Possible Unified Interpretation of Heavy Nuclei

R. F. Casten
Brookhaven National Laboratory, Upton, New York 11973, and Institut für Kernphysik, Universität Köln, Köln, Germany
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The data for five mass regions from $A = 100$–200 are plotted against the valence-nucleon product N_pN_n. They are found to display a remarkable similarity, heretofore unrecognized, which suggests a unified interpretation in terms of the strength of the proton-neutron interaction and which also leads to a better understanding of the $A = 190$ region.

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It was recently suggested[1] that a simplified parametrization of nuclear transition regions could be obtained by plotting of the data as a function of the product, N_pN_n, of the numbers of valence proton and neutron particles (or holes past midshell), instead of against N, Z, or A as is usually done. The motivation behind this suggestion is that the *p-n* interaction, especially in highly overlapping orbits, is the dominant factor in the inducing of phase transitions and deformation, as has been emphasized in the literature.[2-10] Since this interaction is most important among valence particles, a reasonable estimate[4,5,8,10] of its average strength is N_pN_n.

Figure 1 illustrates the simplification brought about by this device in the $A = 150$ region for which the structure indicator $E_{4_1^+}/E_{2_1^+}$ is plotted against Z and N_pN_n. This quantity is < 2 for a multinucleon shell-model configuration, ~ 2 for a vibrational nucleus, and approaches 3.33 for a deformed symmetric rotor. Both sides of Fig. 1 clearly show a spherical-deformed phase transition. However, whereas the plot against Z shows widely dispersed points, the N_pN_n systematics can be well described by a simple smooth universal curve (except for the $N = 90$ points).

The purpose of this Letter is twofold. First, similar systematics for the $A = 190$ region will be shown to lead to a modified interpretation of the onset of deformation analogous to the Federman-Pittel type[4,9] for the $A = 100$ and 150 regions. Secondly, N_pN_n systematics will be presented for five mass regions from $A = 100$–200, comprising ~ 120 nuclei, and it will be shown that they disclose a remarkable similarity that suggests a simple unified interpretation of these apparently diverse regions.

For a long time the $A = 100$ region was thought to be unique for three reasons. First, it is the most rapid spherical-deformed phase transition in heavy nuclei. Second, the fact that Sr and Zr ($Z = 38$ and 40) become deformed at all apparently violates the rule[3] that singly magic nuclei should be resistant to deformation since the valence *p-n* force vanishes for such nuclei ($N_pN_n = 0$). Third, those elements which are most spherical for low-N values not only become deformed but attain the maximum deformation as well. An ex-

