Optical Guiding in a Free-Electron Laser

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By use of two-dimensional approximations for the equations that describe a high-gain free-electron laser (FEL) amplifier, and the properties of optical fibers, it is shown that the coherent interaction between the light and the electron beam in an FEL can optically guide the light. In the exponential-gain regime, the FEL performance in the presence of strong diffraction can be simply described by a cubic equation for the complex gain and the dispersion relation for an optical fiber. The phenomenon of optical guiding is illustrated with two-dimensional numerical simulations. The phenomenon has applications to short-wavelength FEL's, to directing of intense light, and to bending of x rays.

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It has long been known that the coherent interaction between the light and the electron beam in a free-electron laser (FEL) produces a phase shift of the light, and that the sign of the effect can be such that the light is refracted toward the electron beam.[1,2] The electron beam in a high-gain FEL bunches on an optical wavelength; because of the bunching, the beam has an effective index of refraction greater than unity. This is in sharp contrast to the behavior of an unmagnetized (and unbunched) plasma, and is the basis for the optical guiding effects described here. The effect occurs even if there is negligible growth in light intensity and can be important in many situations.

The usual analysis of step-profile optical fibers[3] assumes that the fiber consists of a central core of radius a and index of refraction n, and a cladding of index $n_{\rm cl}$. In our treatment, the core is the electron beam and the cladding is free space, and so $n_{\rm cl}=1$. We can make the assumption that the fiber is weakly guiding:

$$|n-1| << 1. \tag{1}$$

This inequality is quite good for all cases of interest, and is consistent with the assumption of a slowly varying phase ϕ of the optical field,

$$d\phi/dz << k=\omega/c, \tag{2}$$

familiar from FEL theory.

Following Marcuse,[3] we consider guided modes with only one transverse electric field component E_x (but both magnetic and electric longitudinal components), so that E_x varies as $J_\nu(\kappa r)$ inside the fiber and as $H_\nu^1(\gamma r)$ outside, where J_ν and H_ν^1 are Bessel functions and Hankel functions of the first kind, respectively. The arguments of the functions are $\kappa=(n^2k^2-\beta^2)^{1/2}$, $\gamma=(\beta^2-k^2)^{1/2}$, and the field is assumed to vary as $\exp[i(\beta z-\omega t)]$. Continuity of B_z and E_z at the fiber edge yields the dispersion relation:

$$\kappa\frac{J_{\nu+1}(\kappa a)}{J_\nu(\kappa a)}=\frac{\gamma K_{\nu+1}(\gamma a)}{K_\nu(\gamma a)}, \tag{3}$$

with

$$(\kappa^2+\gamma^2)a^2=V^2(n^2-1)k^2a^2. \tag{4}$$

The quantity V is called the "fiber parameter."

The condition for mode cutoff in a fiber is $\gamma\rightarrow 0$. While formally there is no cutoff for the mode LP_{01} (the first index labels the Bessel function, the second labels the zeros), which is the dominant mode for FEL amplification, it is incorrect to think of the mode as bound by the fiber for all $V>0$ since it extends far outside the beam for $V<<1$. For the mode LP_{01} to be considered guided, we somewhat arbitrarily require that the $1/e$ point of E_x be within 5 times the fiber radius. This condition corresponds to demanding that $V^2>1$.

The analysis can be extended to a fiber with gain (or loss) by permitting n to be complex. The dispersion relation, Eq. (3), is unchanged, but κ and γ can now also be complex. From numerical solution of the complex dispersion relation, we find that the above criterion generalizes to

$$\mathrm{Re}(V^2)+|\mathrm{Im}(V^2)|/2.4>1. \tag{5}$$

The index of refraction of an optically bunched beam comes from the FEL equations as formulated by Prosnitz, Szoke, and Neil[4]:

$$\mathrm{Re}(n)-1=\frac{1}{k}\frac{d\phi}{dz}=\frac{2\pi eJa_w}{mc^3ke_s}\left\langle\frac{\cos\psi_i}{\gamma_i}\right\rangle, \tag{6}$$

$$-\mathrm{Im}(n)=\frac{1}{ke_s}\frac{de_s}{dz}=\frac{2\pi eJa_w}{mc^3ke_s}\left\langle\frac{\sin\psi_i}{\gamma_i}\right\rangle. \tag{7}$$

TABLE I. Simulation parameters.

