Evolution of Cosmic Strings

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Results of numerical simulations of the evolution of a network of interacting cosmic strings in an expanding universe are presented and compared with a simple model. The results lend weight to earlier speculations about the evolution of cosmic strings.

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There has been some interest recently in the idea that topologically stable strings or vortex lines formed at a phase transition in the very early universe could later provide the density perturbations needed to initiate the formation of galaxies.

Central to the idea is an observation due to Zel'dovich[1] and later Vilenkin,[2] based on earlier work by Kibble,[3] that if the network of strings evolves so that there are a fixed number of lengths of string crossing each horizon volume at any time, then the fractional density perturbation produced by the strings in the surrounding matter as a given scale falls inside the horizon is naturally scale invariant. If the mass per unit length is μ, the density of string is $\sim \mu/t^2$. Since the density of the surrounding matter or radiation scales in the same way, $\rho \sim 1/Gt^2$, the fractional density perturbation due to strings at the horizon crossing scale is $\sim G\mu \sim (m/m_{\mathrm{Pl}})^2$ where m is the scale of symmetry breaking and m_{Pl} the Planck mass. For those grand unified theories predicting strings, $m \sim 10^{16-17}$ GeV is typical and the density perturbations are of the right magnitude to produce galaxies by today.[4,5] Cosmic strings are also predicted naturally in unified superstring theories.[6]

To study the consequences of cosmic strings it is crucial to understand how they evolve once they are produced. So far, work in this area has made several assumptions about string evolution. It has been assumed, for example, that the string density does scale as t^{-2}. If the string density decreased more slowly, strings would quickly come to dominate, a cosmological disaster. This is the problem we address here.

Strings are initially formed as defect lines in the orientation of the Higgs field Φ mediating the phase transition. In the approximation that the phase transition happens sufficiently rapidly, one may simulate this process by choosing random orientations for Φ in regions of size ξ, the typical domain size after the phase transition. One then smoothes out these orientations along the shortest path (in internal space) in going from one domain to the next. Edges which Φ wraps around are then string segments. The string density is given by a fixed number of segments of length ξ per volume ξ^3.[3] Vachaspati and Vilenkin[7] have discussed the resulting spectrum of strings for cubic lattices. With periodic boundary conditions, all the strings are in the form of closed loops. The loops fall into two categories. Loops with radii much smaller than the box size have a scale-invariant spectrum—the number density of loops with radii between r and $r+dr$ is given by $n(r) \propto r^{-4}\,dr$, independent of ξ for $r >> \xi$. However, there is a constant contribution to the string density from loops with sizes near the box size, amounting to 80% of the total string density. As the box size goes to infinity, these become infinite strings. Both the closed loops and the "infinite" strings are Brownian, the length L along the string between two points being given by $L \propto d^2/\xi$ where d is the distance between them. We have repeated their calculations for lattices up to 70^3 with similar results, although we use rms radius for r while Vachaspati and Vilenkin use the sum of maximum extents. A simulation on a noncubic lattice gave a similar result.[8]

Of course the initial conditions tell us little about the dynamics. Luckily, this problem is largely a classical one—on the scales we are interested in the string width is negligible and the motion is well described by the action for an infinitely thin relativistic string, proportional to the area of the world sheet it sweeps out. The only input from the original field theory is in determining what happens when two strings cross. There is a probability $1-p$ that they pass through each other, and a probability p that they break and reconnect the other way ("intercommute"). In our simulations, we used directional strings so that the reconnection was unambiguous. p is hard to calculate, but recent simulations in a simplified model suggest $p \sim 1$.[9] In this Letter we examine two cases, $p=0$ and $p=1$, with background metric that for a spatially flat, radiation-dominated universe. Other values of p and the case of a matter-dominated universe will be considered elsewhere.[10] The equations governing the string motion are, in a convenient gauge,[11]

$$\ddot{\mathbf{x}} + 2\left(\frac{\dot{R}}{R}\right)\dot{\mathbf{x}}(1-\dot{\mathbf{x}}^2) = \frac{1}{\epsilon}\frac{\partial}{\partial \sigma}\left(\frac{\mathbf{x}'}{\epsilon}\right); \quad \dot{\epsilon} = -2\frac{\dot{R}}{R}\epsilon\dot{\mathbf{x}}^2 \quad (1)$$

where

$$\epsilon = [\mathbf{x}'^2/(1-\dot{\mathbf{x}}^2)]^{1/2};$$

$$\dot{\mathbf{x}} = \frac{\partial x}{\partial \eta}, \quad \mathbf{x}' = \frac{\partial x}{\partial \sigma}, \quad \dot{R} = \frac{dR}{d\eta}, \tag{2}$$