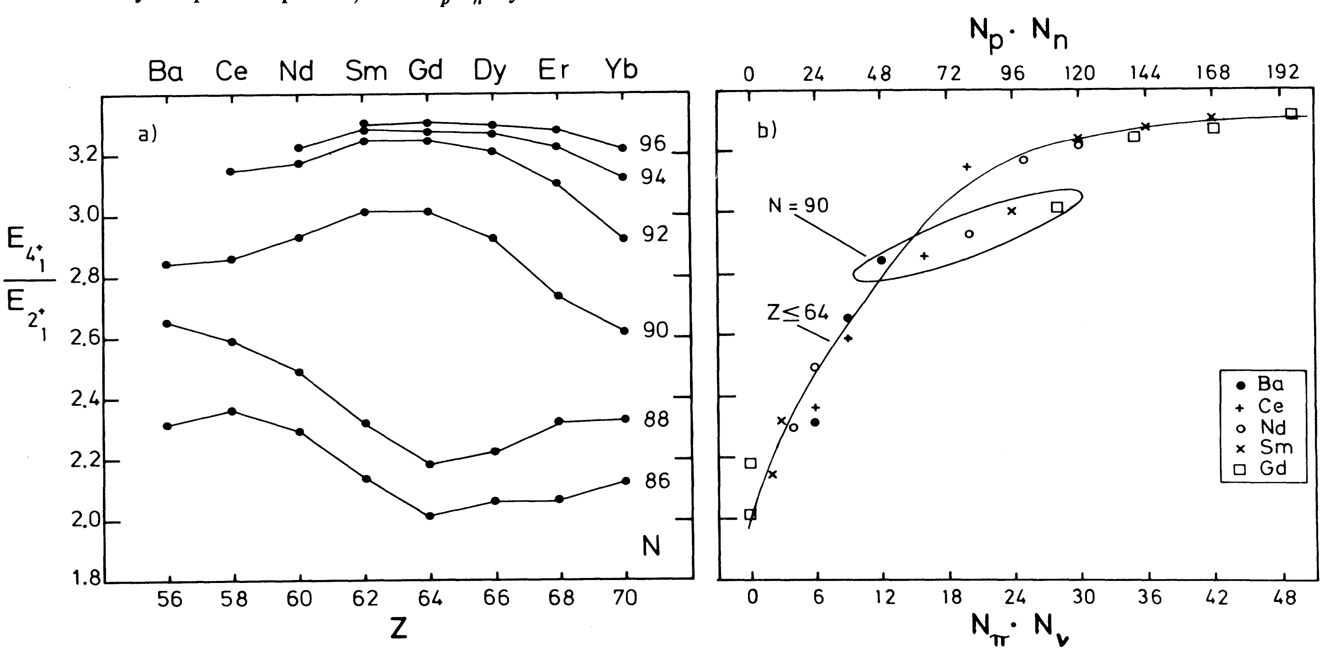

FIG. 1. N_pN_n systematics for the $A = 150$ region. The boson product $N_\pi N_\nu = N_pN_n/4$ is also given.

planation of these features[4] centers on the idea that when neutrons begin to fill the $g_{7/2}$ orbit near $A = 60$, the p-n interaction in highly overlapping orbits is so strong that the $g_{9/2}$ proton single-particle energy is lowered sufficiently so that it is generally feasible to elevate protons into this orbit. This has the effect of eradicating the $Z = 38$ (or 40) shell gap and inducing the configuration mixing that leads to deformation.

When the $Z = 64$ shell gap was discovered,[11] it became apparent[9] that the Federman and Pittel explanation was not only of isolated applicability for an anomalous region but was in fact a model for the $A = 150$ one as well. Here, the $Z = 64$ is eradicated when neutrons begin to fill the $h_{9/2}$ orbit near $N = 90$, which leads to proton occupation of the $h_{11/2}$ spin-orbit partner orbit and the sudden onset of deformation.

To obtain the smooth systematics in the N_pN_n plots, it is essential to take such subshell changes into account.[1] Thus, in Fig. 1(b), the proton shell is defined as $Z = 50$–64 for $N < 90$ and $Z = 50$–82 for $N \geqslant 90$. Therefore, ^{148}Sm, for example, has two proton valence holes, whereas ^{152}Sm has twelve proton valence particles. The $N = 90$ points that lie off the smooth curve reflect the fact that the above prescription, not surprisingly, is too drastic and that the $Z = 64$ gap remains partially intact at $N = 90$ and leads to an intermediate effective number of valence protons.

The $E_{4_1^+}/E_{2_1^+}$ data for the $A = 190$ region are plotted against N and N_pN_n in Fig. 2. When the left side of Fig. 2 is compared with similar plots for the $A = 150$ or 100 regions, little similarity is evident. However, comparison of the $A = 190$ data against N with that for the $A = 150$ and 100 regions against Z [cf., for example, Fig. 1(a)] discloses a striking similarity in structure that has not heretofore been recognized, partially perhaps because of the dual reversal of the roles of protons and neutrons and of particles and holes. Viewed in this light, the $A = 190$ phase transition is not gradual, as often thought, but is, in fact, extremely rapid, although as a function of the number of proton holes rather than neutron particles. Moreover, just as, near $A = 100$ and 150, where those elements which are least deformed for $N < 60$ and $N < 90$ are the most deformed for $N \geqslant 60$ and $\geqslant 90$, here, the isotones ($N \sim 112$–108) which are least deformed for $Z > 76$ are most deformed for $Z \leqslant 76$. This similarity suggests that the disappearance of a shell gap[4] must be again responsible: Here, though, it is a neutron gap affected by the change of proton number. The N_pN_n plot in Fig. 2(b), which assumes no subshell changes, supports this idea since, though it is a simplification of the left-hand plot, the degree of improvement is not nearly as much as in other mass regions [cf. Figs. 1 and 3 (below)].

