Muonic Atoms with Vacant Electron Shells

R. Bacher, D. Gotta, and L. M. Simons
Institut für Kernphysik, Kernforschungszentrum Karlsruhe, Karlsruhe, Federal Republic of Germany, and Institut für Experimentelle Kernphysik, Universität Karlsruhe, Karlsruhe, Federal Republic of Germany

and

J. Missimer and Nimai C. Mukhopadhyay[(a)]
Theory Group, Schweizerisches Institut für Nuklearforschung, CH-5234 Villigen, Switzerland
(Received 31 January 1985)

We show that the cascade in muonic atoms with $Z < 20$ ejects sufficient atomic electrons to ionize an isolated muonic atom completely. In gases, the rates with which electrons refill the atomic shell can be accurately deduced from measured and calculated electron transfer cross sections. Thus, we can conclude that completely ionized muonic atoms can be prepared in gases, and that they remain isolated for long enough times at attainable pressures to facilitate studies of fundamental interactions in muonic atoms.

PACS numbers: 36.10.Dr

The muonic atom completely ionized of atomic electrons is a quantum mechanical system as simple as the hydrogen atom.[1,2] The production of muonic atoms in this ideal state would facilitate studies of numerous fundamental phenomena. Examples are energy shifts due to quantum electrodynamics or exotic interactions,[3] mixing of the $2S$ and $2P$ states in light muonic atoms due to the neutral-weak-current interaction,[4] and muon capture from hyperfine components of the $1S$ state.[5] However, such studies have been hindered by the fact that even an incompletely ionized muonic atom is very attractive to electrons and consequently any vacancies in its electron shell refill.

In this Letter, we consider the ionization of an isolated muonic atom, and deduce which atoms might be prepared in a completely ionized state. There follows a discussion of the times for which muonic atoms remain isolated in gases and solids. We conclude that completely ionized muonic atoms with atomic number $Z < 20$ can be prepared in gases, and that they remain isolated for long enough times at attainable pressures to facilitate the studies specified above.

Atomic electrons are ionized during the atomic capture of the muon[6] and the subsequent cascade.[2] The cascade involves the ejection of electrons due to muonic Auger, electronic Auger, and Coster-Kronig processes. The muonic Auger process, in which the muon deexcites by ejecting an electron from the atomic shell, is the dominant mode of deexcitation for muonic states with principal quantum numbers greater than $n \sim 5$. The highest state at which radiative transitions of the muon dominate increases slowly with atomic number Z; for $Z = 8$ the highest state is $n = 4$ and for $Z = 36$, it is $n = 6$.

The initial distribution of atomic states into which muons are captured is not precisely known; distributions of states with principal quantum number $n \sim 14$ are usually assumed in quantum cascade calculations. In these states, the muon is well localized within the electron K shell and reliable quantum calculations are possible. Conversion of L-shell electrons dominates the muonic Auger transitions for states between $n \simeq 14$ and $n = 10$. Energy conservation permits the conversion of K-shell electrons only for states $n \leq 9$; the ejection of K-shell electrons is then the dominant muonic Auger process.

Holes in the K or L shell of atoms produced by conventional methods[7,8] are observed to produce vacancy cascades; chains of Auger and Coster-Kronig processes refill the holes in the interior shells while ejecting electrons from the outermost shell. A hole produced by a muonic Auger process will induce similar cascades if the rates for the electronic processes[9] are larger than the muonic Auger rates. A summary of the rates relevant for μNe and μAr are presented in Table I.

TABLE I. Representative rates (inverse seconds) of transitions which determine the muonic cascade and the state of the electron shell.

Process	Muonic atom Neon	Argon
Muonic Auger[a]		
L shell: $n > 9$	2×10^{14}	4×10^{14}
K shell: $n > 9$	6×10^{14}	7×10^{14}
L shell: $n > 7$	1×10^{13}	3×10^{14}
Auger and Coster-Kronig[b]		
KLL	4×10^{14}	7×10^{14}
KLM	...	1×10^{14}
$L_1L_{23}M$	...	4×10^{15}
$L_{23}MM$	...	2×10^{14}
Radiative $L \rightarrow K$	5×10^{12}	8×10^{13}

[a]Based on Ref. 10. [b]Reference 9.

