Diverging Characteristic Lengths at Critical Disorder in Thin-Film Superconductors

A. F. Hebard and M. A. Paalanen
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 22 January 1985)

The characteristic lengths associated with the flux-flow resistance and with the critical fields measured at the vortex-antivortex transition temperature of thin-film In/InO_x composites have been found to be approximately equal and to diverge simultaneously near critical disorder where superconductivity disappears. The observed disorder-induced enhancement of the vortex mobility cannot be explained by the dirty-limit formula for the coherence length when used within the context of the Bardeen-Stephen description for vortex dissipation.

PACS numbers: 74.40.+k, 73.60.Ka

Recent experimental efforts to characterize the effect of disorder on the resistive transition of thin-film superconductors[1-4] have been primarily concerned with the high-temperature paraconductivity regime where fluctuations in the magnitude of the order parameter are important. These efforts include measurements of the disorder-induced trends in the mean-field transition temperature T_{c0},[1-3] the upper critical field $H_{c2}(T)$,[1-3] and the inelastic electron scattering rates extracted from magnetoconductance behavior.[4] The prevailing theoretical interpretation is that superconductivity is weakened and T_{c0} lowered by the enhanced Coulomb repulsion and depressed density of states at the Fermi level usually associated with atomic disorder.[5]

Equally interesting are the corresponding disorder-induced effects in the low-temperature region of the resistive transition where fluctuations in the phase of the order parameter are driven by thermally excited vortices[6] giving rise to a vortex-antivortex phase transition at a temperature T_c. In this Letter we address this heretofore ignored problem of how disorder weakens and eventually destroys vortex fluctuations in thin two-dimensional films. For a series of five films with increasing amounts of disorder, as measured by the room-temperature normal-state resistance R_N, we present experimental evidence that the "vortex-core" characteristic length $\xi_c(T_c)$, derived from flux-flow resistance measurements taken at $T=T_c$, undergoes a pronounced divergence near critical disorder where superconductivity disappears ($T_c \to 0$). A second characteristic length, the Ginzburg-Landau length $\xi_{GL}(T_c)$ derived *independently* from H_{c2} measurements at $T=T_c$, is found to behave a similar fashion. Given the assumption of the validity of the Bardeen-Stephen model[7] for vortex dissipation, it is found that the temperature-dependent factors in the dirty-limit formula for the coherence length cannot account for the observed divergence.

Figure 1 is a plot on logarithmic axes of the zero-field ($H=0$) resistive transitions of five different 100-Å-thick In/InO_x films which have been prepared in different oxygen ambients and which have a composite amorphous microstructure.[8] The rapid decrease in T_{c0} (downward-directed arrows) with increasing R_N together with the pronounced crossover from superconducting to insulating behavior near $\hbar/e^2$ have been reported previously.[3] Film *a* of Fig. 1 is film *b* of Ref. 3. The films of Fig. 1 represent a set of films for which measurements of T_{c0}, T_c, R_N, $H_{c2}(T_c)$, and the flux-flow resistance have been obtained. Aslamazov-Larkin theory[9] fits to the zero-field resistive transitions have been used to determine T_{c0}.[10] The omission of the Maki-Thompson contribution in this procedure has been shown in earlier investigations[11,12] to be justified for films with high R_N. The critical field H_{c2} has been defined[13] as that field necessary to create a resistance in the sample equal to the resistance measured at T_{c0}.

