Model for Maximal CP Nonconservation

M. Gronau
Physics Department, Technion, 32000 Haifa, Israel

and

R. Johnson and J. Schechter
Physics Department, Syracuse University, Syracuse, New York 13210
(Received 18 March 1985)

We display a model for the up- and down-quark mass matrices which permits, for any number of generations, the complete determination of the Kobayashi-Maskawa matrix in terms of the quark masses. We discuss in this framework the correlation between the quark-mass spectrum and a maximal CP-nonconserving phase and also give an application to the four-generation case.

PACS numbers: 11.30.Er, 12.10.Ck, 14.80.Dq

By now it is clear[1] that if the standard model is to explain the observed CP nonconservation in the K_L-K_S system the magnitude of this nonconservation must be about as large as possible. We hope that this feature might be a valuable clue to the development of a yet more fundamental theory. Towards this end we would like to transfer the criterion for maximal CP nonconservation from a model-independent statement about the Kobayashi-Maskawa (KM) matrix to a class of models for the (not yet diagonal) up- and down-quark mass matrices. One interesting model for these matrices is the Fritzsch[2] model which (assuming all masses are known) determines the $(N-1)^2$ parameters of the N-generation KM matrix in terms of $N-1$ arbitrary constants. Another interesting model is due to Stech[3]: This model provides $N(N-1)/2$ relations among the KM parameters. Evidently neither of these completely determines the KM matrix[4] in terms of masses. Here we point out that these two different but reasonable models do not necessarily contradict each other. Thus, they may be imposed simultaneously. Remarkably, this results in the complete determination of the KM matrix. The condition for maximal CP-nonconserving phase becomes correlated with the quark mass spectrum.

We will use, for definiteness, the following quark masses[5,6]:

$$\begin{aligned} &m_u = 5.1\ \text{MeV},\quad m_c = 1.35\ \text{GeV},\quad m_t = 45\ \text{GeV},\\ &m_d = 8.9\ \text{MeV},\quad m_s = 175\ \text{MeV},\quad m_b = 5.3\ \text{GeV}. \end{aligned} \tag{1}$$

It is interesting that the up-to-down mass ratios for each generation,

$$\frac{m_u}{m_d} \approx 0.57,\quad \frac{m_c}{m_s} \approx 7.7,\quad \frac{m_t}{m_b} \approx 8.4, \tag{2}$$

indicate that the up-quark mass is unusually low. We also need the ratios of quark masses for successive generations:

$$\begin{aligned} &(m_c/m_u)^{1/2} \approx 16,\quad (m_t/m_c)^{1/2} \approx 5.8,\\ &(m_s/m_d)^{1/2} \approx 4.4,\quad (m_b/m_s)^{1/2} \approx 5.5. \end{aligned} \tag{3}$$

Again, the small value of m_u breaks the pattern.

It is very convenient to adopt a phase convention[7] for the KM matrix U, in which, to practical accuracy,

$$U \approx \begin{bmatrix} 1 & S_{12}e^{i\phi_{12}} & S_{13}e^{i\phi_{13}} \\ -S_{12}e^{-i\phi_{12}} & 1 & S_{23}e^{i\phi_{23}} \\ -S_{13}e^{-i\phi_{13}} + S_{12}S_{23}e^{-i(\phi_{12}+\phi_{23})} & -S_{23}e^{-i\phi_{23}} & 1 \end{bmatrix}. \tag{4}$$

Here the three mixing angles coincide with the measurable transition amplitudes $S_{12} = |U_{us}|$, $S_{23} = |U_{cb}|$, and $S_{13} = |U_{ub}|$. The physical CP nonconservation is measured by the invariant phase[7,8]

$$\Phi = \phi_{12} + \phi_{23} - \phi_{13}. \tag{5}$$

All CP-nonconserving amplitudes will be proportional to $|U_{us}U_{cb}U_{ub}|\sin\Phi$. Thus if one holds $|U_{us}|$, $|U_{cb}|$, and $|U_{ub}|$ fixed, the CP-nonconserving amplitudes will be maximized for $|\Phi| = \pi/2$. We shall call this situation "maximal CP nonconservation."