In muonic atoms with atomic number $Z < 12$, the muonic Auger rates dominate until the muon has reached a state with $n=7$ and ejected the K-shell electrons. If enough L-shell electrons remain, they repopulate the K shell via a KLL Auger process. The resultant K electron is then ejected by a muonic Auger transition. With the assistance of the Akylas-Vogel program,[10] we have verified that the probability is less than 0.25 of an electron remaining bound to μNe by the time that the muon reaches the state $n=5$ after descending through the circular orbitals, which dominate the muonic cascade.

In muonic atoms with atomic numbers, $12 \leqslant Z < 20$, the rate for refilling a hole in the $2s$ state via a Coster-Kronig process $(L_1L_{23}M)$ is almost an order of magnitude larger than the rate for a muonic Auger transition in which a $2s$ electron is ejected. Thus, the $2s$ orbital is refilled and two electrons are removed from the $2p$ and $3s$ or $3p$ orbitals each time that a muonic transition occurs until the muon reaches states with $n=9$. The conversion of K-shell electrons in the succeeding two transitions induces a vacancy cascade of KLL, KLM, and $L_{23}MM$ Auger transitions which depletes the L and M shells. These cascades may be insufficient to ionize μAr completely, if it is effectively a neutral atom when the muon reaches a state with $n=14$. However, assuming the electron's shell to be complete at this state appears to conflict with experimental intensities.[11] A cascade calculation incorporating an initial electron shell consistent with experiment indicates that the probability of an electron remaining bound to μAr by the time that the muon reaches $n=5$ is less than 0.6.

A decisive measure of the ionization of muonic atoms is provided by a ratio of two radiative-transition intensities. The numerator is the intensity of the last transition between states for which muonic Auger processes dominate; the denominator is the intensity of the first transition between states for which radiative transitions dominate. In μNe and μAr, the transitions are those between the $n=6$ and 5 states and between the $n=5$ and 4 states, respectively. The ratio varies as much as 30% with the population of the K shell at the time that the muon occupies states with $n=6$; it varies only 3% with reasonable initial muon distributions. The determination of this ratio is difficult because transitions between the $n=6$ and 5 states are difficult to observe experimentally. However, they have been detected in N_2 at low pressures.[12]

We now discuss the length of time that a muonic atom remains isolated in gases. The electron refilling rate

$$R_e^g = \rho v \sigma_T$$

is determined by the cross section for electron transfer σ_T between neutral atoms and the ionized muonic atoms, by the target-gas density ρ, and by the velocity v of the muonic atom at the time it reaches the state relevant for a given experiment.

The velocity of the muonic atoms is the most difficult of these quantities to specify. Estimates of the kinetic energies of muonic atoms range from thermal energies[6] to the maximum energy which can be transferred in the muon capture process. For a muonic atom of atomic weight $M_A \simeq AM_N$, thermal energies imply average velocities

$$V_A^t \approx \left(\frac{3}{2}\frac{T}{M_A}\right)^{1/2} \approx 1.6\times 10^4 \left(\frac{T}{A}\right)^{1/2} \frac{\text{cm}}{\text{sec}},$$

where T is the temperature in degrees Kelvin. Maximum energy transfer yields velocities

$$V_A^\mu \approx \frac{M_\mu}{M_A}\left(\frac{2T_\mu}{M_\mu}\right)^{1/2} \approx 4.7\times 10^5 \frac{\sqrt{T_\mu}}{A} \frac{\text{cm}}{\text{sec}},$$

where T_μ is the kinetic energy in electronvolts of the muon before capture. A most probable value is 20 eV.[6] For light muonic atoms ($A \lesssim 40$) and for room temperatures, these velocities are of order 10^5 cm/sec, and comparable to the recoil velocity of an atom after a K_α transition,

$$V_A^r \approx \frac{3}{8}(\alpha Z)^2 \frac{M_\mu}{M_A} c \approx 3.4\times 10^4 Z \frac{\text{cm}}{\text{sec}}.$$

Thus, a velocity of 10^5 cm/sec can be assumed in our calculations.

