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Neutron Microscope

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We report successful operation of a neutron microscope using ultracold neutrons at the high-flux reactor at Grenoble. A sharp, achromatic image of an object slit was obtained at a magnification of 50. The measured resolution of 0.1 mm was limited mainly by the available beam intensity, not by aberrations.

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Neutrons interact with matter quite differently than photons (light or x rays) or electrons. Therefore they "sense" a different contrast. Thermal-neutron scattering in combination with controlled proton-deuteron exchange is nowadays established as a unique and extremely powerful method for probing the hydrogen distribution in matter. The availability of ultracold neutrons[1–3] (UCN) has opened the possibility of exploiting the "neutron contrast" also in a neutron microscope. This appears feasible since ultracold neutrons are totally reflected from suitable mirror materials even at large angles of incidence, and thus anastigmatic imaging systems can be developed.

However, in contrast with light, UCN beams are curved by gravity, and the wavelength dependence of curvature gives rise to chromatic aberrations of mirror systems. The properties of a concave mirror as a system for UCN imaging were first described by Frank in 1972.[4] For realistic applications the gravitationally induced chromatic aberrations must be corrected.

In the first neutron imaging system meeting this criterion[5,6] correction of the longitudinal chromatic aberration (in focal length) to first order was achieved by use of a "zone mirror" consisting of a totally reflecting, blazed zone grating deposited on a concave spherical mirror with vertical optical axis. Compensation of gravity by a homogeneous magnetic field gradient was proposed by Skachkova and Frank.[7]

The present paper reports experimental results obtained with a prototype of an achromatic two-mirror neutron microscope with a magnification of 50. As previously described[8,9] it is designed so as to correct both the longitudinal and the transverse chromatic aberration (wavelength dependence of magnification) to first order. Successful operation of a different, four-mirror system with magnifications 1.375 and 0.725, designed for chromatic correction at high enough velocities, was recently reported.[10]

Fermat's principle of stationary phase implies[5,9] that in the presence of a constant force field acting purely in the vertical z direction the imaging equation for a curved mirror with optical axis in the z direction reads,[4,5] for paraxial rays,

$$\frac{2}{R}=\frac{1}{d_1^*}+\frac{1}{d_2^*}=\frac{1}{v}\left(\frac{1}{t_1}+\frac{1}{t_2}\right), \qquad (1)$$

where R is the radius of mirror curvature, v is the neutron velocity at mirror height, and t_1 $(i=1,2)$ are the neutron flight times between mirror center and object or image points. Equation (1) is identical to the imaging equation for straight rays except for a replacement of the acutal flight paths, d_i, by those for straight rays, d_i^*, intersecting the mirror surface at the same angles as the curved particle rays. The magnification is given by $\mu=t_2/t_1$. All the relevant quantities, g (gravitational constant), v, R, d_i, and t_i, can assume positive or negative values, depending on the actual geometrical configuration.

For a concave mirror the object distance $d_1 = R/2$ (at the light-optical focus above the mirror) is distinguished in the sense that the image formed at $d_2 = R/2 = d_1$ by the rays after passing the maximum height of their flight parabola, $H = v^2/2g$, is free of longitudinal chromatic aberration to all orders. However, for this case the magnification $\mu \simeq 8H/R \sim v^2$ (for $H >> R/2$) is strongly chromatic.

A possibility of compensating both first-order chromatic aberrations arises for a system of two mirrors arranged one above the other on their common optical axis. We consider specific configurations subject to the conditions $\partial d_2'/\partial v = 0$ and $\partial M/\partial v = 0$, where the primes refer to the secondary mirror system and $M = \mu\mu'$ is the overall magnification. After extensive algebra we find the following relations for two-mirror achromatic systems:

$$2/\alpha_1' = 2\eta + \eta_1^{-1} - 1 \tag{2}$$

and

$$\mu = \frac{\alpha_1'}{2}\frac{(\eta_1 + 1)(\eta_1\eta - 2\eta_1 + 1)}{\eta_1(\eta_1 - \eta)},$$