Current (I)	270 A
Electron beam radius in the wiggler (a)	0.01 cm
Electron Lorentz factor (γ_0)	2000
Fractional electron energy spread $(\Delta\gamma/\gamma)$	1.2×10^{-3}
Laser wavelength $(2\pi/k)$	2500 Å
Rayleigh length (z_r)	50 cm
Dimensionless rms wiggler vector potential (a_w)	4.35
Wiggler length (L)	30 m
Wiggler period $(2\pi/k_w)$	10 cm
Input laser power (P_i)	30 Mw
Output laser power (P_0)	585 MW

In Eqs. (6) and (7), e_s and a_w are the normalized rms amplitudes, $e_s = e|E_x|/\sqrt{2}mc^2$ (for a linear wiggler), and $a_w = e|B_w|/\sqrt{2}k_w mc^2$, where k_w is the wiggler wave number. The wave number of the optical field is k, the current density is j, ψ_i is the phase of an electron in the ponderomotive potential well, and γ_i is its Lorentz factor. The angular brackets denote an average over the electron distribution. We use Gaussian cgs units.

Numerical simulations were performed with the two-dimensional (2D) FEL code FRED.[5] The code follows an axisymmetric laser beam around an electron beam that bunches longitudinally (in ψ). Axisymmetric diffraction effects are fully included, via the paraxial wave approximation; refractive and gain effects are included through the local source terms provided by the electron beam. The parameters of one simulation are listed in Table I. Figure 1 is a three-dimensional contour plot of laser intensity, while Fig. 2 presents some details of the simulation.

In the exponential-gain regime we can go further in the analysis by extending the linear analysis of Bonifacio, Pellegrini, and Narducci[6] to include the effects of diffraction. To do so, we write the usual longitudinal electron equations derived by Kroll, Morton, and Rosenbluth[1] in complex form. The particle motion equations are unchanged, but the complex field equation has a transverse gradient term added:

$$\frac{\partial e_s}{\partial z} = \frac{2\pi i e a_w}{mc^3} f_B \frac{J}{N} \sum_j \frac{e^{-i\theta_j}}{\gamma_j} + \frac{i\nabla_\perp^2 e_s}{2k}, \tag{8}$$

where e_s is now a complex field amplitude. The total number of electrons is N, and f_B is the difference of

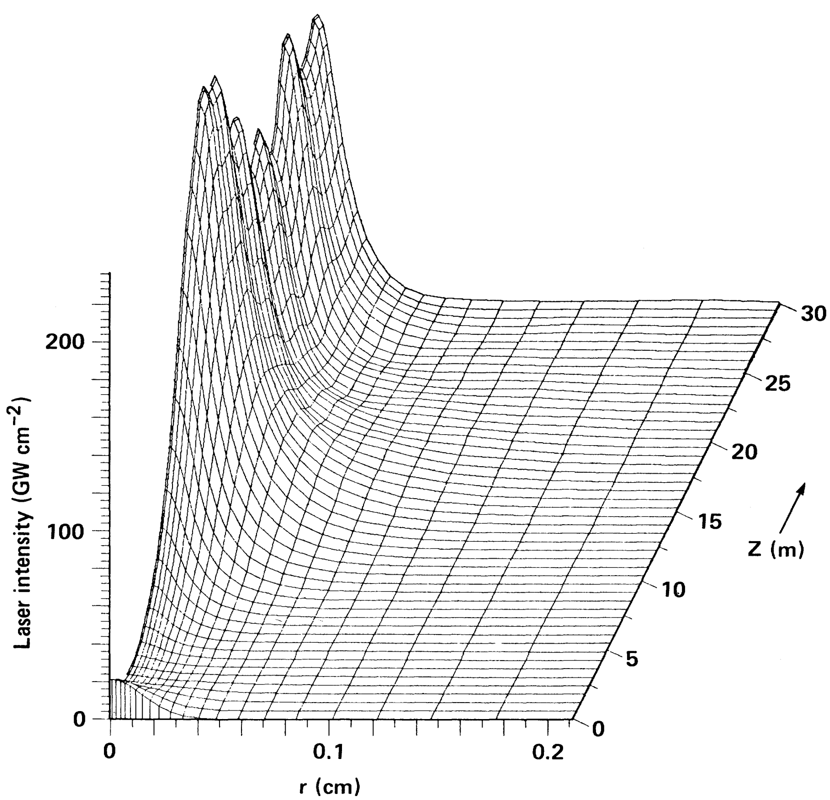

FIG. 1. A three-dimensional plot of laser intensity vs r and z inside the wiggler for case I. Note that the laser profile is nearly constant for 60 Rayleigh lengths.