The cross sections for electron transfer can be estimated by the assumption that the muonic atom is equivalent to a fully ionized atom with atomic number one less than that of the neutral target atom. For the gas densities we consider, the muon reaches the lowest states of the muonic cascade before colliding with another atom. Thus, the muon is localized well within the electronic K shells and the assumption is valid. The necessary electron-transfer cross sections are essential to studies of x-ray lasers, tokamak instabilities, and solar spectra; they have been measured with use of heavy-ion collisions by at least two groups.[13,14] The cross sections can be described by a simple geometrical model,[13] which provides a good approximation to the measured cross sections. The calculations and measurements show a weak dependence of the cross sections on velocity and atomic number for the range of these quantities relevant to us. For example, the effective cross section for electron transfer in the collision $Ne^{+10} + Ne$ has been measured[14] at a velocity of $v = 6.8\times 10^5$ cm/sec to be

$$\sigma_T = 2.8\times 10^{-15}\ \text{cm}^2.$$

This cross section yields for the refilling rate in muonic neon

$$R_e^g = 10^{10} P\ \text{sec}^{-1},$$

where P is the target-gas pressure in atmospheres. The weak dependence of the cross sections on atomic number implies that this equation yields the correct order of magnitude for all monoatomic gases.

The equation for the refilling rate can be used to understand the relationship between pressure and intensity of the transition (K_α) between $2P$ and $1S$ states in diatomic nitrogen and in the noble gases by Ehrhart *et al.*[11] According to the equation, refilling supplies electrons at pressures of 10^5 atm as fast as muonic Auger transitions can convert them. Since the dominant electric dipole modes change the principal quantum number by only one unit, muonic Auger transitions tend to populate the suborbitals (l) of upper muonic states ($n > 5$) with equal probability. The nP states among them then yield the more energetic components of the K series: $8 \rightarrow 1$, $7 \rightarrow 1, \ldots,$ etc. The contribution of the K_α transition to the total intensity of the K series decreases to less than 75% in N_2.

At pressures less than 10 atm, the refilling rate is less than that of the slower muonic Auger transitions. Refilling does not occur during the cascade, unless it proceeds through the $2S$ state. Therefore, the relative frequency of radiative transitions increases. Since the dominant electric dipole modes can change the principal quantum number by several units, the radiative transitions tend to direct the cascade through the circular orbits ($l = n-1$). The contribution of the K_α transition to the total intensity of the K series thereby attains a maximum.

The increases of the K_α intensity with decreased pressure is evident in the measurements of Erhart *et al.* In the muonic atoms of N_2, Ne, and Ar, the K_α transitions exhibit an almost universal relative intensity of about 91% at 0.5 atm. This approximate universality indicates that the maximum relative K_α intensity has been attained: The refilling is too slow at this pressure to influence the relative intensities of the K series. The magnitude of the maximum is predicted by the cascade calculations which show that muonic neon and argon are completely ionized at low pressures by the cascade. The intensities decrease slowly with pressure in Ne and Ar. The much more rapid variation in N_2 can be ascribed to higher effective velocities produced by the Coulomb explosion,[7] which scales the refilling rates by a factor of 20.[11] The refilling rate in N_2 is then of order 10^{13} $\sec^{-1}$ at the maximum pressure of 50 atm; the experimental curve has flattened noticeably at this pressure, although the asymptote predicted for high pressure has not yet been reached. In the muonic atoms, Kr and Xe, the K_α intensity varies little, if at all. Gas pressure and the resultant electron refilling can have little influence on the relevant K_α intensity, because enough electrons remain bound to these heavy atoms to direct the cascade via muonic Auger transitions.