$$\mu' = \frac{2}{\alpha_1'}\frac{\eta - 2\eta_1 + 1}{\eta(\eta_1 + 1) - 5\eta_1 + 3}, \tag{3}$$

where we have used the dimensionless quantities $\xi = v/v'$, $\xi_i = v_i/v'$, $\alpha_i^{(1)} = gt_i^{(1)}/v'$, and $\eta_{(1)} = (1 + \xi_{(1)})/(1 - \xi_{(1)})$. The connection between the two mirror subsystems via the common intermediary image/object is expressed by the relation $\alpha_2 = 1 - \xi - \alpha_1'$.

We may regard the total magnification, $M = \mu\mu'$, and η_1 (a measure of object position) as independent parameters determining the possible geometries of the optical system. For given M and η_1 two values for η may be found from the quadratic equation $M = M(\eta, \eta_1)$. Only a small subset of solutions satisfies the following criteria: (a) η must be real valued and such that both object and final image will be real and the mirrors are hit by real, not virtual, rays. (b) The mirrors, object or image scanning device, supports, etc., should not block a substantial fraction of the possible flight paths. (c) It is preferable to arrange the object at a short distance above the primary mirror, and below the secondary one, in order to make optimum use of the source UCN spectrum which in all practical cases is depleted at the lowest energies.

A common feature of all systems meeting these criteria is the following: The neutrons pass the highest point of their flight parabola on their way between the two mirrors. Only for such configurations can the first-order chromatic aberrations of one mirror be compensated by those of the other mirror. For this class of instruments it was found impossible to correct chromatic aberrations of higher order since terms like $\partial^2 d_2'/\partial v^2$ or $\partial^2 M/\partial v^2$ remain finite. Large magnification, $|M| >> 1$, can be achieved either if $|\mu| >> 1$ (hence $\eta \simeq \eta_1$) or if $|\mu'| >> 1$, but only the former case corresponds to a system with a real object. Furthermore, $M \to +\infty$ (the + sign indicating that the intermediary image is real) may be excluded on account of postulate (c).

Finally, we may choose v (or v') and thus the common scale factors: v' for velocities, v'/g for flight times, and v'^2/g for lengths. For systems utilizing total reflection from the mirrors, $|v|$ and $|v'|$ must lie below the critical value v_l. (For instance, for ^{58}Ni the lower of the two values for the two spin states is $v_l = 7.5$ m/s.)

The following design parameters were chosen for the present experiment: $M = -51.94$, $\eta_1 = -0.14294$, and $v = 5.73$ m/s. The associated geometry is as follows (see Fig. 1): $d_1 = 293.1$ mm (object above primary mirror); $R = 585.2$ mm (concave) and $R' = -1209.8$ mm (convex); mirror separation, 897.5 mm (second above first); intermediary (virtual) image distance below, and final image distance above, the second mirror, $d_1' = -415.1$ mm and $d_2' = 662.7$ mm; partial magnifications, $\mu = 20.10$ and $\mu' = -2.58$; velocities, e.g., $|v_1| = 5.204$ m/s (at the object) and $|v'| = 3.903$ m/s (at second mirror).

A detailed analysis of higher-order aberrations was

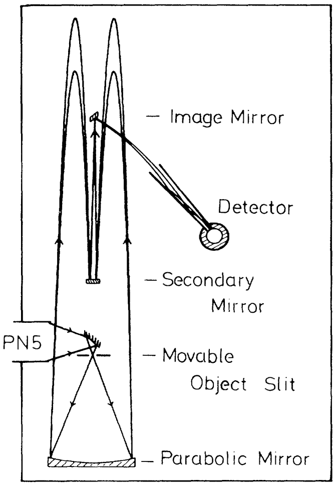

FIG. 1. Scheme of the two-mirror neutron microscope setup. Neutron trajectories are shown for two different velocities. The neutrons pass the highest point of their flight parabolas between the imaging mirrors. This is a necessary condition for compensation of both chromatic aberrations (in focal length and in magnification) to first order. The instrument is designed for a wide entrance aperture (1:1.5) and for magnification $\simeq 50$.