More central to the purpose of this paper is the determination of T_c which for each of the five films in

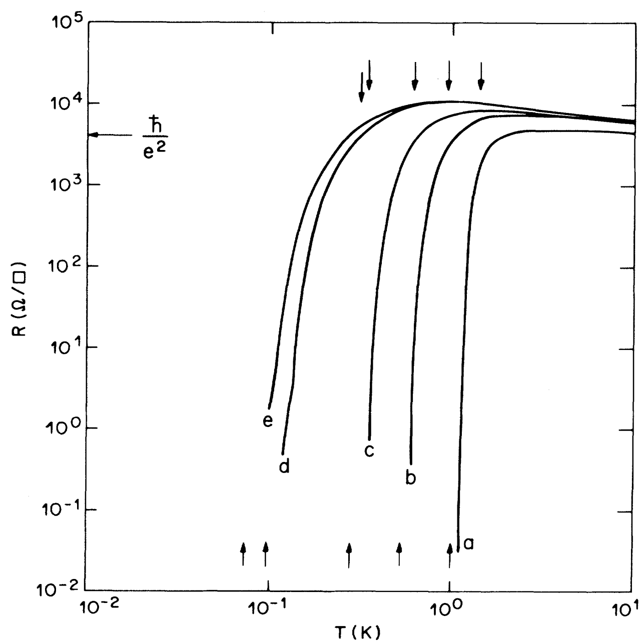

FIG. 1. Logarithmic plot of resistive transitions at $H=0$ for five films with T_{c0} (↓) and T_c (↑) indicated for each film.

Fig. 1 is represented as an upward-directed arrow. In earlier work[10,14] on similar films with less disorder ($T_c \gtrsim 1.8$ K) excellent agreement with the predictions of the Kosterlitz-Thouless theory was found by extraction of power-law exponents from the nonlinear current-voltage characteristics taken at $H=0$. These techniques are precluded in the present films with lower T_c because of the rapid onset of heating effects associated with the weaker superconducting coupling. We have observed, however, that for *all* the films of Fig. 1 there is a unique temperature, which we identify as T_c, at which the resistance is linear in H. Experimentally our justification for making this identification is based on earlier work on the flux-flow characteristics of higher-T_c films[10] together with $H=0$ current-voltage measurements on films c and d at T_c which give power-law exponents over a limited range (only one decade in voltage for film d not inconsistent with the expected theoretical value of ~ 3. The uniqueness of this identification is common to all of our films and is shown, for example, in the R vs H isotherms for film d in Fig. 2. The inset is a plot of the corresponding slopes as a function of T with the unity (dashed line) intercept defining T_c (vertical arrow). For $T > T_c$ the measurements have been taken for T sufficiently close to T_c so that the resistance due to thermally excited vortices is negligible. The power-law exponent is greater than unity for $T < T_c$ because of the screening of the interaction between bound pairs by the field-induced vortices. The low-field contributions to the magnetoresistance of the Maki-Larkin and Aslamazov-Larkin terms[4] are quadratic in H and hence appear to have no effect on the near-linear flux-flow characteristics in Fig. 2 close to T_c.

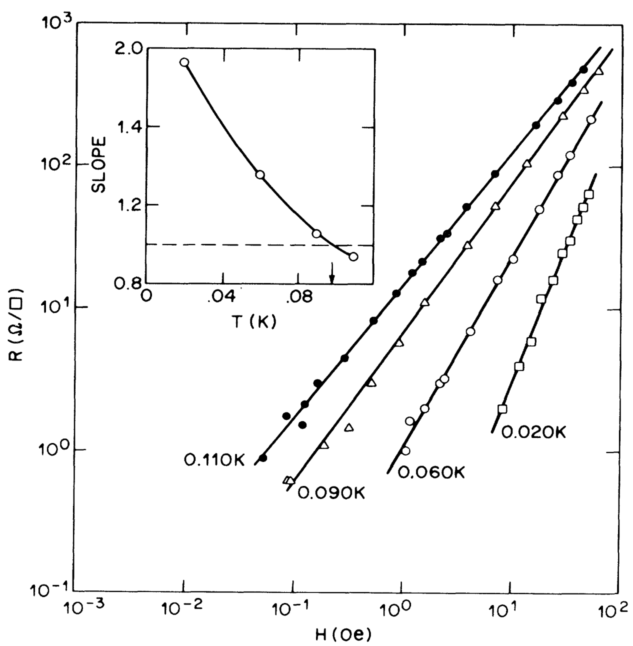

FIG. 2. Logarithmic plot of flux-flow isotherms for film d. The temperature-dependent slopes are shown in the inset, with T_c occurring at unity slope (dashed line).