Although suggestive, the K_α intensity is too sensitive to the difficult-to-determine initial distributions of the cascade and proceeds too rapidly to reflect electron refilling very clearly. Transitions of the muonic $2S$ state provide much more decisive tests of the state of the electron shell. Two of the important Auger decay modes of the $2S$ state in light muonic atoms are the electric dipole transition to the $2P$ state and the electric monopole transition to the $1S$ state. The energy difference between $2S$ and $2P$ states is insufficient to permit the ejection of a K-shell electron for $Z < 12$; the electric dipole Auger transition monitors, then, the population of the electron L shell. The electric monopole Auger transition, on the other hand, is significant only when the K shell is occupied.

Three types of effects are in principle observable. First, the intensity of electric monopole or dipole electrons must decrease and then vanish with decreasing pressure. Second, the intensities of radiation due to transitions between the $2S$ and $2P$ states and between the $2P$ and $1S$ states must vary with pressure. The intensity of the former transition will increase with decreasing pressure as the corresponding muonic Auger transitions extinguish; that of the latter must decrease with decreasing pressure. The third effect is the appearance and increasing intensities of the two-photon and magnetic dipole transitions of the $2S$ state as the pressure decreases.

The detection of Auger electrons would indicate most directly the state of the electron shell, but is exceedingly difficult as a result of scattering in the gas. Similarly absorption of the radiation impedes measurement of the intensity of the transition between $2S$ and $2P$ states. The pressure variation of the intensity of the transition between $2P$ and $1S$ states is about 2% when no coincidence condition is imposed to assure that the $2P$ state was fed by the $2S$ state; such precise measurements of intensities are feasible experimentally. The two-photon transition is difficult to observe experimentally.[15]

The radiative transition most sensitive to the state of the electron shell is the magnetic dipole transition. Observing the transition requires that it be distinguished from the high-energy part of the two-photon spectrum and from the transition between $2P$ and $1S$ states; such discrimination appears feasible in μB, μNe, and μAr.[4] The intensity of the magnetic dipole transition increases by factors of 10 when the electron shell is completely ionized, but the maximum intensities achieved are 10^{-4} in μAr and 4×10^{-5} in μNe.

In this Letter, we ground three propositions. First, the muonic cascade is very likely to completely ionize isolated light muonic atoms ($Z < 20$) via the muonic Auger transitions and the resultant vacancy cascades.

The atom will be ionized by the time that the muon reaches the state $n = 5$. Second, the electron refilling rates in monoatomic gases are sufficiently small at atmospheric pressure that the muonic atom remains ionized during the remaining muonic cascade, unless the cascade proceeds through the $2S$ state. Finally, the decay modes of the $2S$ state provide a good monitor of electron refilling.

The conditions required to study fundamental interactions in completely ionized muonic atoms can now be discussed with some precision. In measurements of energy shifts due to quantum electrodynamics or exotic interactions, the effect of electron screening is for several transitions and regions of Z the overwhelming uncertainty in the calculations used to interpret the experiments.[3] Since light muonic atoms are ionized by the time that the muon reaches the state $n = 5$ and remains so in monoatomic gases at atmospheric pressure until the muon reaches the $n = 2$ state, experiments are conceivable in which the effects of electron screening may be neglected. In molecular gases, the pressures must be reduced to accommodate the Coulomb explosion; the enhanced refilling rates in this case require that the pressure be reduced by at most a factor of 20.

Measurements of the asymmetries resulting from weak neutral currents in muonic atoms require detection of the magnetic dipole transition of the $2S$ state.[4] The presence of atomic electrons precludes the detection of this transition, as explained above. The lifetimes of the $2S$ state vary between 4×10^{-8} sec in μB and 5×10^{-12} sec in μAr; therefore pressures as low as 10^{-3} atm might be required to measure asymmetries in μB, if the effect of the Coulomb explosion is included. A $2S$ state lifetime in μNe of 10^{-9} sec implies that the pressure required to measure weak neutral-current asymmetries in this muonic atom is approximately 10^{-1} atm. These conclusions are consistent with the experimental results of Carter *et al.*,[15] who found no indication of a $2S$ state in N_2, O_2, or B_2H_6 at pressures in the neighborhood of 1 atm.

