Classification of Axisymmetric Vortices in ^{3}He-A

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Singular vortex lines are predicted to occur in the rotating A phase when the superfluid is confined in a parallel-plate geometry. A symmetry classification is introduced for the possible singly and doubly quantized axisymmetric vortices in ^{3}He-A in terms of the separate symmetries of the vortex hard-core and soft-core matters.

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In rotating superfluid ^{3}He-A, two different classes of quantized vortices may arise,[1] corresponding to topological charge N which may assume the values $N=0$ and $N=1$. The vortices in the class with $N=1$ are always singular; these vortices possess a hard core with radius of the order of the Ginzburg-Landau coherence length ξ, which contains a core fluid in a different state—the normal or another superfluid phase—than the bulk liquid. On the contrary, the hard core of the vortices with $N=0$ may be continuously dissolved and these vortices acquire only a soft core of the order of several dipole lengths $\xi_D \simeq (10^2-10^3)\xi$, with the bulk liquid phase everywhere. The Anderson-Toulouse-Chechetkin[2] vortex with two quanta of superfluid circulation exemplifies the continuous vortices in the class $N=0$.

Even though the singly quantized vortices belonging to the class with $N=1$ were estimated to possess a slightly lower free energy[3] than the continuous doubly quantized vortices, all the NMR signatures to date on the vortices in the rotating bulk superfluid A phase[4] are only consistent with theoretical calculations which assume continuous vortices.[5] This applies both to experiments carried out with rotation started after cooldown to the superfluid A phase and to those with continuous rotation through the superfluid transition temperature, which were performed in a search for the singular vortices.

In this Letter we suggest a way to generate singular vortices in the A phase: singly quantized vortices could be nucleated in rotating ^{3}He-A in a parallel-plate geometry, where the plate spacing is of the order of ξ_D; this serves to make the free-energy difference of the vortices more pronounced. We introduce a symmetry classification for the possible vortex lines with integer number m of superfluid circulation quanta. As for the B-phase vortices[6,7] and the continuous A-phase vortices,[5] the most important symmetries which govern the properties of the singular A-phase vortices are discrete, including the time T and space P parities, and space rotation U_2 of the vortex line about a perpendicular axis. For the B-phase vortices which possess hard cores this gives five types of axisymmetric vortices (o,u,v,w,uvw) with the same m, but with different broken discrete symmetries in the core.[6,7]

The continuous A-phase vortex (which in an open geometry has no axial symmetry as a result of the superflow orientating the orbital anisotropy vector $\mathbf{l}$ in the x-y plane outside the soft core) may exist in two different states[5]: v and w. Since parity P is broken in the soft core of these vortices in different ways, the v vortex possesses a spontaneous electric polarization, while the w vortex has a spontaneous superflow along the vortex axis.

A classification of the singular vortices in the A phase is more complicated than in ^{3}He-B as a result of the two length scales (ξ and ξ_D) in the vortex-core structure. We find that the discrete symmetry may be broken in the hard core and in the soft core of the singular vortices independently and in a different manner: This produces a rich variety of the possible vortex structures. The discrete-symmetry classification of the vortices does not depend on the continuous symmetry in an essential way. Therefore, for the sake of simplicity and clarity, we here confine ourselves to the vortices with axial symmetry: It is likely that axisymmetry is indeed realized in the parallel-plate geometry, where the orientation of the $\mathbf{l}$ vector by superflow is prevented by the boundary conditions.

In the parallel-plate geometry, with the plates normal to the axis of rotation $\hat{\mathbf{\Omega}}$, the $\mathbf{l}$ vector may be orientated in two directions: parallel or antiparallel with $\hat{\mathbf{\Omega}}$. The vortex structure depends essentially on the orientation of $\mathbf{l}$ with respect to $\hat{\mathbf{\Omega}}$. Therefore, for fixed ($\hat{\mathbf{l}}=\hat{\mathbf{z}}$) direction of $\mathbf{l}$, we shall consider the vortices with both positive $m=1,2$ which arise for $\hat{\mathbf{\Omega}}=\hat{\mathbf{l}}$, and negative $m=-1,-2$ corresponding to $\hat{\mathbf{\Omega}}=-\hat{\mathbf{l}}$.