performed by ray tracing. The differential aberrations (like coma) are identified with specific terms in a Taylor series for the total aberration A. $A(r_1, \phi_1, \psi_1, w_1)$ denotes the distance within the image plane by which a specific ray misses the nominal image point if it leaves the object plane at distance r_1 from the axis, at off-axis angles ϕ_1 (in the sagittal plane) and ψ_1 (in the meridional plane), and at a velocity $v_1 + w_1$, where v_1 is the design value. The chromatic aberrations

$$\partial^2 A/\partial r\, \partial w|_0 = \partial M/\partial v$$

and

$$\partial^2 A/\partial \phi\, \partial w|_0 \sim \partial d_2'/\partial v$$

were corrected for the present system. All the nonzero expansion coefficients up to fifth order were calculated by numerical differentiation, and terms of higher order were shown to be of less importance by comparison with the total aberration, even if wide apertures are admitted. This analysis provided the basis for the choice of mirror diameters and mirror shapes. Favorable results were obtained for a parabolic shape of the primary, strongly magnifying mirror. We chose a diameter of 20 cm, i.e., an entrance aperture of 1:1.5. The geometrical aberrations (mainly coma, $\partial^3 A/\partial r\, \partial \phi^2$, and spherical aberration, $\partial^3 A/\partial \phi^3$) then lie within tolerable limits of < 1 mm. The smaller secondary mirror of 5 cm in diameter can be spherical without loss in imaging quality. For the full velocity band $5.5 < v/(1\ \mathrm{m/s}) < 6.7$ (as determined by the image mirror height and the overall height of the apparatus) the uncompensated chromatic aberrations of higher order are dominant (several millimeters).

The image field width is limited by the requirement that the scanning device, an "image mirror" reflecting the neutrons into a detector, should constitute a negligible obstacle for the neutrons on their descent to the secondary mirror. An image field width of 17×17 mm^2 was chosen, corresponding to an object of 0.35×0.35 mm^2. In principle, the image could be resolved into two-dimensional elements of a size limited by the aberrations by use of a facet-type mirror reflecting neutrons from different image zones into different detectors. For the present prototype of a microscope we chose a single fixed-image mirror of 17×17 mm^2, inclined 15° to the horizontal plane, and spherically concave. This ensures collection of nearly all neutrons arriving at the image mirror in an inclined, convergent, guide tube of entrance section 2.5×4.5 cm^2 which conducts them to the detector (BF_3 with reduced ^{10}B content). The detector is placed 0.7 m below the image plane, and thus the slowest neutrons gain enough kinetic energy for passing through the detector window (0.1-mm Al with open section 2.5×2 cm^2; for Al, $v_l = 3.2$ m/s).

We used the guide-tube facility PN5[11] at the high-flux reactor at Grenoble, supplying a current density of ≈ 100 UCN/cm$^2 \cdot$s. Since the microscope utilizes a very wide beam divergence, we installed a convergent guide section in front of a slit serving as the object. The primary beam profile can be scanned with use of a monitor detector mirror inserted below the object slit. The imaging mirrors were fabricated from Zerodur and coated with nickel (the primary mirror with ^{58}Ni).

Computer simulations for maladjusted configurations showed that very precise adjustment, especially of the primary mirror and of the object position, was a crucial requirement. Sufficient precisions of $\approx 10^{-4}$ rad in mirror horizontality and of ≈ 0.1 mm in object distance were achieved with use of light-optical methods. The system was pumped to $< 10^{-4}$ mbar.

The microscope was designed for object scanning. The scan perpendicular to the object slit was performed by use of a stepping motor controlled by a microcomputer (MACAMAC) which served also for data collection and reduction.