The Stech model is defined by the following *Ansatz* for the up-quark mass matrix M_u and the down-quark mass matrix M_d:

$$M_u = M_u^\dagger = M_u^{\mathrm{T}}, \tag{6a}$$

$$M_d = M_d^\dagger = \alpha M_u + A, \tag{6b}$$

where α is a constant and A is an antisymmetric matrix. Such relations might arise in a wide class of theories. In the limit $A = 0$, $U = 1$ since M_u and M_d would be diagonal in the same basis and all the ratios in Eq. (2) would become equal. In fact, all mixing angles are small experimentally and the second and third generations do have roughly the same mass ratio. The matrix A is responsible for correlating the nonzero mixing angles and CP-nonconserving phases with the realistic mass spectrum. Bringing M_u and M_d to diagonal form one finds the relation

$$QU\hat{M}_d U^\dagger Q^\dagger = \alpha \hat{M}_u + A', \tag{7}$$

where[9] $\hat{M}_u = \mathrm{diag}(m_u, -m_c, m_t)$, $\hat{M}_d = \mathrm{diag}(m_d, -m_s, m_b)$, and A' is another antisymmetric matrix. Also the "trivial" phase matrix $Q = \mathrm{diag}(e^{i\gamma_1}, e^{i\gamma_2}, e^{i\gamma_3})$, with $\sum_i \gamma_i = 0$, is present to express the deviation of (4) from the most general unitary unimodular matrix. The diagonal elements of (7) yield

$$\alpha = \frac{m_d - m_s + m_b}{m_u - m_c + m_d} \approx \frac{m_b}{m_t} \tag{8}$$

as well as the two predictions

$$(S_{12})^2 \approx m_d/m_s = (0.23)^2, \tag{9a}$$

$$(S_{23})^2 \approx m_s/m_b - m_c/m_t = (0.055)^2. \tag{9b}$$

The numerical value of S_{23} clearly is sensitive to the quark masses. These agree with the experimental values[10] $S_{12} = 0.23$ and $S_{23} = 0.05 \pm 0.01$. In addition, the condition $\det A' = 0$ yields the approximate maximality of the invariant CP-nonconserving phase Φ.

The generalized Fritzsch model assumes that M_u and M_d each have the form

$$M_{ij} = e^{i\lambda_i} F_{ij} e^{i\rho_j}, \quad F = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix}, \tag{10}$$

where A, B, C are positive constants and λ_i and ρ_j are some phases. This model describes the situation where nearest-neighbor generations "interact" with each other and where the dominant scale is set by the heaviest quark mass. The matrix elements of F can be written in terms of the quark masses[11]:

$$A \approx (m_1 m_2)^{1/2}, \quad B \approx (m_2 m_3)^{1/2}, \quad C \approx m_3 \quad (m_3 >> m_2 >> m_1). \tag{11}$$

F is diagonalized by the orthogonal matrix, R;

$$R^{\mathrm{T}} F R = \hat{M},$$

$$R \approx \begin{pmatrix} 1 & (m_1/m_2)^{1/2} & 0 \\ (m_1/m_2)^{1/2} & -1 & (m_2/m_3)^{1/2} \\ -(m_1/m_3)^{1/2} & (m_2/m_3)^{1/2} & 1 \end{pmatrix}. \tag{12}$$

Finally the KM matrix is given[2] by

$$U = P^{-1} R_u^{\mathrm{T}} P R_d, \quad P = \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}), \tag{13}$$

where $\beta_i = \rho_{ui} - \rho_{di}$. Because of the arbitrariness of overall phases we may set $\sum_i \beta_i = 0$. Notice that the factor P^{-1} is required to bring U to the form (4). Equating U_{12} from Eqs. (4) and (13) yields

$$S_{12} e^{i\phi_{12}} \approx \left(\frac{m_d}{m_s}\right)^{1/2} - e^{i(\beta_2 - \beta_1)} \left(\frac{m_u}{m_c}\right)^{1/2}. \tag{14}$$