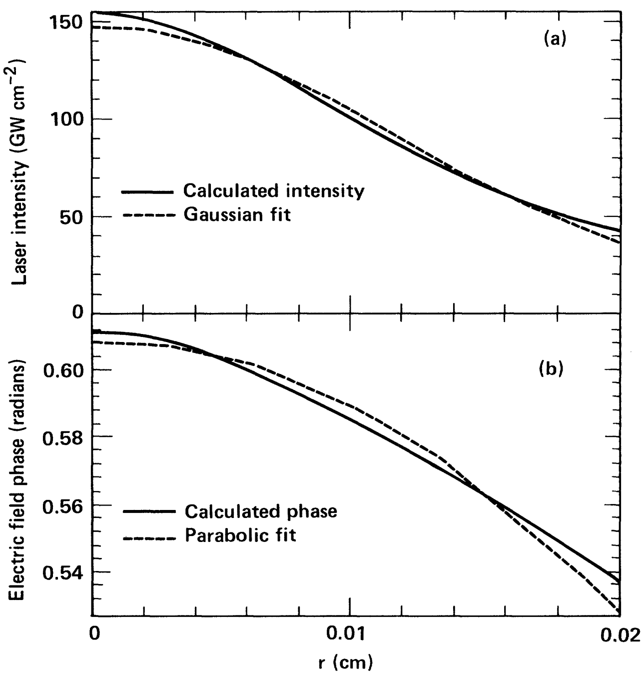

FIG. 2. A cross section (a) of the laser intensity and (b) the laser phase ϕ at the end of the wiggler. The Gaussian fit to the intensity decreases by $1/e^2$ at 0.024 cm. The decrease of ϕ with increasing r indicates that the light is *converging* to a focus 8 cm past the end of the wiggler.

Bessel functions:

$$f_B \equiv J_0\left[\frac{a_w^2}{2(1+a_w^2)}\right] - J_1\left[\frac{a_w^2}{2(1+a_w^2)}\right].$$

We approximate the transverse gradient term by

$$\frac{\nabla_\perp^2 e_s}{2k} \simeq -\frac{e_s}{z_r}, \tag{9}$$

where $z_r = 2k/|\kappa|^2$ with κ defined before Eq. (3). The quantity z_r is approximately the Rayleigh range of the light. If we take the electron distribution function as uniform between $\gamma_0 - \Delta\gamma$ and $\gamma_0 + \Delta\gamma$, to replicate a warm beam, then the result of linearization is a cubic in the complex, dimensionless parameter λ:

$$\lambda^3 + \lambda^2[1 + 2\Delta k_0 z_r] + \lambda\left[2\Delta k_0 z_r + (\Delta k_0 z_r)^2 - 4(k_w z_r)^2 \frac{\Delta\gamma^2}{\gamma_0^2}\right] + A k_w z_r + (\Delta k_0 z_r)^2 - 4(k_w z_r)^2 \frac{\Delta\gamma^2}{\gamma_0^2} = 0. \tag{10}$$

Here $\Delta k_0 \equiv k_w - k/2\gamma_0^2(1+a_w^2)$ is a parameter that measures the departure from resonance, γ_0 is the central energy of the electron beam, and

$$A \equiv \frac{2\pi e J}{mc^3} \frac{a_w^2 f_B^2}{\gamma_0^3} z_r^2 \tag{11}$$

is the dimensionless parameter that measures coupling between the electron beam and the light.

The expression for the fiber parameter V of the electron beam in terms of γ is

$$V^2 = (2ka^2/z_r)(1+\gamma). \tag{12}$$

For the parameters of the simulation, the cubic yields $V_2 = 1.01 - 0.12i$, $\lambda = 0.01 - 0.12i$. Our criterion for guiding Eq. (5) is satisfied, although the laser beam is somewhat more tightly confined to the electron beam than $|V| \simeq 1$ would predict. In terms of V, the discrepancy is only about 20%.

The general procedure for evaluating the importance of the guiding of laser light by an electron beam is iterative. The cubic, Eq. (10), is solved with an assumed value for κ. From the solution for λ, Eq. (12) gives V. The value of V determines, through Eqs. (3) and (4), new values for γ and κ. Iteration produces a consistent solution for the laser beam size and the growth rate, if a guided solution exists. Thus, in the exponential growth regime we have a complete analytic solution of the problem and there is no need to employ the 2D numerical code.

Because of optical guiding one can contemplate very long FEL's. In this way, it appears possible to have a small electron-beam radius and a very long wiggler (hence a very-high-gain FEL) even in the vacuum ultraviolet (vuv) range.

Because of the effect of optical guiding it is possible to direct and focus the FEL-generated optical beam. This is of interest for very intense beams, such as are contemplated for laser inertial fusion, where lenses and mirrors of conventional materials would be destroyed by the light. Use of optical guiding appears to be relatively straightforward since a simple magnetic deflection of the electron beam will result in a deflection of the light.

It should be noted that optical guiding applies, also, to very-short-wavelength light. Application of this to the vuv and to soft x rays, which do not interact coherently with normal material, would appear to make possible some interesting devices.

Optical guiding will be effective in an inverse free-electron laser (IFEL) as well as in an FEL,[7] and hence can be important in the operation of an IFEL. Finally, we note that optical guiding may make possible resonant ring FEL's.[8]

A more extensive treatment of this topic is given by Scharlemann, Wurtele, and Sessler.[9] After the completion of this work our attention was drawn to work by Moore which nicely complements that presented here.[10]

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