Hierarchal Mass Scales in Lattice Gauge Theories with Dynamical, Light Fermions

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SU(2) lattice gauge theory with two Dirac species of light, adjoint fermions included in the dynamics is simulated at finite temperatures by use of the microcanonical algorithm. The theory contains two natural mass scales, the string tension for heavy, fundamental quarks and the chiral-symmetry–breaking scale of light, adjoint quarks. The two scales are found to be distinct. This result generalizes earlier results obtained in the quenched approximation and should encourage builders of hierarchal models.

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An important and challenging problem in high-energy theoretical physics is the nature of chiral-symmetry breaking. In quantum chromodynamics it plays a decisive role in the spectroscopy of mesons and baryons and in low-energy scattering processes involving pions. In theories of the weak interactions it is believed to be essential to the fermion mass problem, dynamical Higgs mechanisms, and hierarchal mass spectra in general. Since these are nonperturbative phenomena, they have eluded detailed predictions for some time. However, with the emergence of lattice gauge theory and computer simulation methods, some of the simpler questions in the field can be attacked numerically. For example, in an earlier work Kogut *et al.*[1] considered chiral-symmetry breaking in pure SU(2) gauge theory in the quenched approximation and found evidence for the Casimir scaling hypothesis[2] which plays a central role in hypercolor models of the weak interactions.[3] In particular, we measured the critical effective couplings necessary to trigger chiral-symmetry breaking for fermions in the $l=\frac{1}{2}$, 1, $\frac{3}{2}$ and 2 color representations of SU(2) and found evidence for Casimir scaling,[2]

$$C_2(l)g^2_{\rm mom} \cong 4.0, \qquad (1)$$

where $C_2(l)$ is the quadratic Casimir constant for the fermion and $g^2_{\rm mom}$ is the critical effective coupling in the momentum-space subtraction scheme. The interesting points about Eq. (1) are that it implies (1) a hierarchal sequence of characteristic mass scales for the condensates of the fermions in different color representation, through asymptotic freedom of the running coupling constant,[2] and (2) a rather small critical coupling necessary to drive chiral-symmetry breaking. In particular, it shows that for fermions in high representations of the color group, short-distance forces alone are adequate to drive chiral-symmetry breaking. Apparently nonperturbative dynamics such as confinement, string formation, and instanton and/or vortex dynamics are not always needed for dynamical mass generation.

Of course Eq. (1) was obtained in the quenched approximation which ignores the impact of the condensing light fermions themselves on the gauge field dynamics.[4] Since the primary influence of light fermions is expected to be screening of strong gauge forces, their inclusion into the dynamics is expected to inhibit the formation of a condensate. It is then natural to ask whether the evidence for chiral-symmetry breaking found in Ref. 1 is limited to the quenched approximation or if it is a feature of the untruncated theory. In this Letter, we shall present some evidence that the phenomenon survives the inclusion of fermions into the dynamics.

Using the microcanonical simulation methods[5] we studied SU(2) lattice gauge theory with two Dirac species of adjoint quarks. The microcanonical algorithm has been discussed elsewhere[5] and the only new feature employed here is the inclusion of $l=1$ light fermions. This leads us to replace the usual fermion contribution to the lattice action of fundamental staggered fermions[6] with the expression

$$S_f^{(l=1)} = \sum_n \bar{\phi}(n)\left\{\frac{1}{2}\sum_{\mu=1}\eta_\mu(n)[D_\mu(n)\phi(n+\mu) - D_\mu^T(n-\mu)\phi(n-\mu)] + m\phi(n)\right\}, \qquad (2)$$

where $\phi_i(n)$, $i=1$, 2, or 3, is a three-component Grassmann field defined on the lattice sites $n=(n_0,\mathbf{n})$, $D_\mu(n)$ is the $l=1$ representation matrix of the gauge field $U_\mu(n)$ on the link connecting the nearest-neighbor sites n and $n+\mu$, $n_\mu(n)$ are the phases of staggered fermions, and m is the bare fermion mass in lattice units. With the hopping matrix of Eq. (2) the microcanonical Lagrangian is written down and the equations of motion used in the