By setting of the viscous drag force equal to the Lorentz force, the equation for flux-flow sheet resistance[9] can be written in the form

$$R(T) = \mu_v(T) N_v(T) \phi_0^2 / c^2, \tag{1}$$

where μ_v is the vortex mobility, N_v the areal density of thermally excited and/or field-induced vortices, and ϕ_0 the flux quantum. At $T = T_c$, where H is sufficiently high so that thermally excited vortices can be ignored (i.e., $N_v = H/\phi_0$), Eq. (1) can be used to infer a vortex mobility $\mu_v(T_c) = c^2 \phi_0^{-1} (dR/dH)|_{T_c}$ directly from the flux-flow resistance measurements. The rapid increase in $\mu_v(T_c)$ with decreasing T_c shown in Fig. 3 (open squares) is thus directly related to the observed rapid increase of $(dR/dH)|_{T_c}$. The physical picture suggested by these data is that localization and interaction effects are causing the vortex cores to become more insulating, hence reducing the dissipation in the cores for fixed vortex velocity.

A surprising aspect of our data is revealed by a comparison of the two characteristic lengths $\xi_{\rm GL}(T_c)$ and $\xi_c(T_c)$, defined respectively by the relations

$$\xi_{\rm GL}^2(T_c) = \phi_0 / 2\pi H_{c2}(T_c) \tag{2}$$

and

$$\xi_c^2(T_c) = \phi_0 e^2 (dR/dH)|_{T_c} / 2\pi\hbar . \tag{3}$$

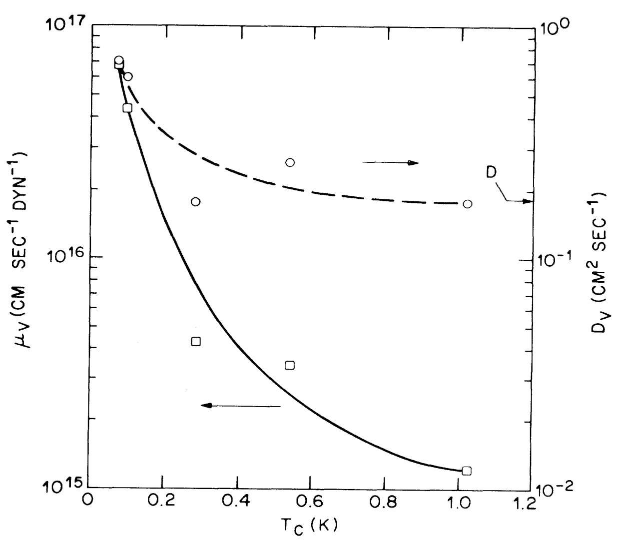

FIG. 3. Vortex mobility $\mu_v(T_c)$ (open squares) and diffusivity $D_v(T_c)$ (open circles) plotted vs T_c for films a–e. The electron diffusivity D is indicated on the right-hand axis. The solid and dashed lines are guides to the eye.

Equation (2) is the well-known relation[9] between the Ginzburg-Landau length $\xi_{GL}(T_c)$ and $H_{c2}(T_c)$. Equation (3) is suggested by the Bardeen-Stephen model[7] result for the vortex mobility

