^{12}C(p,p')^{12}C^*(12.71\text{ MeV}, 1^+, T=0)$ spin-difference data. We emphasize that this new feature is a consequence of the Dirac structure of the nuclear bound states and exists independent of the nature of the binding potentials.

We illustrate the importance of nuclear currents in nucleon-nucleus inelastic scattering by considering the plane-wave impulse approximation. Consider a nucleon of initial (final) momentum $\mathbf{k}$ ($\mathbf{k}'$) exciting a 0^+ target nucleus to a JM final state. If $\mathbf{q}=\mathbf{k}'-\mathbf{k}$ is the momentum transfer, the relativistic plane-wave amplitude can be written as

$$\tilde{T}^{s's}_{JM}=u_p^\dagger(\mathbf{k}',s')\gamma^0(p)\,{}_{\rm D}\langle JM|\sum_{i=1}^{A}\gamma^0(i)\,\tilde{t}_{nn}(p,i)\,e^{-i\mathbf{q}\cdot\mathbf{r}_i}|00\rangle_{\rm D}\,u_p(\mathbf{k},s) \tag{2}$$

where the u_p's are the free projectile spinors and $\tilde{t}_{nn}$ is the relativistically invariant nn t matrix:

$$\tilde{t}_{nn}(p,i)=t_s+\gamma^\mu(p)\gamma_\mu(i)\,t_V+\gamma^5(p)\gamma^5(i)\,t_p+\gamma^5(p)\gamma^\mu(p)\gamma^5(i)\gamma_\mu(u)\,t_A+\sigma^{\mu\nu}(p)\sigma_{\mu\nu}(i)\,t_T. \tag{3}$$

Where necessary a tilde distinguishes comparable quantities in the relativistic (with tilde) or nonrelativistic (without tilde) approaches. The nuclear wave functions, $|JM\rangle_{\rm D}$, are four-component Dirac spinors which, upon neglect of the binding potentials, are related to the usual two-component Pauli-Schrödinger, $|JM\rangle_S$, by

$$|JM\rangle_D = N\begin{pmatrix} 1 \\ -i\boldsymbol{\sigma}\cdot\nabla/(E+m)\end{pmatrix}|JM\rangle_S \tag{4}$$

where N is the normalization constant. The relativistic impulse-approximation amplitude based on this relation we call the free relativistic impulse-approximation amplitude or FRIA. It can be shown that the nonrelativistic impulse approximation is recovered from Eq. (2) if the lower component of the rest-frame wave function is set equal to zero in Eq. (4), thus setting the currents equal to zero as well.[8]

To contrast the relativistic and nonrelativistic approaches we review the simplest nonrelativistic plane-wave impulse approximation which employs a local nucleon-nucleon t matrix, usually written in Wolfenstein form. Using the standard kinematic triad, $\mathbf{p}=(\mathbf{k}+\mathbf{k}')/2$, $\mathbf{q}=\mathbf{k}'-\mathbf{k}$, and $\hat{\mathbf{n}}=\hat{\mathbf{p}}\times\hat{\mathbf{q}}$, one writes

$$t_{nn}(p,i) = a + b\boldsymbol{\sigma}(p)\cdot\boldsymbol{\sigma}(i) + iqc[\boldsymbol{\sigma}(p)+\boldsymbol{\sigma}(i)]\cdot\hat{\mathbf{n}} + d\boldsymbol{\sigma}(p)\cdot\mathbf{q}\boldsymbol{\sigma}(i)\cdot\mathbf{q} + e\boldsymbol{\sigma}(p)\cdot\hat{\mathbf{p}}\boldsymbol{\sigma}(i)\cdot\hat{\mathbf{p}}, \tag{5}$$

or, focusing on the target spin dependence,

$$t_{nn}(p,i) = a + \mathbf{b}\cdot\boldsymbol{\sigma}(i), \tag{6}$$

where a and $\mathbf{b}$ are operators in the projectile spin space with an obvious relationship to the usual Wolfenstein parameters of Eq. (5). The nonrelativistic plane-wave impulse-approximation transition amplitude then becomes

$$T_{JM} = a\rho_{JM} + \mathbf{b}\cdot\boldsymbol{\Sigma}_{JM}, \tag{7a}$$

where

$$\rho_{JM} = {}_S\langle JM|\sum_{i=1}^{A} e^{-i\mathbf{q}\cdot\mathbf{r}_i}|0^+0\rangle_S, \tag{7b}$$

$$\boldsymbol{\Sigma}_{JM} = {}_S\langle JM|\sum_{i=1}^{A} \boldsymbol{\sigma}(i) e^{-i\mathbf{q}\cdot\mathbf{r}_i}|0^+0\rangle_S. \tag{7c}$$