and $\mathbf{x}(\sigma,\eta)$ are the comoving spatial coordinates of the string, σ parametrizes its length, and η is the conformal time, given by $dt = R\, d\eta$ where R is the scale factor occurring in the metric

$$ds^2 = dt^2 - R^2\, d\mathbf{x}^2$$

with $R \propto t^{1/2} \propto \eta$. In this gauge the motion is perpendicular to the string's length, i.e., $\dot{\mathbf{x}} \cdot \mathbf{x}' = 0$. The energy in the string is given by

$$E = \mu R \int \epsilon\, d\sigma. \tag{3}$$

First, we evolved single loops using Eq. (1), with a second-order method for improved accuracy and stability. We imposed the conditions $\dot{\mathbf{x}} \cdot \mathbf{x}' = 0$, $\epsilon = [\mathbf{x}'^2/(1-\dot{\mathbf{x}}^2)]^{1/2}$ initially and then used them as a check on the subsequent evolution of (1). Figure 1 shows the evolution of an initially static ($\dot{\mathbf{x}} = 0$) flat Brownian loop, started with the initial coherence length $\xi_0 \sim h$. $h = 2t$ is the horizon distance. As can be seen, the loop evolves by straightening out on the scale of h.

In our simulations we dealt with the case $\xi_0 \sim h$ initially. Causality demands that $\xi_0 \lesssim h$, and the brief period of heavy damping of the strings tends to increase ξ up to the horizon scale before the strings evolve freely.[3] Different initial conditions ($\xi_0 << h$) have been discussed by Kibble[12] who concludes that the resulting string network evolves in a qualitatively different way. We will examine this question elsewhere.[10]

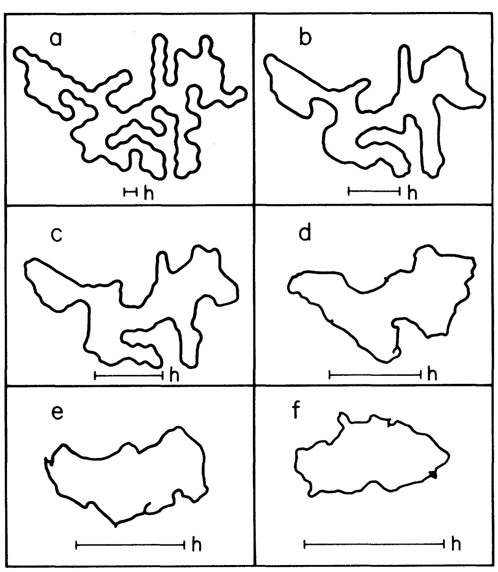

FIG. 1. Evolution of a Brownian loop.

For our dynamical simulations we used an initial $(16\xi_0)^3$ cubic lattice, and produced strings in the manner explained above. There were typically 100 or so loops initially, with 1 or 2 "infinite" ones (i.e., of size comparable to the box size). We evolved the strings using Eq. (1). We found that on smaller lattices with five points per initial segment ξ_0 we obtained results for the energy in the strings within 5% of the value to which the results converged for higher numbers of points (8,10,12) and we therefore used five points per ξ_0 in all subsequent work.

First we examined the case $p = 0$ (no exchange of partners). Here the total energy remained roughly constant. This may be understood as follows. The energy of loops with $r << h$ is almost constant, the expansion of the universe having negligible effect on them.[11] Loops with $r >> h$ are stretched by the expansion, the distance between two points growing with the scale factor, $d \propto t^{1/2}$. They are also straightened out (Fig. 1) so the coherence length $\xi \propto t$. Thus $L \propto d^2/\xi$ remains constant. The "infinite" strings are a disaster for cosmology—they cannot radiate away as small loops can[2,5] and their density scales like matter, quickly coming to dominate over the radiation. Even though they straighten out, $\xi \propto t$, the number of lengths of string crossing each horizon grows as $t^{1/2}$.

Next we examined the case $p = 1$. Here the situation is more complicated, due to the intersection of strings and the resulting production and annihilation of loops. To check for intersections, we divided space into comoving cubes of size $\xi_0/5$. If two points on the string were found in the same box, we checked the maximum distance from the points reached by going from one to the other in each direction along the string. If this distance was greater then $\alpha\xi_0/5$, we exchanged partners. We found the results were insensitive to α for α around 1.5. This is probably the simplest possible procedure for checking for crossing of strings.