The possible elements of symmetry for the m-quantum vortex can be found from the asymptotic form for the vortex: The asymptotic form possesses the maximal symmetry, which may then be broken in the soft or hard cores. Outside the soft (dipole) core, where $\mathbf{l}$ is fixed by the boundary ($\hat{\mathbf{l}}=\hat{\mathbf{z}}$), and the magnetic anisotropy axis $\mathbf{d}$ is parallel with $\mathbf{l}$ as a result of the spin-orbital (dipole) coupling, the order-parameter matrix, which for the A phase has a factorized form

$$A^A_{\alpha i}=\Delta_A d_\alpha \psi_i,\quad \mathbf{d}^2=1,\quad \mathbf{d}=\mathbf{d}^*,\quad \boldsymbol{\psi}^2=0,\quad |\boldsymbol{\psi}|^2=1, \tag{1}$$

where Δ_A is the A-phase energy gap, and i and α denote the spin and orbital indices, is given by

$$A^0_{\alpha i}=\Delta_A \hat{\mathbf{z}}_\alpha \frac{\hat{\mathbf{x}}_i+i\hat{\mathbf{y}}_i}{\sqrt{2}}e^{im\phi}. \tag{2}$$

Here ϕ is the azimuthal angle in the x-y plane.

Equation (2) has two continuous-symmetry groups, with the generators $\hat{\mathbf{S}}_z$ and $\hat{\mathbf{L}}_z-(m+1)$, where $\hat{\mathbf{S}}_z$ is the generator of spin rotations about the $\hat{\mathbf{z}}$ axis ($\hat{\mathbf{S}}_z d_\alpha = ie_{\alpha\beta z}d_\beta$), and $\hat{\mathbf{L}}_z$ is the generator of orbital rotations, including the internal and external rotations ($\hat{\mathbf{L}}_z=\hat{\mathbf{L}}_z^{\rm int}+\hat{\mathbf{L}}_z^{\rm ext}$; $\hat{\mathbf{L}}_z^{\rm ext}=-i\,\partial/\partial\phi$, $\hat{\mathbf{L}}_z^{\rm int}\psi_i=ie_{ijz}\psi_j$). However, because of the spin-orbital coupling in the soft core, the two symmetries cannot be conserved separately. Therefore, the axisymmetric vortex solution should satisfy only one equation:

$$\hat{\mathbf{Q}}_m A_{\alpha i}=0,\quad \hat{\mathbf{Q}}_m=\hat{\mathbf{S}}_z+\hat{\mathbf{L}}_z-(m+1). \tag{3}$$

The general solution of Eq. (3) is

$$A_{\alpha i}=\Delta_A \sum\nolimits_{\mu\nu} a_{\mu\nu}\lambda^\mu_\alpha\lambda^\nu_i,\quad a_{\mu\nu}=C_{\mu\nu}(r)e^{i(m+1-\mu-\nu)\phi}, \tag{4}$$

where $\lambda^\pm_{i,\alpha}=\frac{1}{2}\sqrt{2}(\hat{\mathbf{x}}_{i,\alpha}\pm i\hat{\mathbf{y}}_{i,\alpha})$, $\lambda^0_{i,\alpha}=\hat{\mathbf{z}}_{i,\alpha}$ are the spherical eigenfunctions of $\hat{\mathbf{L}}_z$ and $\hat{\mathbf{S}}_z$ with eigenvalues ν and μ, and the $C_{\mu\nu}(r)$ are functions of radial distance r from the vortex axis. In the soft-core region, but outside the hard core, $r>>\xi$, the A-phase state of Eq. (1) is undisturbed. Hence the parameters $C_{\mu\nu}$ may be factorized as follows:

$$C_{\mu\nu}=a_\mu b_\nu,\quad \mathbf{d}=\sum\nolimits_\mu a_\mu(r)\boldsymbol{\lambda}^\mu e^{i\mu\phi},\quad \boldsymbol{\psi}=\sum\nolimits_\nu b_\nu(r)\boldsymbol{\lambda}^\nu e^{i(m-\nu+1)\phi}, \tag{5}$$

where the functions $a_\mu(r)$ and $b_\nu(r)$ fulfill the conditions $a_\mu=a^*_{-\mu}$, $\sum_\mu |a_\mu|^2=1$, $\sum_\nu b_\nu b_{-\nu}=0$, and $\sum_\nu |b_\nu|^2=1$, following from Eq. (1).