For a test of the specific one-mirror system described above the secondary, small mirror was removed and an image mirror, inclined at 45° and intercepting 1.5×1.5 cm^2 of the beam, was installed at a short distance of 80 mm above the object slit. The object slit was placed just above the optical focus of the primary mirror to compensate for the slight deviation from the ideal geometry where object and image should coincide in the light-optical focus. The detector and a short feeding guide tube were placed so as to collect the neutrons reflected from the image mirror. The magnification is 25 at $v = 6.2$ m/s but varies as $\sim v^2$. Figure 2(a) shows the data obtained in a scan of the 1.06-mm-wide object slit. The edges are blurred as a result of the wide velocity range accepted. The measured curve agrees well, even in absolute height, with a computer simulation (dashed curve) where the broad incident neutron spectrum was taken into account and the monitor data for the beam profile were used.

Figure 2(b) shows the result of a similar scan for the two-mirror microscope setup of Fig. 1. The data points are consistent with an edge width of ≈ 0.3 mm, as expected for a magnification of 52 and the image mirror width of 17 mm (see the solid curve).

We have reported successful operation of a novel neutron microscope characterized by a magnification of ≈ 50 and full achromatism to first order. The resolution of ≈ 0.1 mm observed was limited by course-grained image detection, and this, in turn, was dictated by the modest UCN source strength. Future UCN sources are expected to be $\approx 10^2$ times stronger (e.g., see Ageron and Mampe[12]). The present aberration limit to the resolution (in two dimensions) is ≈ 0.02 mm, but can be reduced, say, to several micrometers, by coarse incident-beam monochromatization. This would require higher magnifications which are possible

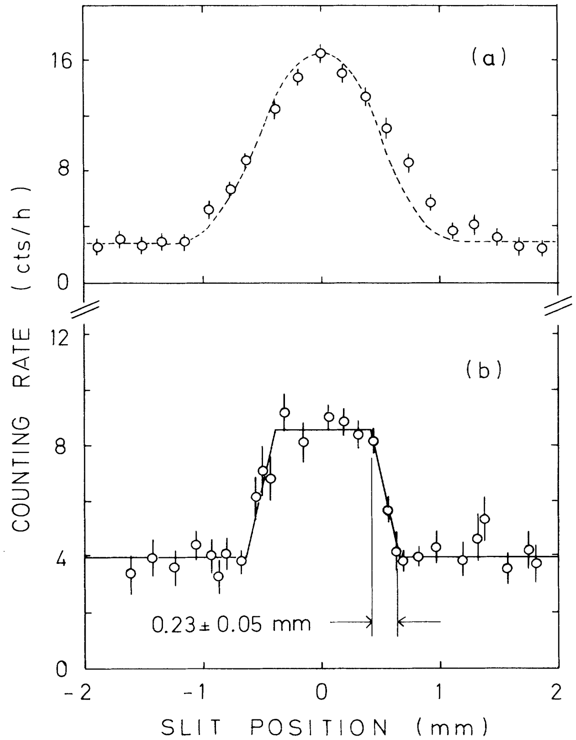

FIG. 2. Data for scans of a 1.06-mm-wide object slit (a) for the specific one-mirror system where (ideally) object and image coincide at the light-optical focus, for any arbitrary neutron velocity; and (b) for the two-mirror microscope setup of Fig. 1. In (a) the magnification is strongly chromatic, and this leads to the observed edge blurring for the wide UCN spectrum used. The trapezoidal image in (b) implies a resolution of ≈ 0.1 mm. It was limited by coarse-grained image detection rather than aberrations. The measured data are consistent with calculations (dashed and solid curves). Measurement (a) was performed at a reduced primary intensity (factor of 6).

for two-mirror schemes of the present topology. The problems of chromatism and intensity can be relaxed by choosing higher speeds around 12 m/s and multilayer mirrors, as proposed previously.[8]

We point out an interesting by-product of the present research. At the classical reversal point for vertical particle motion in the gravitational field the velocity vanishes. In the present experiment the minimum neutron velocity at the highest point of their flight parabola between the mirrors is of order 1 cm/s. This is close to the range $\approx 10^{-5}$ m/s for the "snail's-pace" neutrons introduced by Shull.[13]

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