How does the string density scale for $p = 1$? To see this we must include all loops and the effect of their interactions. However, as is easy to check, the probability for a loop with $r << t$ to interact with another loop is negligible. These smallest loops have almost constant energy and their density scales like matter. They radiative graviational waves[5,7] at a rate $\dot{M} \sim -100 G\mu^2$ and since their mass is $M \sim \mu \times 2\pi r_0$, they have a lifetime $\tau \sim r_0/10G\mu >> r_0$ (r_0 is their initial radius). This gives a cutoff in the distribution of loops at any particular time t, at $r_c << t$. In our simulations we did not have the range for this, but since these small loops hardly interact at all, we did not need to. Instead, we used a radius cutoff r_c at a fraction of h, and then varied this cutoff to see the effect of the smaller loops. With a simple model we could then extrapolate to include smaller loops for any value of $G\mu$.

In Fig. 2, the quantity $\rho t^2/\mu$ is plotted against $\theta=2(t_0 t)^{1/2}/16\xi_0$. ρ is the density in string loops with radii larger than $r_c=0.1(2t)$. θ is the ratio of the horizon distance to the size of the box. We ran our simulations until $\theta\sim h$ and finite-volume effects became obviously important. The range shown $\frac{1}{16}<\theta<\frac{5}{8}$, represents a factor of 100 in time t.

The three different plots correspond to different choices of initial values θ_0. For $\theta_0=\frac{1}{16}$, so that $\xi_0=2t_0$, $\rho t^2/\mu$ rose smoothly as $t^{1/2}$ as in the $p=0$ case. For $\theta_0=\frac{5}{16}$, $\xi_0=\frac{2}{5}t_0$, $\rho t^2/\mu$ dropped abruptly and then leveled off. For $\theta_0=3.7/16$, $\xi_0=2t_0/3.7$, an intermediate case, $\rho t^2/\mu$ rose slightly, fell slightly and then leveled off.

The results may be understood in terms of the following simple picture. The string network has a "stable" state with ξ some fraction of the horizon distance. If ξ_0 is larger than this, the rate of intersection of strings is lower than at the stable point and the number of string lengths per horizon grows as it did in the $p=0$ case. If ξ_0 is smaller than the stable value, the rate of intersection is larger than the stable value, and more closed loops are produced by the self-intersection of long strings, reducing their density to the stable value. In fact we find that the energy density due to loops with $r>2t$ is given in the stable regime by

$$\rho_{r>2t}=(2.5\pm 0.5)\mu/t^2, \tag{4}$$

which corresponds to roughly one segment of length t per volume t^3 in qualitative agreement with the behavior in Fig. 1 and the above picture.

Now we turn to the energy-density contribution of the smaller loops. To understand this note that since ξ grows as t on the network of strings larger than h, the radius of the loops produced at a time t should be $\sim t$. In fact since the network of larger loops does not come to dominate, a large part of their energy must flow out through loops with $r\sim t$. These loops may intersect and self-intersect thereafter. One expects the self-intersection process to be completed in a few expansion times. This is because a loop's motion is almost exactly periodic inside the horizon,[11,13] and if it intersects at all it must do so within a period $T\sim\pi r$. If it splits into two, these have a period $\sim\pi r/2$. Continuing this argument, the process should be over by $\sim 2\pi r$, with the result either that it has completely disappeared or that it has produced a number of non-self-intersecting loops. Our simulations show the latter behavior.

We choose to model the resulting energy density distribution in loops with the following formula:

$$\rho_{r>r_c}(t)=\alpha\int_{r_c}^{2t}dr'\,\mu r'\frac{1}{r'^4}\left(\frac{r'}{t}\right)^{3/2}+\beta\int_{2t}^{\infty}dr'\mu\left(\frac{r'^2}{t}\right)\frac{1}{r'^4}+\frac{\gamma\mu}{t^2}, \tag{5}$$

where the three contributions are from loops smaller than the horizon, loops larger than the horizon, and "infinite" loops (i.e., loops as large as the box we use). Small loops of radius r' were produced at a time $t'\sim r'$ and at a rate $1/t'^4$ per unit volume. Their mass is $\sim\mu r'$ and their density has been reduced by $(t'/t)^{3/2}$ since they were produced. The model ignores the breakup of these loops due to self-intersections and the fact that it fits the data well suggests that soon after loops fall inside the horizon self-intersections be-

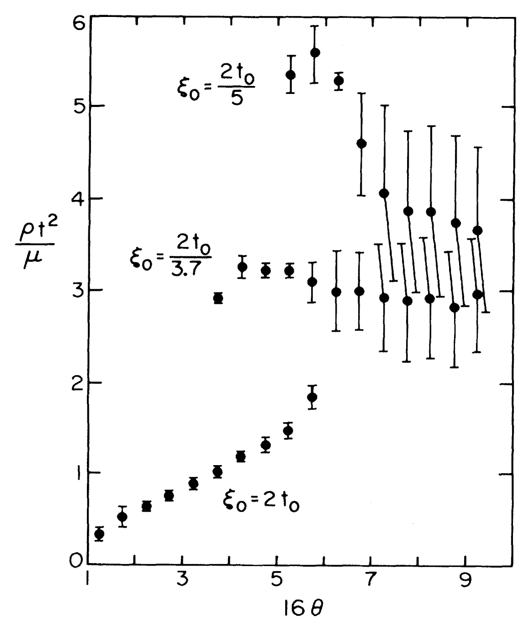

FIG. 2. Energy density in loops with radii greater than $r_c=0.1(2t)$ vs $\theta\propto t^{1/2}$ for different initial conditions.