Experiments conceived to measure muon capture from hyperfine components of the $1S$ state[5] are confronted by the lifetime of this state, which varies between 2×10^{-6} sec in Li and 5×10^{-7} sec in Cl. Electron refilling of the L shell during the lifetime of the $1S$ state permits a muonic Auger transition between the upper and lower hyperfine components to occur before capture. In order to prevent electron refilling during the lifetime of the $1S$ state, pressures of order 10^{-4} atm in monoatomic gases are required.

The pressures required to perform the three experiments mentioned above are now attainable. The cyclotron trap is designed to provide a high stopping density of muons at pressures as low as 10^{-2} atm[16]; the magnetic bottle has measured x-ray spectrum of muonic hydrogen at pressures as low as 3×10^{-4} atm.[17] With these and similar devices, the studies of the muonic cascade and electron refilling necessary to verify experimentally our arguments are feasible, and investigations of fundamental processes in muonic atoms are possible with new standards of precision.

We are pleased to acknowledge useful discussion with F. Kottmann and D. Taqqu. One of us (N.C.M.) acknowledges support by the U.S. Department of Energy under Contract No. DE-AC02-83ER40114, and is grateful to F. Lenz for the hospitality at the Swiss Institute for Nuclear Research.

(a)Permanent address: Department of Physics, Rensselaer Polytechnic Institute, Troy, N.Y. 12181.

1C. S. Wu and L. Wilets, Annu. Rev. Nucl. Sci. **19**, 527 (1969); S. Devons and I. Dueroth, Adv. Nucl. Phys. **2**, 527 (1969); C. J. Batty, Sov. J. Part. Nucl. **13**, 71 (1982).

2J. Hüfner, F. Scheck, and C. S. Wu, in *Weak Interactions,* Muon Physics, Vol. I, edited by V. Hughes and C. S. Wu (Academic, New York, 1977), p. 202.

3H.-J. Leisi, in *Exotic Atoms: Proceedings of the International School of Physics of Exotic Atoms, Erice, Italy,* edited by G. Fiorentini and G. Torelli (Laboratoire Nazionale di Frascati, Pisa, Italy, 1977), Fig. 1.

4J. Missimer and L. M. Simons, to be published.

5Nimai C. Mukhopadhyay, Phys. Rep. **30C**, 1 (1977).

6P. Vogel, P. K. Haff, V. Akylas, and A. Winther, Nucl. Phys. **A254**, 445 (1975); P. Vogel, A. Winther, and V. Akylas, Phys. Lett. **70B**, 39 (1977).

7T. A. Carlson, Radiat. Res. **64**, 53 (1975), and references therein.

8*Atomic Inner Shell Processes,* edited by B. Crasemann (Academic, New York, 1975).

9O. Keski-Rahkonen and M. Krause, At. Data Nucl. Data Tables **14**, 139 (1974), and references therein.

10V. R. Akylas and P. Vogel, Comput. Phys. Commun. **15**, 291 (1978).

11P. Ehrhart, Doctoral dissertation, Technische Universität München, 1982 (unpublished); P. Ehrhart *et al.*, Z. Phys. A **311**, 259 (1983), and Phys. Rev. A **27**, 575 (1983).

12F. Kottmann, private communication.

13H. F. Beyer, R. Mann, and F. Folkmann, J. Phys. B **15**, 1083 (1982), and Gesellschaft für Schwerionenforschung Report No. 82-24 (to be published).

14C. R. Vane, M. H. Prior, and R. Marrus, Phys. Rev. Lett. **46**, 107 (1981), and Phys. Rev. A **28**, 141 (1983).

15A. L. Carter *et al.*, Phys. Lett. **124B**, 465 (1983).

16L. M. Simons, "The Cyclotron trap: Status of Preparation and Planned Experiments," in Proceedings of the Low-Energy Antiproton Ring Workshop, Erice, 1982 (unpublished).

17H. Anderhub *et al.*, Phys. Lett. **143B**, 65 (1984).