Surface Thickness in the Eden Model

Plischke and Rácz (PR)[1] have recently studied the surface thickness of growing aggregates. They have found an unusual scaling, even for the Eden model.[2] Here, we consider three versions of this model and we show that the one used by PR exhibits strong finite-size effects. In a strip geometry, we find that the surface thickness simply grows as the square root of the cluster width.

In our simulations the aggregate grows on a square lattice, in a strip of width l, with periodic boundary conditions at the edges.[3] We start with all sites occupied up to $z=0$. Then new particles are added one by one. In version A (considered by PR), the new particle is added, equiprobably, on any *unoccupied* site adjacent to the surface. In version B (introduced by Eden himself[2]), an open bond is chosen equiprobably, and the new particle is added at the edge of this bond. In version C, an *occupied* site of the surface is first chosen equiprobably, and the new particle is added, equiprobably, at the edge of any open bond connected to this site. One can be convinced that these prescriptions give different results on short length scale. However, such differences must not affect the scaling behavior, for sufficiently large sizes.

We have calculated the thickness of the surface, by $\sigma^2=\sum_i(z_i-\bar{z})^2/n_s$, with $\bar{z}=\sum_i z_i/n_s$, where the sum covers the n_s sites of the surface. In model A, we have also calculated σ', by performing instead the sum over the unoccupied sites adjacent to the surface (as PR did). The surface of holes (if any) is included. σ depends on two parameters, the width l and an effective height $h=N/l$, N being the number of particles. For large l and h, we recover the scaling $\sigma(l,h) \sim l^\alpha f(h/l^\gamma)$, with $\gamma>1$ and with $f(x)\rightarrow$ const for $x\rightarrow\infty$ and $f(x)\sim x^{\alpha/\gamma}$ for $x\rightarrow 0$, so that $\sigma(\infty,h)\sim h^{\alpha/\gamma}$ for $h<<l$ and $\sigma(l,\infty)\sim l^\alpha$ for $h>>l$ (steady state). When comparing with the PR geometry, our α/γ becomes the exponent relating σ to the radius, $\sigma\sim R^{\alpha/\gamma}$. $\alpha/\gamma=\bar{\nu}/\nu$, with PR notations. We report here on some results for the exponent α.

For given l, we have precisely determined $\sigma(l,\infty)$ (going up to $h=20l$) by averaging over 1000 trials. Results are given in Fig. 1. While the version-C curve is nicely linear, leading to $\alpha=0.50\pm0.02$, the others exhibit an S shape. In case A the slope decreases first but, as expected, the curve cannot cross the others and, for larger sizes, the slope then increases, tending very slowly to $\alpha\sim 0.5$. Striking differences between the three cases may also be seen in Fig. 2, where one observes many holes in case A. Finite-size effects also affect other quantities, such as the length n_s of the surface.[4] Many more results, with estimates of both α and γ and extensions to higher dimensions, will be given elsewhere.[5] In two dimensions, we have found $\alpha/\gamma=0.30\pm0.03$, a value smaller than the PR result ($\bar{\nu}/\nu\sim 0.37$).

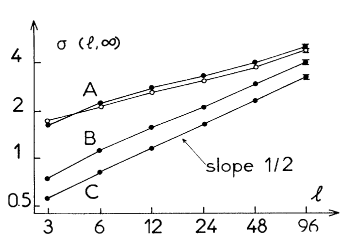

FIG. 1. Numerical results for σ (solid circles) and σ' (open circles) in the steady state, as a function of l (log-log plot).

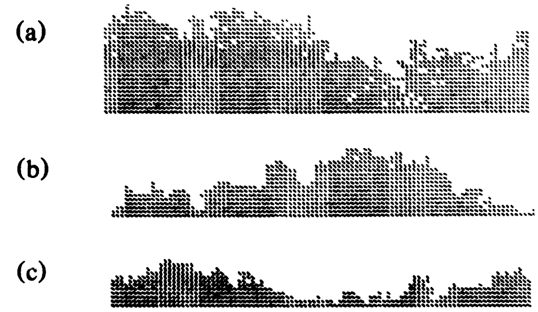

FIG. 2. Typical examples obtained for $l=96$. The figure only shows the last top rows, containing surface sites.

Even if we admit, as PR, that there are two characteristic lengths, we think that our square-root behavior may be considered as a trivial scaling.[6] Thus, we hope that the Eden model will continue to be a simple reference, but we greatly encourage people to use our version C!

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[1]M. Plischke and Z. Rácz, Phys. Rev. Lett. **53**, 415 (1984).

[2]M. Eden, in *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probabilities,* edited by F. Neyman (Univ. of California Press, Berkeley, 1961), Vol. IV, p. 233.

[3]Z. Rácz and T. Vicsek, Phys. Rev. Lett. **51**, 2382 (1983); R. Jullien, M. Kolb, and R. Botet, J. Phys. (Paris) **45**, 395 (1984).

[4]P. Meakin and T. A. Witten, Jr., Phys. Rev. A **28**, 2985 (1983).

[5]R. Jullien and R. Botet, to be published.

[6]F. Family and T. Vicsek, J. Phys. A **18**, L75 (1985).