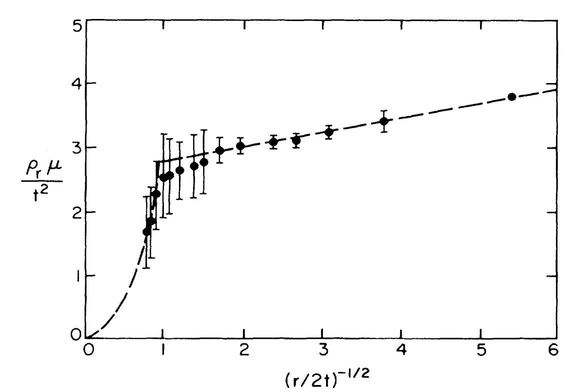

FIG. 3. Energy density in loops with radii greater than r, ρ_r, vs $(r/2t)^{-1/2}$ at "steady state." Dashed lines show fit with model in Eq. (5).

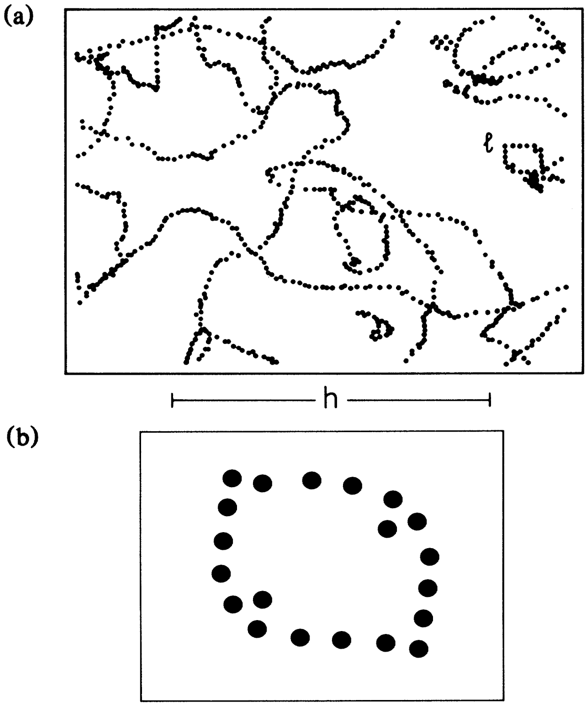

FIG. 4. (a) Evolved network of strings on a $(6\xi_0)^3$ lattice. Dots show the points on the string used in numerical evolution. h shows the horizon scale. The comoving initial coherence length ξ is $\sim h/4$. (b) Loop marked l in (a) is enlarged.

come rare. In a more complicated model α would depend on r_c too. Larger loops have a size distribution dr'/r'^4 as they did when they were formed, r' growing like R while the volume goes like R^3. They are still Brownian, with length $\sim r'^2/t$. Finally, infinite strings have a density $\sim \mu/t^2$. α, β, and γ are the overall numerical factors for each term.

We check this model in Fig. 3. Here the energy density in loops with radii greater than r is plotted against $(r/2t)^{-1/2}$ at a fixed time, when the string density appears to have reached a steady state (when $16\theta = 7$ on the middle curve in Fig. 2). For $r << 2t$, the distribution is well modeled by the $r^{-1/2}$ power law predicted in Eq. (5). Close to $2t$, the distribution tilts over and at $r = 2t$ abruptly drops, in surprisingly good agreement with (5). Fitting this part of the curve with a r^{-1} law we see that data are actually consistent with $\gamma = 0$. It seems possible therefore that the string network evolves into a distribution composed entirely of closed loops.

In Fig. 4(a) we show a simulation on a $(6\xi_0)^3$ lattice. Several loops were formed by the self-intersection of lengths of string. The loop marked l and enlarged in Fig. 4(b) has two cusps and looks very similar to the exact lowest-mode solutions found in Ref. 5.

In conclusion, we have given strong evidence that for $p = 1$ the string energy density scales as radiation and does not come to dominate. More detailed predictions will be discussed elsewhere.

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Note added.—According to our most recent simulations, the constants in (5) are $\alpha = 0.21 \pm 0.03$, $\gamma + \beta/2 = 2.4 \pm 0.1$.

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