The plane-wave free relativistic impulse-approximation amplitude (PW-FRIA) can be written in a similar form:

$$\tilde{T}_{JM}^{s's} = \sum_{j=1}^{4} [f_j u_p^\dagger(\mathbf{k}',s')\Gamma_j(p)u_p(\mathbf{k},s)\rho_{JM}^{(j)} + g_j u_p^\dagger(\mathbf{k}',s')\boldsymbol{\sigma}(p)\Gamma_j(p)u_p(k,s)\cdot\boldsymbol{\Sigma}_{JM}^{(j)}], \tag{8a}$$

where

$$\rho_{JM}^{(j)} = {}_D\langle JM|\sum_{i=1}^{A}\Gamma_j(i)e^{-i\mathbf{q}\cdot\mathbf{r}_i}|00\rangle_D, \tag{8b}$$

$$\boldsymbol{\Sigma}_{JM}^{(j)} = {}_D\langle JM|\sum_{i=1}^{A}\boldsymbol{\sigma}(i)\Gamma_j(i)e^{-i\mathbf{q}\cdot\mathbf{r}_i}|00\rangle_D. \tag{8c}$$

In these definitions we have made use of the matrices

$$\{\Gamma_j\} = \left\{\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}1&0\\0&-1\end{pmatrix},\begin{pmatrix}0&1\\1&0\end{pmatrix},\begin{pmatrix}0&1\\-1&0\end{pmatrix}\right\} \tag{9}$$

which act only in the space of upper-lower components. The f_j's and g_j's are given by the relativistic nn t matrix: $\{f_j\}=\{t_v, t_s, t_A, t_p\}$, and $\{g_j\}=\{-t_A, 2t_T, -t_v, 2t_T\}$. The form of the transition amplitude in Eq. (8) is analogous to that encountered in the nonrelativistic impulse approximation, except that there are now four times as many transition densities.

Spin-transfer observables are defined by

$$ID_{ij} = \tfrac{1}{2}\mathrm{Tr}\textstyle\sum_M \theta_i\tilde{T}_{JM}\theta_j\tilde{T}_{JM}^\dagger \tag{10}$$

where $i(j)$ is the initial (final) spin orientation and

$$I = \tfrac{1}{2}\mathrm{Tr}\textstyle\sum_m \tilde{T}_{JM}\tilde{T}_{JM}^\dagger \tag{11}$$

and $\{\theta\}=\{1, \boldsymbol{\sigma}\cdot\hat{\mathbf{n}}, \boldsymbol{\sigma}\cdot\hat{\mathbf{p}}, \boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\}$ for $\{i\}=\{0,n,p,q\}$.

Of particular interest is the spin-difference function, Δ_s, which is defined in Eq. (1) ($D_{n0}=P$ and $D_{0n}=A_y$). For Δ_s to be nonzero, projectile operators $\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}$ and $\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}$ must contribute to the same magnetic substate of the target. In the nonrelativistic impulse approximation, all terms of the transition amplitude are proportional to $\sigma_i\delta_{m,i}$ or $\delta_{m,n}$ and therefore Δ_s vanishes.

In contrast, the PW-FRIA amplitude contains the needed additional structure for nonzero Δ_s. The term arising from the purely longitudinal timelike axial vector interaction, $\rho_{JM}^{(3)}$, is a particularly simple example. Quantizing along $\hat{\mathbf{q}}$, we write

$$\rho_{JM}^{(3)} = \delta_{m,0}\,{}_D\langle J0|\sum_{i=1}^{A}\begin{pmatrix}0&1\\1&0\end{pmatrix}_i e^{-i\mathbf{q}\cdot\mathbf{r}_i}|00\rangle_D. \tag{12}$$

The corresponding projectile spin dependence is given by

$$u^{\dagger}(\mathbf{k}',s')\begin{pmatrix}0&1\\1&0\end{pmatrix}u(\mathbf{k},s)=\chi^{\dagger}_{s'}\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{m}\chi_{s} \tag{13}$$

so that the resulting term of the inelastic amplitude is proportional to $\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\delta_{m,0}$. This type of term is new to the impulse-approximation amplitude and was previously generated only through nonlocal processes like knock-on exchange. It will interfere with the standard $\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\delta_{m,0}$ piece of the amplitude (which also arises in PW-FRIA) to give a nonzero spin-difference function. The new feature is the appearance of the nuclear current operator which arises from the lower component of the bound nucleon. Using Eq. (4) to replace the Dirac wave functions with Pauli-Schrödinger wave functions in Eq. (12), we find

$$\rho^{(3)}_{JM}\xrightarrow[\text{Eq. (4)}]{}\rho^{\sigma j}_{JM}=\delta_{m,0}N_f N_i\,{}_{S}\langle J0|\,i\sum_{i=1}^{A}\boldsymbol{\sigma}(i)\cdot\frac{(\overleftarrow{\nabla}_i-\overrightarrow{\nabla}_i)}{2m}[e^{-i\mathbf{q}\cdot\mathbf{r}_i}]|00\rangle_{S},$$

or

$$\rho^{\sigma j}_{JM}=\delta_{m,0}N_f N_i\,{}_{S}\langle J0|\sum_{i=1}^{A}\boldsymbol{\sigma}(i)\cdot\mathbf{j}(i)[e^{-i\mathbf{q}\cdot\mathbf{r}_i}]|00\rangle_{S}, \tag{14}$$