Since the first term on the right-hand side already accounts for the magnitude of S_{12}, (14) shows that $\beta_2 - \beta_1$ must be very roughly around $\pm 90°$ and that the phase ϕ_{12} is small. Similarly, equating U_{23} from (4) and (13) yields

$$S_{23} e^{i\phi_{23}} \approx -\left(\frac{m_s}{m_b}\right)^{1/2} + \left(\frac{m_c}{m_t}\right)^{1/2} e^{i(\beta_3 - \beta_2)}. \tag{15}$$

Since the magnitude of S_{23} is considerably smaller than the magnitudes of each of the two terms on the right-hand side, the latter must partially cancel and so the phase $\beta_3 - \beta_2$ must be small. Comparing U_{23} and U_{13} from (13) yields the simple relation

$$U_{13}/U_{23} \approx -(m_u/m_c)^{1/2} e^{i(\beta_2 - \beta_1)}. \tag{16}$$

Actually, because of cancellations, U_{13} is sensitive to corrections which are of higher order in the mass ratios in (12). Taking these into account gives a prediction $|U_{13}| \simeq 0.0034 \pm 0.0014$ which is consistent with the experimental bound[10] $|U_{13}| < 0.007$. For the invariant CP-nonconserving phase defined in (5) we see from the approximation (16) that

$$\begin{aligned} \Phi &= \arg U_{12} + \arg(U_{23}/U_{13}) \\ &\approx \phi_{12} + \beta_1 - \beta_2 - \pi, \end{aligned} \tag{17}$$

which, upon insertion of numbers, is found to be maximal.

Notice that the predictions of the Stech model and of the Fritzsch model are complementary to each other. Both agree that Φ is maximal. The Stech model predicts S_{12} and S_{23} in agreement with experiment but does not yield a prediction for S_{13}. On the other hand the Fritzsch model does not predict S_{12} and S_{23} but does predict S_{13}.

At first glance one might think that the underlying

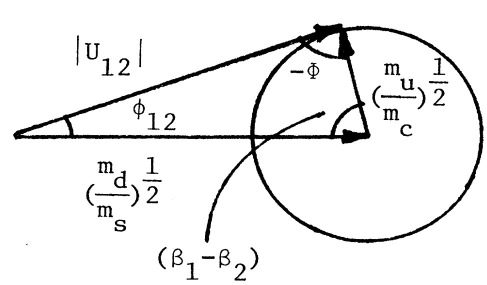

FIG. 1. Schematic vector diagram illustrating Eqs. (14) and (17). The maximum value of ϕ_{12} occurs when the vector $U_{12}e^{i\phi_{12}}$ is tangent to the circle which makes the invariant phase $(-\Phi)$ ninety degrees.

assumptions are rather different. M_d for the Stech model in (6b) has an antisymmetric piece while M_d for the Fritzsch model of the form (10) apparently differs only in an unessential way from a symmetric matrix. However, it is possible to write down unique mass matrices which satisfy both assumptions (6) and (10):

$$M_u = \begin{pmatrix} 0 & A_u & 0 \\ A_u & 0 & B_u \\ 0 & B_u & C_u \end{pmatrix}, \tag{18a}$$

$$M_d = \alpha M_u + \begin{pmatrix} 0 & ia & 0 \\ -ia & 0 & ib \\ 0 & -ib & 0 \end{pmatrix} = P \begin{pmatrix} 0 & (\alpha^2 A_u^2 + a^2)^{1/2} & 0 \\ (\alpha^2 A_u^2 + a^2)^{1/2} & 0 & (\alpha^2 B_u^2 + b^2)^{1/2} \\ 0 & (\alpha^2 B_u^2 + b^2)^{1/2} & \alpha C_u \end{pmatrix} P^{-1} \equiv P \begin{pmatrix} 0 & A_d & 0 \\ A_d & 0 & B_d \\ 0 & B_d & C_d \end{pmatrix} P^{-1}, \tag{18b}$$