The number of these parameters may be reduced by taking the discrete symmetry into account. The discrete symmetries of the asymptotic form, Eq. (2), include the three elements

$$P_1=P(U_\pi)^{m+1},\quad P_3=TU_2U_\pi,\quad P_2=P_1P_3. \tag{6}$$

Here P, with $PA_{\alpha i}=-A_{\alpha i}(-\mathbf{r})$, and T, with $TA_{\alpha i}=A^*_{\alpha i}$, are the space and time inversions; $U_2=O_{x,\pi}$ is a rotation (by angle π around axis $\hat{\mathbf{x}}$) which reverses the vortex axis, and U_π denotes a gauge transformation, which changes the sign of the order parameter $A_{\alpha i}$ but does not influence any measurable physical quantities. The amplitudes $C_{\mu\nu}$ in Eq. (4) are transformed under the symmetry elements in Eq. (6) as follows:

$$P_1C_{\mu\nu}=(-1)^{\mu+\nu+1}C_{\mu\nu},\quad P_2C_{\mu\nu}=C^*_{\mu\nu},\quad P_3C_{\mu\nu}=(-1)^{\mu+\nu+1}C^*_{\mu\nu}. \tag{7}$$

Let us first consider the structure of the most-symmetric o vortex, with all of the discrete symmetries, Eqs. (6) and (7), conserved. In the soft-core region, but outside the hard core, $r>>\xi$, where the order parameter is given by Eq. (5), the symmetry constraints yield for the o vortex the same structure as outside the soft core; i.e., the asymptotic form of Eq. (2) is carried all the way through the soft core and becomes the asymptotic form for the hard-core structure. In the hard-core region where the dipole energy may be neglected, both of the continuous-symmetry groups [with the generators $\hat{\mathbf{S}}_z$ and $\hat{\mathbf{L}}_z-(m+1)$] are separately conserved, and one has the additional constraint $\hat{\mathbf{S}}_zA_{\mu\nu}=0$. In conjunction with the constraints of discrete symmetry, this yields the following structure for the o vortex with m quanta of circulation in the entire range of r:

$$A_{\alpha i}=\Delta_A\hat{\mathbf{z}}_\alpha\left(C_{0+}\frac{\hat{\mathbf{x}}_i+i\hat{\mathbf{y}}_i}{\sqrt{2}}e^{im\phi}+C_{0-}\frac{\hat{\mathbf{x}}_i-i\hat{\mathbf{y}}_i}{\sqrt{2}}e^{i(m+2)\phi}\right), \tag{8}$$

where C_{0+} and C_{0-} are real with $C_{0+}\to\sqrt{2}$ and $C_{0-}\to 0$ for $r>>\xi$. Note that the fluid is normal on the vortex axis [$C_{\mu\nu}(0)=0$] for all the o vortices, except the o vortex with $m=-2$: Only in the latter vortex the singular phase factor $e^{i(m+2)\phi}$ becomes regular and C_{0-} is not forced to vanish on the axis. This corresponds to the A phase in the vortex core with reversed direction of $\hat{\mathbf{l}}=-\hat{\mathbf{z}}$.

As a rule, it is more advantageous[7,8] to break one of the discrete symmetries in Eq. (6). This gives rise to the u, v, or w vortices with respective symmetries P_1, P_2, or P_3, and to the uvw vortex which has no discrete symmetry. However, this classification is very rough for the A-phase vortices with two different cores and is only rigorous for the continuous vortex which has just one core, the soft core, where the A-phase state is not distorted. The axisymmetric continuous vortex may be constructed only from the o vortex with $m=-2$, which contains A phase with $\hat{\mathbf{l}}=\hat{\mathbf{z}}$ outside the soft core, and with $\hat{\mathbf{l}}=-\hat{\mathbf{z}}$ on the vortex axis. To match the A phase in the intermediate region, one has to construct a nonuniform $\mathbf{l}$ texture and therefore break space-parity P. Thus the continuous axisymmetric vortex may be in three states: the P_2-symmetric v vortex, the P_3-symmetric w vortex, and the least-symmetric uvw vortex.