It has been conjectured for some time[4,6,8] that neutron number $N = 114$ might represent a spherical shell gap. However, evidence for this is obliterated for deformed nuclei and such a gap is not normally considered to be effective. However, as Z increases beyond 76, the last protons would enter the strongly upsloping $\frac{11}{2}$ [505] and $\frac{3}{2}$ [402] Nilsson orbits. This is energetically unfavorable and a more nearly spherical shape is instead preferred, thus reestablishing the efficacy of the $N = 114$ gap, which leads to nearly filled $h_{11/2_p}$ and $i_{13/2_n}$ orbits and accounts for the suddenness of the phase transition. These sudden shifts in orbit energies are indeed also evident in neighboring odd-mass nuclei. For example, the $\frac{11}{2}$ [505] proton Nilsson orbit is a reasonably high-lying particle excitation near $N = 114$ for $Z = 75, 77$, but suddenly becomes a hole excitation for $Z = 79$. The reason the deformation maximizes for Os and W near $N = 108$ instead of at midshell is also analogous to that for the $A = 100$ and 150 regions: for $N \sim 108$, both protons and neutrons are filling high K ($h_{11/2}$ and $i_{13/2}$) orbits whereas, for smaller N, the neutrons fill the $K = \frac{5}{2}$ and $\frac{7}{2}$ $i_{13/2}$ and $h_{9/2}$ orbits and, consequently, have reduced overlaps with the valence protons in the deformed field, leading to lower deformations.

It is interesting to test these ideas by reconstruction of the N_pN_n plot. For $Z \leqslant 76$ the neutrons are still considered as before to fill a single $N = 82$–126 shell,

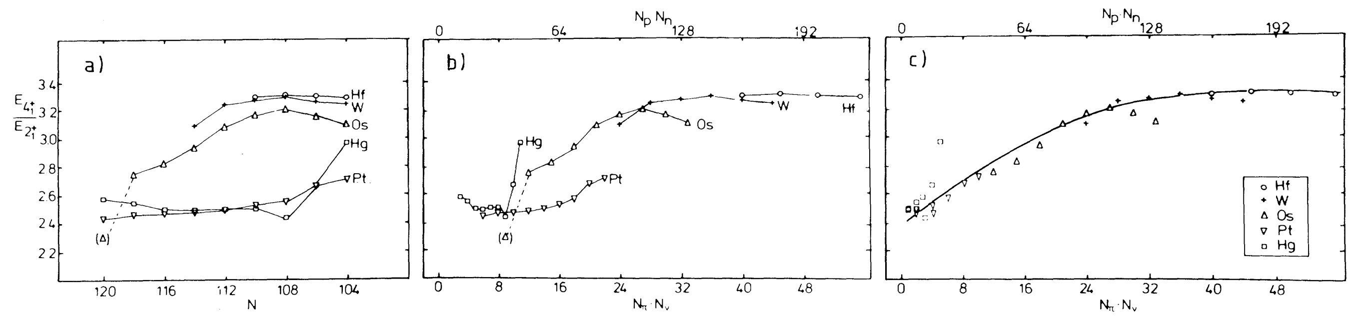

FIG. 2. Similar to Fig. 1 except for the $A = 190$ region. The two N_pN_n plots correspond to different neutron subshell assumptions (see text).

but for $Z \geqslant 78$, this now breaks into two shells, $N = 82$–114 and $N = 114$–126. Thus, the Pt and Hg isotopes have lower valence neutron numbers. For example, ^{196}Pt ($N = 118$) has only four valence neutron particles now (relative to $N = 114$) instead of eight valence holes (relative to $N = 126$). The result of this is Fig. 2(c) which is much smoother than Fig. 2(b) and, clearly, an improved characterization of the region. The only deviant point is ^{184}Hg: However, here the onset of deformation has been explained[5,7] as due to the descent of the $h_{9/2}$ "intruder" configuration which destroys the $Z = 82$ gap, and therefore increases the effective number of valence protons. If this had been taken into account in the construction of Fig. 2(c) the ^{184}Hg point would have been shifted to the right. That such details in subshell structure could also have been deduced from the N_pN_n systematics alone demonstrates that closer inspection of such plots may indeed be fruitful.