where the phase matrix P which was defined in terms of unknown phases β_i (satisfying $\sum_i \beta_i = 0$) in (13) is now explicitly specified by

$$\cos(\beta_1 - \beta_2) = \alpha A_u / A_d, \tag{19a}$$

$$\cos(\beta_2 - \beta_3) = \alpha B_u / B_d. \tag{19b}$$

Note that A_u, B_u, and C_u are given in terms of the three up-quark masses. The parameters A_d, B_d, and C_d are similarly given by the three down-quark masses and in turn determine α, a, and b. Thus the phases in (19) are specified up to a trivial sign ambiguity. The KM matrix for our model is similarly specified in terms of the quark masses for an arbitrary number N of generations. Note that the generalized M_u for N generations has nearest-neighbor interactions with a diagonal entry only for the heaviest quark.

For three generations the consistency constraints $a^2 = A_d^2 - \alpha^2 A_u^2 > 0$ and $b^2 = B_d^2 - \alpha^2 B_u^2 > 0$ are equivalent to the positivity of $(S_{12})^2$ and $(S_{23})^2$ as predicted in (9). Note that Eqs. (19b) and (15) for S_{23} can be seen to agree with each other when (19b) is used. The present model predicts all KM parameters, $S_{12} \approx 0.23$, $S_{23} \approx 0.055$, $S_{13} \approx 0.0034 \pm 0.0014$, and $|\Phi| \approx 90°$.

This model also enables us to discuss the concept of a maximal CP-nonconserving phase in terms of the physical mass spectrum. Note from (14) that, because of the smallness of m_u, the phase ϕ_{12} is small. Then the invariant phase Φ in (17) is roughly measured by $\beta_1 - \beta_2$. From (19a) we predict

$$\cos(\beta_1 - \beta_2) \approx \left(\frac{m_u m_c}{m_d m_s} \right)^{1/2} \left(\frac{m_b}{m_t} \right). \tag{20}$$

In the limit where all ratios of up- and down-quark masses are about the same [see Eq. (2)], (20) would tell us that $\cos(\beta_1 - \beta_2) \approx 1$. However, the unusually low value of m_u drastically distorts the pattern in (2) and we actually have $\cos^2(\beta_1 - \beta_2) \approx 0.06$ which gives a practically maximal invariant phase. Note that the limit $m_u \to 0$ which corresponds to zero "strong" CP nonconservation[12] considerably reduces S_{13} [see Eq. (16)] and hence the weak CP-nonconserving amplitudes. For the actual value $m_u = 5.1$ MeV we see that the allowed solutions give $\phi_{12} = \pm 16°$ corresponding to $\beta_1 - \beta_2 = \pm 76°$. Thus the invariant phase is $\Phi \approx 88°$, illustrating that the first two terms of (17) work together to keep Φ close to maximal. This may be understood from Stech's model since (7) yields, when we note that the phases in Q cancel,

$$\frac{U_{12} U_{23}}{U_{13}} \approx -\left(\frac{1}{m_s} \right) \frac{(U \hat{M}_d U^\dagger)_{12} (U \hat{M}_d U^\dagger)_{23}}{(U \hat{M}_d U^\dagger)_{13}} = -\frac{A'_{12} A'_{23}}{A'_{13} m_s}. \tag{21}$$

The invariant phase $\Phi \approx \arg(U_{12} U_{23} / U_{13})$ is then directly given by $\arg(A'_{12} A'_{23} / A'_{13})$ which is clearly $\pm \pi/2$.