The continuous v vortex with $m=-2$ is nothing but the Anderson-Toulouse-Chechetkin texture. In terms of $\mathbf{v}_s$ and $\mathbf{l}\parallel\mathbf{d}$, this texture has the following representation:

$$\mathbf{l}=\hat{\mathbf{z}}\sin\eta(r)\pm\hat{\mathbf{r}}\cos\eta(r), \quad \mathbf{v}_s=(\hbar/2m_3r)[1+\sin\eta(r)]\hat{\boldsymbol{\phi}}. \tag{9}$$

Here $\eta(r)$ varies from $-\pi/2$ on the vortex axis to $\pi/2$ outside the soft core. The signs $\pm$ indicate the two-fold degeneracy of the vortex.

As distinct from the v vortex, the continuous w vortex with $m=-2$ displays a twisted $\mathbf{l}$ texture:

$$\mathbf{l}=\hat{\mathbf{z}}\sin\eta(r)\pm\hat{\boldsymbol{\phi}}\cos\eta(r), \quad \mathbf{v}_s=-(\hbar/2m_3r)[1+\sin\eta(r)]\hat{\boldsymbol{\phi}}. \tag{10}$$

To facilitate a further fine classification of the vortices with two types of cores one needs to specify the region where the symmetry breaking takes place. There are four important regions to consider in such vortices: Region I of the asymptotic form, $r\gg\xi_D$, with total o symmetry; region II of the soft core, $r\simeq\xi_D$; region III of the intermediate asymptotic form, $\xi_D\gg r\gg\xi$; and, finally, region IV of the hard core, $r\simeq\xi$. The discrete symmetry which is broken in the soft core, may in principle be restored in region III and then again broken in region IV, but in a different way. Therefore, there exists a large number of different vortex substates,[8] which we specify by a code of three letters, corresponding to the symmetry in regions II, III, and IV, respectively. For example, the least-symmetric uvw vortex state may be in the substate v-o-w (which has the symmetry P_2 in the soft core, the total symmetry o in region III, and symmetry P_3 in the hard core) or in the substates w-o-v, v-v-uvw, etc. The P_2-symmetric v-vortex state may contain four different substates: v-v-v, v-o-o, v-o-v, and o-o-v, and the P_3-symmetric state w has the four substates w-w-w, w-o-o, w-o-w, and o-o-w.

However, the most interesting classification is found for the vortices with $m=-1$. Since $S_z+L_z=0$ for this vortex class [see Eq. (3)], new elements of discrete symmetry may appear in region III of the intermediate asymptotic form. As elements of the o symmetry in Eq. (6), there also occur new elements due to different combinations of the three basic transformations T, P, and U_2 (here we do not consider U_π which does not influence any of the observable variables: $\mathbf{l}$, $\mathbf{v}_s$, electric polarization, etc.). Time-inversion symmetry is broken in the A phase because $T\mathbf{l}=-\mathbf{l}$. Therefore, T may enter in a combined symmetry only. Hence there exist three different sets of possible discrete symmetries for the asymptotic form of linear defects in the A phase, which we denote the o_1, o_2, and o_3 symmetries:

$$P,\quad TU_2,\quad PTU_2, \tag{11a}$$

$$U_2,\quad PT,\quad PTU_2, \tag{11b}$$

$$PU_2,\quad TU_2,\quad PT. \tag{11c}$$

The o_1 symmetry in Eq. (11a) is our o symmetry [Eq. (6)], which characterizes the vortices in the A and B phases, while the symmetry o_2 in Eq. (11b) and the symmetry o_3 in Eq. (11c) characterize a radial disgyration ($\hat{\mathbf{l}}=\hat{\mathbf{r}}$, $\mathbf{v}_s=\mathbf{0}$) and a tangential disgyration ($\hat{\mathbf{l}}=\hat{\boldsymbol{\phi}}$, $\mathbf{v}_s=\mathbf{0}$), respectively. Both have continuous symmetry $S_z+L_z=0$ and may thus serve as the intermediate asymptotic form in region III of the vortex with $m=-1$. We denote the most-symmetric defect with the asymptotic form of a radial disgyration as the o_2 defect and that with the asymptotic form of a tangential disgyration as the o_3 defect; then, by x, y, and z we denote the defect structures, obtained by breaking the symmetries o_2 or o_3, i.e., the U_2-symmetric, the TP-symmetric, and the PU_2-symmetric linear defects, respectively.

These new symmetries, Eqs. (11b) and (11c), give two additional substates for the v-vortex state with $m=-1$: v-o_2-o_2 and v-o_2-v; two substates for the w vortex: w-o_3-o_3 and w-o_3-w; and many additional uvw vortices: v-o_2-x, w-o_3-z, etc. For example, the axisymmetric $m=-1$ vortex, calculated by Fetter, Sauls, and Stein,[9] which transforms to the radial disgyration in region III and then to the polar phase[10] in region IV is—within our classification—the substate v-o_2-o_2 of the v vortex. In regions III and IV the constraints imposed by the symmetry o_2 in Eq. (11b) [$U_2C_{0\nu}=(-1)^\nu C_{0,-\nu}=C_{0\nu}$, $TPU_2C_{0\nu}=C^*_{0\nu}=C_{0\nu}$] require the following form for this v-o_2-o_2 vortex:

$$A_{\alpha i}=\Delta_A\hat{\mathbf{z}}_\alpha\sum_\nu\lambda_i^\nu C_{0\nu}e^{-i\nu\phi} =\Delta_A\hat{\mathbf{z}}_\alpha(a\hat{\mathbf{z}}_i+ib\hat{\boldsymbol{\phi}}_i), \tag{12}$$

where the $a(r)$ and $b(r)$ are real parameters, both tending to unity for $r\gg\xi$. While the singularity in $\hat{\boldsymbol{\phi}}$ at the vortex axis forces the prefactor $b(r)$ to vanish in the limit $r=0$, the coefficient $a(r)$ need not vanish. Therefore, the v-o_2-o_2 vortex always contains polar phase on the axis. Other superfluid phases may appear if the continuous symmetries with the generators $\hat{\mathbf{S}}_z$ and $\hat{\mathbf{L}}_z$ are broken in the hard core.

To summarize, axisymmetric vortex lines may appear in rotating ^{3}He-A only in the parallel-plate geometry, and we presented a symmetry classification of the possible vortex states. In addition to the continuous symmetries of the vortex, there are three discrete-vortex-symmetry operations which lead to the possible existence of five different vortex states (o, u, v, w, and uvw) for each m ($m = \pm 1$, $m = \pm 2$), as in the B phase.[6,7] However, for each of these vortex states there emerges a new spectrum of fine structure, which has been classified in terms of the separate symmetries of the soft and hard cores, producing a rich variety of the possible vortex substates.[8] The physical properties of the vortices that are associated with the broken discrete symmetries, such as the vortex magnetization, the electric dipole moment of the vortex, and the possibility of spontaneous axial supercurrents, are discussed in detail elsewhere[7,8]; these properties can help identify the vortices in an experiment.

In practice, one would employ a stack of maybe several thousand parallel plates. The **l** vector in a given spacing is independent from that in the other spacings, and thus equally probable to point up or down. Where **l** points down, one could find singular polar-core vortices with $m = -1$, or continuous axisymmetric Anderson-Toulouse-Chechetkin vortices with $m = -2$. However, whenever the **l** vector points up, axisymmetric vortices with $m = 1$ and $m = 2$ both have normal-fluid core. To destroy the hard core of the $m = 2$ vortex, which belongs to the topological class with $N = 0$, axial symmetry should be broken.

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