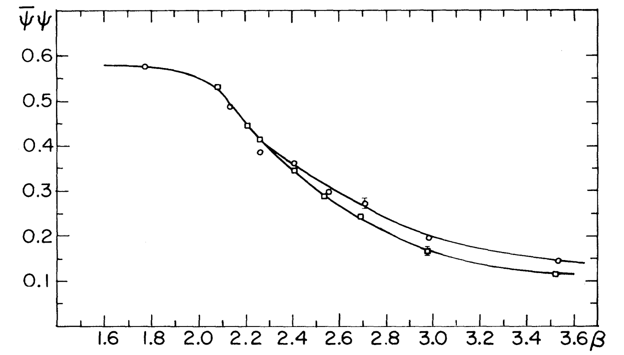

FIG. 1. Raw data for $\langle\bar{\psi}\psi\rangle$ at $m = 0.10$ (circles) and 0.075 (squares) vs $\beta = 4/g^2$. Typical statistical uncertainties are shown.

computer simulation follow. The second-order formalism for the fermion hopping matrix Eq. (2) is used, and following the notation of the third entry to Ref. 5 the real pseudofermion field $P(n)$ is set equal to zero on every other lattice site to reduce the number of fermions characterizing the continuum limit of the theory to two Dirac species.

To probe the theory for multiple mass scales we simulated it at finite temperature.[1] A lattice 4×8^3 ($N_\tau = 4$, $N = 8$) was chosen so that the physical temperature T was $aT = \frac{1}{4}$, where a is the lattice spacing. There are two mass scales of interest: the temperature T_D at which the string tension for heavy fundamental quarks vanishes and the temperature T_c at which chiral symmetry is restored for the adjoint quarks. The first temperature can be measured from the Wilson line

$$W = \left\langle \prod_{n_0=1}^{N_t-1} U_0(n_0, \mathbf{n}) \right\rangle,$$

which is the exponential of minus the free energy of a static fundamental quark in units of the physical temperature[7] or the gluonic contribution ϵ/T^4 to the theory's internal energy.[8] At the temperature where the gluons form a plasma, W and ϵ/T^4 should rise

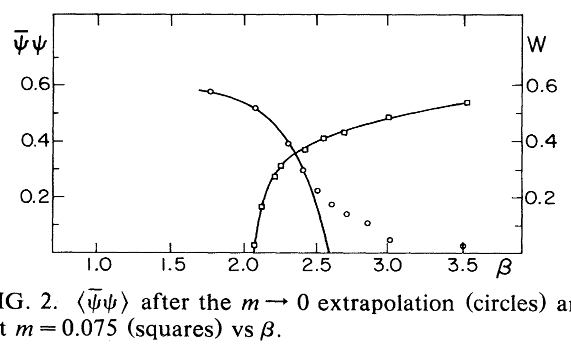

FIG. 2. $\langle\bar{\psi}\psi\rangle$ after the $m \to 0$ extrapolation (circles) and W at $m = 0.075$ (squares) vs β.

from zero. The second temperature is that where $\langle\bar{\psi}\psi\rangle$ vanishes for $l = 1$ massless quarks.

In Figs. 1–3 we show the results of the computer simulations. At each β 2000–4000 time steps of the microcanonical equations were made and the observables were measured every 25 steps. Statistical errors of less than 3% resulted from $\langle\bar{\psi}\psi\rangle$ and W while ϵ/T^4 had greater uncertainty. Typical error bars are shown in the figures. Figure 1 shows the raw $\langle\bar{\psi}\psi\rangle$ data. At each β value $\langle\bar{\psi}\psi\rangle$ was measured at the masses 0.10 and 0.075 and the results were extrapolated to zero mass. The result of that extrapolation and W for $m = 0.075$ are shown in Fig. 2. W vanishes at $\beta = 2.10 \pm 0.05$ and $\langle\bar{\psi}\psi\rangle$ vanishes at $\beta \geq 2.60$. Since larger β corresponds to larger physical temperature, the two temperatures T_D and T_c are clearly distinct. In fact, it was found that W for $m = 0.075$ was slightly larger than W for $m = 0.10$ at a given β. This result is sensible since decreasing m should increase the effects of fermion screening. So in the zero-mass limit T_D is presumably even smaller than that predicted by Fig. 2. The "tail" in the plot of $\langle\bar{\psi}\psi\rangle$ vs β in Fig. 2 is presumably due to the crudeness of the zero-mass extrapolation—smaller m values are probably necessary to measure $\langle\bar{\psi}\psi\rangle$ well when it is numerically small—and the solid curve is simply meant to "guide the eye." In past studies[9] better data at smaller m values have served to remove the "tails" in such curves and thus to leave critical curves as shown in the figure. Even with these relatively crude data, however, the difference in the two temperatures is quite firm. Finally, in Fig. 3 we show the gluonic internal energy ϵ/T^4. It jumps from zero at $\beta = 2.10 \pm 0.05$, in agreement with the Wilson-line data, and approaches the Stefan-Boltzmann limit for free gluons on a 4×8^3 lattice at large temperatures.