$$\mu_v(T) = 2\pi c^2 \xi_c^2(T) R_N/\phi_0^2, \tag{4}$$

where we have, however, replaced R_N by the combination of fundamental constants $\hbar/e^2 = 4114\ \Omega/\square$. This replacement emphasizes the occurrence of critical disorder at $R_N \simeq \hbar/e^2$ (Fig. 1 and Ref. 3) and appears justified because of the slow variation (< 20%) in R_N shown in Table I. By contrast the respective variations from film *a* to film *e* of $H_{c2}(T_c)$ from 12 960 to 414 Oe and of $(dR/dH)|_{T_c}$ from 0.25 to 13.95 $\Omega\ \square^{-1}\ \mathrm{Oe}^{-1}$ are much more rapid. In spite of these rapid variations in $H_{c2}(T_c)$ and $(dR/dH)|_{T_c}$ the characteristic lengths calculated from Eqs. (2) and (3) and shown in Table I turn out to be roughly equal and to diverge simultaneously with increasing disorder $(T_c \rightarrow 0)$. This equality is essentially a consequence of the *experimental result* that $H_{c2}(T_c)$ decreases and $(dR/dH)|_{T_c}$ increases with increasing disorder in such a way that the product of these quantities is approximately constant and on the order of $\hbar/e^2$.

It is instructive to explore the consequences of the assumption that the experimentally determined length $\xi_c(T_c)$ discussed above is a measure of the vortex-core size and that the Bardeen-Stephen result for $\mu_v(T_c)$ [Eq. (4) evaluated at $T = T_c$] is appropriate near critical disorder. Accordingly, when the formula[9] for the square of the dirty-limit coherence length ξ_{dl},

$$\xi_{dl}^2(T_c) = 0.70\hbar D\Delta(0)^{-1}(1 - T_c/T_{c0})^{-1}, \tag{5}$$

together with the BCS relationship[9] $\Delta(0) = 1.76 k_B T_{c0}$ for the zero-temperature energy gap $\Delta(0)$ of the disordered superconductor, is inserted into Eq. (4), then the resulting temperature-dependent factor $(T_{c0} - T_c)^{-1}$ calculated by use of the Table I entries is found to account for only 3% of the total rise in $\mu_v(T_c)$ plotted in Fig. 3. This argument assumes constant electron diffusivity *D*. Although the *ad hoc* replacement of $\Delta(0)$ by $k_B T_c$ in Eq. (5) gives a more satisfactory agreement it still accounts for less than 50% of the rise. The dirty-limit values for the core size $\xi_{dl}(T_c)$, calculated from Eq. (5) with $\Delta(0) = 1.76 k_B T_{c0}$ and $D = 0.18\ \mathrm{cm^2/sec}$[3] and listed in the final column of Table I, reveal explicitly the significantly weaker dependence of $\xi_{dl}(T_c)$ on disorder than found by the directly measured dependences of $\xi_{GL}(T_c)$ and $\xi_c(T_c)$. To our knowledge there is at present no theory which deals with a disorder-induced enhancement of $\xi_c(T_c)$.[15] One possible interpretation, consistent with the use of Eq. (5), is that the BCS relation $\Delta(0) = 1.76 k_B T_{c0}$ does not hold and hence the disorder-induced reduction in $\Delta(0)$ is greater than the disorder-induced reduction in T_{c0}.

In the easily understood limit of weak disorder where $(T_c \lesssim T_{c0})$ we can comfortably use the BCS relation $\Delta(0) = 1.76 k_B T_{c0}$ and the Kosterlitz-Thouless prediction[6,10] $T_c/T_{c0} = (1 + 0.173\epsilon_c e^2 R_N/\hbar)^{-1}$, together with Eqs. (4) and (5), to evaluate the vortex diffusivity $D_v(T_c) = \mu_v(T_c) k_B T_c = 1.46 D/\epsilon_c$. The experimental value of the vortex dielectric constant ϵ_c at T_c has been found in previous work to be slightly larger than unity.[10] Interestingly, the experimental values for $D_v(T_c)$ determined from the entries of Table I and plotted as circles in Fig. 3 are in fact quite close to the value $D = 0.18\ \mathrm{cm^2/sec}$ (indicated by an arrow in Fig. 3) inferred from H_{c2} measurements,[3] hence supporting the notion that ξ_c is a measure of the vortex-core size and that these cores can diffuse no faster than their normal electron constituents. A similar result has been reported for ^{4}He films.[16]