A particularly powerful advantage of the N_pN_n parametrization is that it gives an appropriate *unit* with which to compare the rapidity of *different* transition regions. To see this, N_pN_n plots for five transition regions $A = 100$, 130, 150 ($Z \leqslant 64$, proton particles), $A = 150$ ($Z \geqslant 66$, protons holes), and $A = 190$ are shown in Fig. 3. In order to compare these regions, the smooth curves drawn through the points in this figure are collected, all on the same scale, in Fig. 4. Inspection of this figure, which compares regions long thought to be widely different, reveals that, on the contrary, the curves are nearly identical in structure. In particular, in the crucial transitional region (dashed box) from $E_{4_1^+}/E_{2_1^+} \sim 2.2$–$3.0$, the slopes are almost unchanged from one region to another. Closer inspection shows that the curves actually fall into *two classes,* one for the $A = 100$ and two rare-earth groups and one for the $A = 130$ and 190 regions. Within a class, the curves are *essentially parallel* and differ *only* in a *displacement* value $(N_pN_n)_0$.

It is possible to develop a simple interpretation of

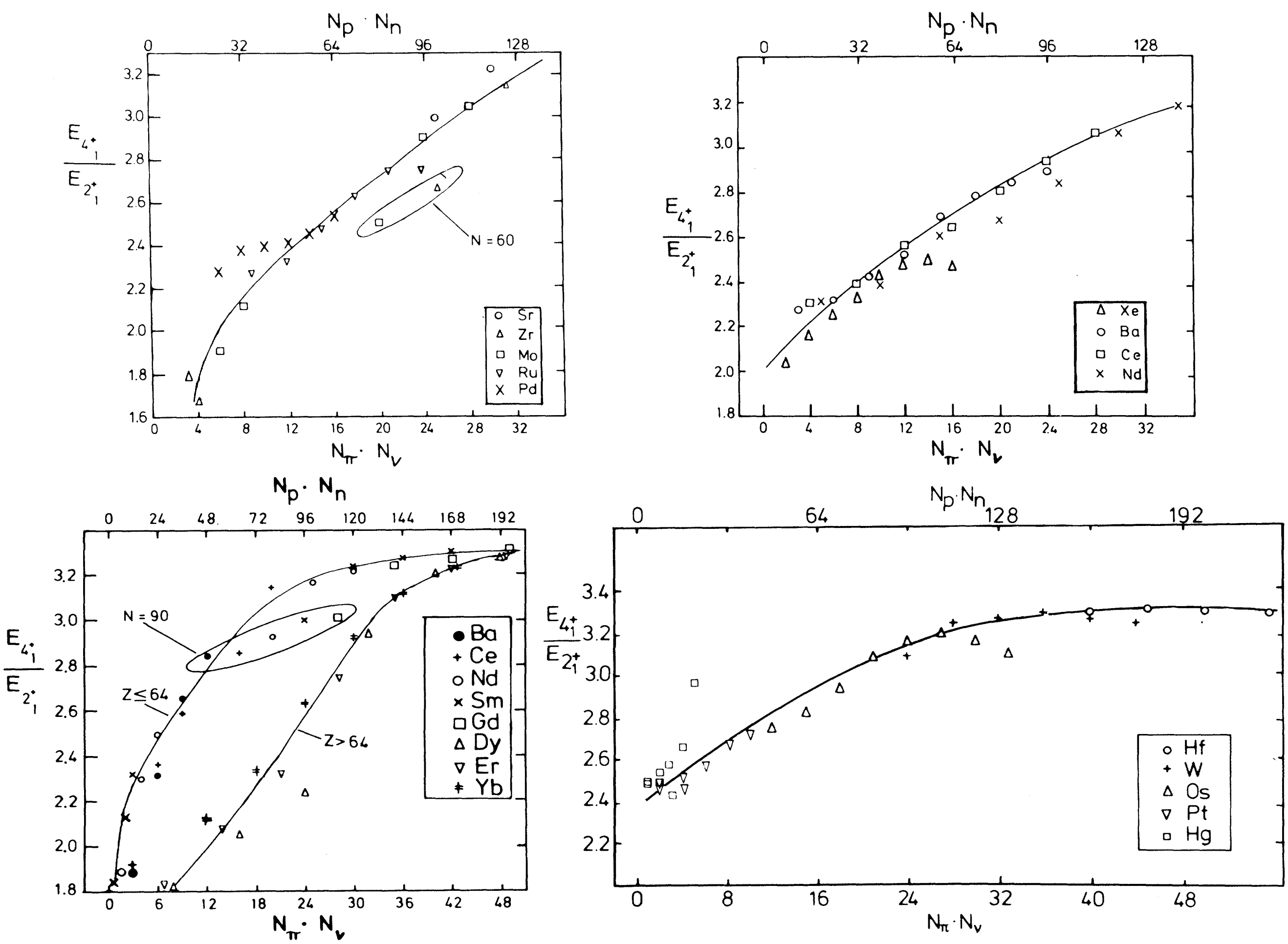

FIG. 3. N_pN_n curves for five mass regions. Data from Sakai (Ref. 12) and very recent Manchester-Daresbury data (Ref. 13) on the light Ce and Nd nuclei.