where the gradients of the current operator, $\mathbf{j}$, *do not* act on the exponential. The full PW-FRIA calculation reveals two other nuclear-structure matrix elements involving the current operator:

$$\mathbf{J}_{JM}={}_{S}\langle JM|\sum_{i=1}^{A}\mathbf{j}(i)[e^{-i\mathbf{q}\cdot\mathbf{r}_i}]|00\rangle_{S}, \tag{15a}$$

$$\boldsymbol{\Sigma}^{\sigma j}_{JM}={}_{S}\langle JM|\sum_{i=1}^{A}\boldsymbol{\sigma}(i)\times\mathbf{j}(i)[e^{-i\mathbf{q}\cdot\mathbf{r}_i}]|00\rangle_{S}, \tag{15b}$$

which come from the vector and tensor interactions, respectively.

In this relativistic plane-wave approximation we obtain closed-form expressions for the spin-difference function which are explicitly linear in the new terms:

$$I\Delta_s=t_A(p/m)\rho^{\sigma j}_{J}[(b+q^2d)\Sigma^{L}_{J}+\cdots]+2t_T(P/m)\Sigma^{\sigma j}_{J}[(b+e)\Sigma^{T}_{J}+\cdots], \tag{16}$$

where $\rho^{\sigma j}_{J}$ and Σ^{L}_{J} vanish for natural-parity transitions, Σ^{T}_{J} is the transverse spin-transition density, and b, d, and e are Wolfenstein t-matrix amplitudes defined in Eq. (5) (see Ref. 8 for details).

Inclusion of distortion does not qualitatively alter the plane-wave result for Δ_s at low q. In Fig. 1 the FRIA results, which now include distortion, are compared with the $P-A_y=\mathrm{Im}(\Delta_s)$ data for the $^{12}C(p,p')^{12}C^{*}(12.71$ MeV, $1^{+},T=0)$ transition at 150 MeV.[1] The agreement with the data is good, but little quantitative significance should be given this agreement because of the uncertain nature of the input information. For example, since no relativistic shell model yet exists, we have simply used the nonrelativistic structure amplitudes of Lee and Kurath.[9] As we mentioned earlier, the nonrelativistic impulse approximation gives identically zero for this observable. Nonzero values may be obtained through inclusion of nonlocal effects such as knock-on exchange, as shown for example by the DW81 calculation (dashed curve).[11]

Contemporary relativistic models of the nucleon-nucleus interaction are characterized by scalar and timelike vector potentials with strengths of several hundred megaelectronvolts.[11] The effect of such potentials on nuclear currents has already been addressed in the context of inelastic electron scattering.[12] The dash-dot curve in the figure demonstrates that such potentials also have an appreciable effect on $P-A_y$. When more refined calculations can be performed, we may be able to probe the nuclear currents in sufficient detail to determine if such strong potentials are in fact present.

We stress that the significance of the present result is not primarily in the level of agreement with data. In fact the $P-A_y$ data for the 1^{+}, $T=1$, 15.11-MeV state in ^{12}C taken at 150 MeV is poorly described by both relativistic and nonrelativistic approaches. Clearly, comparison with a broad range of data yet to be measured as well as more sophisticated relativistic calculations which include exchange and other nonlocalities is desirable. The primary significance of the present work is demonstration of the natural appearance of nuclear currents in a local Dirac formulation of the hadronic scattering problem as well as demonstration of their importance in understanding a specific combination of spin observables, the spin-difference

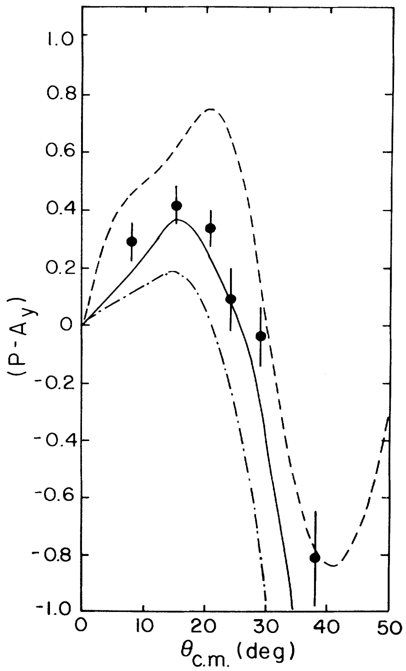

FIG. 1. Polarization minus analyzing power data for 150-MeV proton excitation of the $1^+, T=0$, 12.71-MeV state in ^{12}C, compared with the free relativistic impulse calculation (solid curve), and the nonrelativistic calculation using DW81 (dashed curve). The dash-dot curve shows the effect of including the strong scalar and vector binding potentials in the relativistic calculation. The data are from Ref. 1.

function. The natural appearance of nuclear currents at the level of impulse approximation demonstrates that the Dirac approach provides a particularly efficient theoretical framework for the description of proton-nucleus inelastic scattering.

The authors thank R. D. Amado for useful conversations. This work was supported in part by the National Science Foundation and the Department of Energy.

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