It is interesting to see what role the invariant phase plays in the Fritzsch model, where the phases β_i are still free parameters. The CP-nonconserving amplitudes to lowest order are proportional to the *small* angle ϕ_{12}. ϕ_{12} may be determined geometrically, as shown in Fig. 1, from Eqs. (14) and (17). It is seen that for fixed masses and variable $\beta_1 - \beta_2$, ϕ_{12} is maximum at the point of tangency; this corresponds to a maximal invariant phase $|\Phi| = \pi/2$ rather than a maximal $\beta_1 - \beta_2$. The latter angle going through 90° has been used as a maximality criterion in the Fritzsch model by Georgi, Nelson, and Shin.[2] They also advocate constraining $\beta_2 - \beta_3$ to be zero. In the present model $\cos^2(\beta_2 - \beta_3) \approx m_c m_b / m_s m_t$. This does not involve m_u and so the pattern in (2) shows $\beta_2 - \beta_3$ to be small. Explicitly, $\beta_2 - \beta_3 \approx \pm 18°$ in agreement with the determination from (15) using $s_{23} \approx 0.05$.

The CP-nonconservation parameter ϵ for three generations in the standard model is estimated[1] to be

$$|\epsilon|_{\text{theor}} / |\epsilon|_{\text{expt}} \approx 290 S_{13} B \sin\Phi. \tag{22}$$

This indicates that, with S_{13} given by either the

Fritzsch model or the present model, one requires the factor[13] B to be around unity rather than around $\frac{1}{3}$. If the latter value for B turns out to be correct we must search for additional contributions to ϵ. A natural possibility is to consider a fourth generation. If a fourth quark generation (t',b') existed and had masses in agreement with the pattern (3) one would expect $m_{b'}$ to be roughly in the 130-GeV range and $m_{t'}$ to be in the 1-TeV range. It is then easy to see that, to a good approximation, the formulas (14), (15), and (16) for U_{12}, U_{23}, and U_{13} continue to hold. The complete parametrization[14] of U in analogy to (4) requires the specification of three additional mixing angles which may be taken as $|U_{14}|$, $|U_{24}|$, and $|U_{34}|$. Two more invariant CP-nonconserving phases beyond $\Phi \equiv I_{123}$, say $I_{124}=\arg(U_{12}U_{24}/U_{14})$ and $I_{234}=\arg(U_{23}U_{34}/U_{24})$, are also required. These may be read off from the relations

$$U_{24} \approx \left(\frac{m_c}{m_t}\right)^{1/2} e^{i(\beta_3-\beta_2)} U_{34}, \tag{23a}$$

$$U_{14} \approx -\left(\frac{m_u}{m_c}\right)^{1/2} e^{i(\beta_2-\beta_1)} U_{24}, \tag{23b}$$

$$U_{34} \approx \left(\frac{m_b}{m_{b'}}\right)^{1/2} - \left(\frac{m_t}{m_{t'}}\right)^{1/2} e^{i(\beta_4-\beta_3)}, \tag{23c}$$

as well as

$$\cos(\beta_1-\beta_2) \approx \left(\frac{m_u m_c}{m_d m_s}\right)^{1/2} \left(\frac{m_{b'}}{m_{t'}}\right), \tag{24a}$$

$$\cos(\beta_2-\beta_3) \approx \left(\frac{m_c m_t}{m_s m_b}\right)^{1/2} \left(\frac{m_{b'}}{m_{t'}}\right), \tag{24b}$$

$$\cos(\beta_3-\beta_4) \approx \left(\frac{m_t m_{b'}}{m_{t'} m_b}\right)^{1/2}. \tag{24c}$$

Comparing (24a) with (20) shows that $m_{b'}/m_{t'}$ should be about the same as m_b/m_t if we wish the previous prediction for S_{12} to be unchanged. Then (24b) will reproduce the three-generation results for S_{23} and S_{13}. Substituting (24c) into (23c) gives