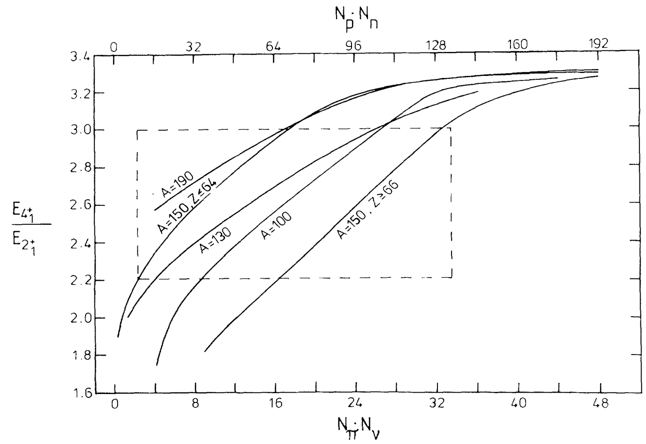

FIG. 4. Summary of the N_pN_n plots.

this which leads to a unified view of these phase transitions. The *p-n* interaction is known[2] to be orbit sensitive and, nearly independent of the details of the interaction, is largest for orbits of large spatial overlap. A crude but useful measure of its strength can be provided by the differences in the shell-model quantum numbers *nlj* of the orbits involved. A convenient quantity is

$$1/(1+\langle \Delta n + \Delta l + \Delta j \rangle_{\text{av}}), \qquad (1)$$

which is 0.5 for spin-orbit partner orbits (e.g., $g_{9/2}$-$g_{7/2}$), and 0 for widely different ones. Table I lists the important orbits in each region and evaluates the above quantity. It is clear that the interaction is, indeed, roughly constant for the three steeply sloping regions in Fig. 4 and somewhat lower for the others. Thus, one infers from Fig. 4 and Table I that *the strength of the proton-neutron interaction in highly overlapping orbits determines the rate of passage (measured* in *units* of N_pN_n) through each transition region. Moreover, these rates seem to be *remarkably similar* and in fact the principal differences from one mass region to another rather concern the *horizontal displacements* which are a measure of the orbital *point of departure* for the transition region: That is, in each region, certain numbers of protons and neutrons must fill orbits of modest overlap before the crucial, highly overlapping, orbits are reached. The differing numbers of such "inert" nucleons give rise to the various horizontal shifts in Fig. 4.

Figure 4 also suggests a particularly simple approach to detailed collective-model calculations in which the *same* set of parameters is used for all the regions of a given class, except for the displacement value $(N_pN_n)_0$, which is *region* dependent. This can lead to substantial reductions in the number of parameters, as will be discussed in a forthcoming publication.[14]

To summarize, N_pN_n provides an appropriate unit to compare the rapidity of different transition regions. When parametrized in terms of N_pN_n, the data for heavy nuclei display a remarkable similarity which suggests a unified interpretation in terms of the *p-n* interaction in highly overlapping orbits. Frequently, this leads to the disappearance of subshell gaps, and thus the Federman-Pittel mechanism[4] provides a widely applicable prototype for the onset of deformation *throughout* heavy nuclei.

TABLE I. Simple estimates [from Eq. (1)] of the *p-n* interaction in key orbits.

Region *A*	Orbits (*p-n*)	Interaction strength
100	$1g_{9/2}$-$1g_{7/2}$	0.5
130	$(1g_{7/2}, 2d_{5/2}) - 1h_{11/2}$	0.17
$150(Z \leq 64)$	$1h_{11/2}$-$1h_{9/2}$	0.5
$150(Z \geq 66)$	$1h_{11/2}$-$1h_{9/2}$	0.5
190	$1h_{11/2}$-$1i_{13/2}$	0.33

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