The difference between the critical β values in Fig. 2 is $\Delta\beta \approx 0.50$–0.55. With asymptotic freedom this

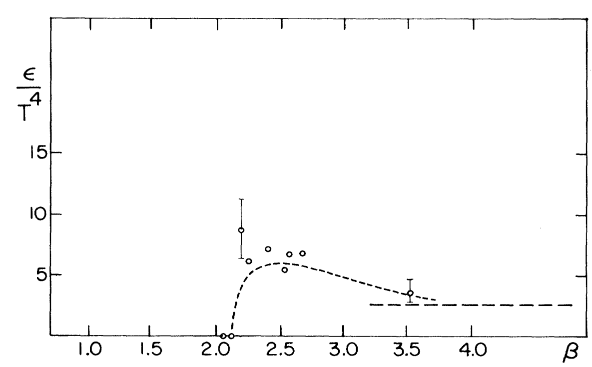

FIG. 3. Gluon internal energy in units of the physical temperature vs β. Typical statistical errors are shown and the dashed line is the Stefan-Boltzmann value for free gluons on a 4×8^3 lattice.

change in coupling can be translated into a ratio of critical temperatures. For two massless Dirac adjoint fermions, the ratio of the temperature to one-loop order is

$$T_c/T_D = \exp(-\pi^2 \Delta_\beta) = 175 \pm 50, \tag{3}$$

a very large number indeed! This rough estimate is much larger than that found in the quenched approximation ($T_c/T_D = 8.6 \pm 4.5$, quenched) and should not be treated as quantitative until larger lattices are simulated and asymptotic freedom with fermion feedback is verified. However, the result is certainly encouraging motivation for more ambitious calculations.

In summary, the disparity in mass scales observed in simulations of the quenched approximation to lattice gauge theory appears to survive the inclusion of fermion loops. However, quantitative and reliable numerical results on this difficult problem must await larger-scale simulations or more sophisticated theoretical techniques. Simulations on larger lattices, 6×12^3, are planned, and pseudofermion simulations in which the number of $l = 1$ fermion species can be better controlled are also underway.[10] It would also be fascinating to simulate larger gauge groups which are relevant to unified gauge theories. The possibility of first-order transitions and their impact on the two scales would be interesting to understand and might have cosmological implications.

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1J. Kogut, M. Stone, H. W. Wyld, S. H. Shenker, J. Shigemitsu, and D. K. Sinclair, Nucl. Phys. **B225** [FS9], 326 (1983).

2W. J. Marciano, Phys. Rev. D **21**, 2425 (1980).

3S. Weinberg, Phys. Rev. D **13**, 974 (1976), and **19**, 1277 (1979); L. Susskind, Phys. Rev. D **20**, 2619 (1979); S. Raby, S. Dimpoulos, and L. Susskind, Nucl. Phys. **B169**, 373 (1980).

4H. Hamber and G. Parisi, Phys. Rev. Lett. **47**, 1792 (1981); E. Marinari, G. Parisi, and C. Rebbi, Phys. Rev. Lett. **47**, 1795 (1981); D. Weingarten, Phys. Lett. **109B**, 57 (1982).

5D. Callaway and A. Rahman, Phys. Rev. Lett. **49**, 613 (1982); J. Polonyi and H. W. Wyld, Phys. Rev. Lett. **51**, 2257 (1983); J. Polonyi, H. W. Wyld, J. B. Kogut, J. Shigemitsu, and D. K. Sinclair, Phys. Rev. Lett. **53**, 644 (1984).

6See Ref. 1 and references included therein for discussions of staggered fermions.

7See, for example, L. McLerran and B. Svetitsky, Phys. Lett. **98B**, 195 (1981); J. Kuti, J. Polonyi, and K. Szlachany, Phys. Lett. **98B**, 199 (1981).

8See, for example, J. Kogut, M. Stone, H. W. Wyld, W. R. Gibbs, J. Shigemitsu, S. H. Shenker, and D. K. Sinclair, Phys. Rev. Lett. **50**, 393 (1983), and references included therein.

9J. B. Kogut, J. Shigemitsu, and D. K. Sinclair, Phys. Lett. **138B**, 283 (1984).

10In collaboration with F. Karsch.