$$|U_{34}|^2 \approx \frac{m_b}{m_{b'}} - \frac{m_t}{m_{t'}}, \tag{25}$$

which is the analog of (9b). The new CP-nonconserving phases are seen to be $I_{124} \approx I_{123}$ and $I_{234} \approx \arg U_{23} + \beta_2 - \beta_3$. I_{124} is obviously maximal since I_{123} has already been shown to be maximal. I_{234} is also easily seen to be maximal. The maximality of the three invariant phases clearly arises in the same way as in the three-generation case. Note that m_u appears in (24a) but not in either (24b) or (24c). From Eqs. (23) we obtain $|U_{24}| \approx 0.17|U_{34}|$ and $|U_{14}| \approx 0.01|U_{34}|$. Equation (25) shows that, accepting the rough pattern (3), one would not expect $|U_{34}|$ to be larger than about 0.1. A value of $|U_{34}|$ of this size would lead to a contribution to ϵ about the same as that of the previous three generations. A more detailed discussion of this point will be given elsewhere.

We would like to thank A. P. Balachandran, K. C. Wali, and L. Wolfenstein for discussions. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-76ER03533.

[1]For a recent review see L. Wolfenstein, CERN Report No. TH 3925/84, 1984 (unpublished). See also F. Gilman and J. Hagelin, Phys. Lett. **133B**, 443 (1983); L. L. Chau and W. Y. Keung, Phys. Rev. D **29**, 592 (1984).

[2]H. Fritzsch, Phys. Lett. **73B**, 317 (1978), and Nucl. Phys. **B155**, 189 (1979); L. F. Li, Phys. Lett. **84B**, 461 (1979); H. Georgi and D. V. Nanopoulos, Nucl. Phys. **B155**, 52 (1979); A. Davidson, V. P. Nair, and K. C. Wali, Phys. Rev. D **29**, 1513 (1984); M. Shin, Phys. Lett. **145B**, 285 (1984); H. Georgi, A. Nelson, and M. Shin, Phys. Lett. **150B**, 306 (1985).

[3]B. Stech, Phys. Lett. **130B**, 189 (1983); see also G. Ecker, Z. Phys. C **24**, 353 (1984).

[4]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[5]J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[6]C. Rubbia, talk at the Stanford Linear Accelerator Center Topical Conference on Particle Physics, Stanford, California, August, 1984, unpublished.

[7]M. Gronau and J. Schechter, Phys. Rev. Lett. **54**, 385, 1209(E) (1985).

[8]Alternative approaches are given by M. Roos, University of Helsinki Report No. HU-TFT-84-38, 1984 (unpublished); L. Wolfenstein, Carnegie-Mellon University Report No. CMU-HEP 84-20, 1984 (unpublished).

[9]We have chosen the signs of the quark masses in the Stech model (which are not *a priori* predicted) to agree with those of the Fritzsch model. Notice that a sign change of a right-handed quark field can alter the sign of a quark mass without changing anthing else.

[10]L. L. Chau and W. Y. Keung, Brookhaven National Laboratory Report No. BNL-34643, 1984 (unpublished); E. Fernandez *et al.*, Phys. Rev. Lett. **51**, 1022 (1983); N. S. Lockyer *et al.* (Mark II Collaboration), Phys. Lett. **130B**, 444 (1983); A. Chen *et al.*, Phys. Rev. Lett. **52**, 1084 (1984); D. E. Klem *et al.* (DELCO Collaboration), Stanford Linear Accelerator Center Report No. SLAC-PUB-3379, 1984 (unpublished).

[11]The exact diagonalization for the three-generation case is given by H. Georgi and D. V. Nanopoulos in Ref. 2. The simple approximation employed here agrees, within practical accuracy, with the exact results. It has the advantage of displaying the essential physical aspects of the model.

[12]F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978); S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); R. D. Peccei and H. R. Quinn, Phys. Rev. D **16**, 1791 (1977).

[13]For recent discussions see J. Donoghue, E. Golowich, and B. Holstein, Phys. lett. **119B**, 412 (1982); J. Bijnens, H. Sonoda, and M. Wise, Phys. Rev. Lett. **53**, 2367 (1984).

[14]See, for example, M. Gronau and J. Schechter, Phys. Rev. D **31**, 1668 (1985).