Finally, at critical disorder, pair-breaking processes occurring on a time scale comparable to the inelastic electron scattering time τ_i should suppress vortex fluctuations when $\hbar/\tau_i \simeq k_B T_c$. One might expect that this occurs when there is a crossover in length scales, that is when $\xi_c^2 = D\tau_i$, where $(D\tau_i)^{1/2}$ is the inelastic electron diffusion length, a quantity which *decreases* with increasing disorder. This crossover in length scales

TABLE I. Experimentally determined parameters for the five films discussed in the text. The Ginzburg-Landau length ξ_{GL}, determined from H_{c2} measurements, the vortex-core size ξ_c, determined from flux-flow measurements, and the dirty-limit coherence length ξ_{dl}, calculated from Eq. (5), are all evaluated at $T = T_c$.

Film	T_c (K)	T_{c0} (K)	R_N ($\Omega\ \square^{-1}$)	ξ_{GL} (Å)	ξ_c (Å)	ξ_{dl} (Å)
a	1.025	1.451	2900	159	142	113
b	0.542	0.973	3394	210	238	113
c	0.286	0.636	3240	290	269	125
d	0.098	0.352	3440	713	854	147
e	0.075	0.326	3454	892	1057	148

implies that at critical disorder the maximum vortex areal density $N_v \simeq (D\tau_i)^{-1}$, so that the maximum flux-flow resistance, calculated from Eq. (1) together with the approximations $D \simeq D_v$ and $\hbar/\tau_i \sim k_B T_c$, is on the order of $\hbar/e^2$, as found by experiment (cf. horizontal arrow in Fig. 1).

The authors acknowledge useful discussions with R. N. Bhatt, A. T. Fiory, and D. S. Fisher and appreciate the technical assistance of H. W. Dail and R. H. Eick.

[1]S. Okuma, F. Komori, Y. Ootuka, and S. Kobayashi, J. Phys. Soc. Jpn. **52**, 2639 (1983).

[2]J. M. Graybeal and M. R. Beasley, Phys. Rev. B **29**, 4167 (1984).

[3]A. F. Hebard and M. A. Paalanen, Phys. Rev. B **30**, 4063 (1984).

[4]G. Bergmann, Phys. Rev. B **29**, 6114 (1984), and references therein.

[5]S. Maekawa and H. Fukuyama, J. Phys. Soc. Jpn. **51**, 1380 (1982).

[6]M. R. Beasley, J. E. Mooij, and T. P. Orlando, Phys. Rev. Lett. **42**, 1165 (1979).

[7]J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965).

[8]A. F. Hebard and S. Nakahara, Appl. Phys. Lett. **41**, 1130 (1980).

[9]For an introductory treatment, see M. Tinkham, *Introduction To Superconductivity* (McGraw-Hill, New York, 1975).

[10]A. T. Fiory, A. F. Hebard, and W. I. Glaberson, Phys. Rev. B **28**, 5075 (1983).

[11]For a review, see W. J. Skocpol and M. Tinkham, Rep. Prog. Phys. **38**, 1049 (1975).

[12]M. Strongin, R. S. Thompson, O. F. Kammerer, and J. E. Crow, Phys. Rev. B **1**, 1073 (1970).

[13]M. A. Paalanen and A. F. Hebard, Appl. Phys. Lett. **45**, 794 (1984).

[14]A. F. Hebard, and A. T. Fiory, Phys. Rev. Lett. **50**, 1603 (1983).

[15]Work by A. Kapitulnik and G. Kotliar [Phys. Rev. Lett. **54**, 473 (1985)] predicts the opposite effect for the zero-temperature coherence length.

[16]M. Kim and W. I. Glaberson, Phys. Rev. Lett. **52**, 53 (1984).