Parametric Self-Enhancement of the Spontaneous Decay of Sound in Superfluid Helium

J. S. Foster
Edward L. Ginzton Laboratory, Stanford University, Stanford, California 94305

and

S. Putterman
Department of Physics, University of California, Los Angeles, California 90024
(Received 26 December 1984)

The spontaneous decay of coherent monochromatic sound is dramatically self-enhanced by parametric amplification. We observed this effect for the first time with 3.25-GHz sound in superfluid helium at 85 mK. Starting with a typical intensity of 10 W/m^2 the sound beam is depleted by more than 99.9% over just 1 mm of propagation distance. In addition we observed, in contrast to Landau-Rumer processes, an exponential decay coefficient proportional to the square root of the initial intensity.

PACS numbers: 43.25.+y, 42.65.Cq, 67.40.Mj

We report the observation of the rapid nonlinear depletion of a sound wave due to enhanced spontaneous decay using pulsed 3.25-GHz sound waves of intensities ~ 10 W/m^2 launched into superfluid helium at $T = 85$ mK. The observed depletion length is about 1 mm in contrast to the decay length of 20 m which would be calculated from the kinetic equation that describes spontaneous decay[1–7] with neglect of the parametric enhancement described in this Letter. The sound propagation is characterized by two regions. In the first region the impressed sound wave becomes a pump for the spontaneous decay products, which act as signals and idlers in a parametric amplifier. The pump exhibits a rollover depletion which is qualitatively characteristic of three wave parametric processes.[8] In the second region, the intensity of the pump decays exponentially. Furthermore, we observe for the first time a coefficient of exponential decay proportional to the square root of the input intensity. This decay is the result of the one-dimensional scattering of sound by noise and is in contrast to three-dimensional Landau-Rumer processes[9] wherein attenuation is proportional to noise intensity.

Spontaneous decay of incoherent phonons has been observed in CaF_2 by Baumgartner.[10] Spontaneous decay of photons[11–14] (known as parametric fluorescence or spontaneous parametric emission in nonlinear optics) is well known. Pump depletion in the optical case has been inferred at intensities high enough to create subharmonic intensity (typically called stimulated parametric fluorescence[15] or parametric superluminescence).[16] For pump depletion of only 0.01% however, these experiments must use laser powers of 3×10^8 W/cm.^{2} At these powers stimulated Raman scattering, self-focusing, and crystal breakdown complicate the precise interpretation of the results. The huge conversion efficiences (up to 99.9%) for very low input powers (10^{-3} W/cm^2) in the acoustic case described here demonstrate a new experimental system which can be used to study new regimes in strong nonlinear interactions.

Spontaneous decay occurs in a medium that has an anomalous dispersion relation for frequency w as a function of wave number k (i.e., speed increases with frequency). The decay products have frequencies w_1, w_2, and wave numbers $\mathbf{k}_1$, $\mathbf{k}_2$ satisfying

$$w_1 + w_2 = w_p; \quad \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_p, \tag{1}$$

where the subscript $p(\equiv \text{pump})$ refers to the impressed wave field.

Calculations[1–7] of the spontaneous decay of sound in a fluid, neglecting any parametric enhancement, yield exponential decay of the intensity $I = I(0)\exp(-\alpha_0 t)$ where

$$\alpha_0 = \hbar\beta^2 k_p^5/(240\pi\rho), \tag{2}$$

where $\beta = 1 + (\rho/c)dc/d\rho$ (~ 3.84 for helium) with c the sound velocity and ρ the density. In these calculations, interactions between the decay products, at frequencies w_1 and w_2, and the initial sound beam, at frequency w_p have always been ignored. For incoherent sources this is valid, but for a coherent source the decay products have exactly the proper frequencies and wave numbers to scatter resonantly with the beam [see Eq. (1)]. Under conditions discussed below there can be a strong parametric coupling between the beam and its quantum decay products which lead to a dramatic amplitude dependent decay of the beam on a length scale much smaller than c/α_0.

The most important condition for strong parametric amplification of traveling waves is anomalous dispersion. Helium-4 exhibits this property at low pressure. Anomalous dispersion allows conservation of energy and momentum [Eq. (1)] for three-phonon interactions,[17] and limits the stronger processes of higher harmonic and sum frequency generation[18] characteristic of linear dispersion. The characteristic distance over which there is appreciable harmonic generation for linear dispersion is the shock discontinuity length $L_s = 1/(\beta M k)$, where M is the Mach number of the coherent sound wave (particle velocity divided by sound velocity). With some dispersion (normal or anomalous) the harmonic generation can become reversible.[19,20] In fact at low power levels, second harmonic intensity is periodic with length

$$2L_c = 2\pi/\Delta k, \tag{3}$$

where L_c is called the second harmonic coherence length, $\Delta k = k_2 - 2k_1$, and k_1 and k_2 are the wave numbers of the fundamental and second harmonic, respectively. With greater dispersion, of course, Δk becomes larger. A rough condition for limiting significant harmonic generation requires L_c to be shorter than L_s or

$$\Delta k/k \gtrsim \pi\beta M. \tag{4}$$

The anomalous dispersion in helium increases with frequency, satisfying Eq. (4) at or above gigahertz frequencies.

Another criterion for the observation of the pump depletion is that the depletion time due to nonlinear effects must be less than the $1/e$ decay time due to all friction effects or

$$\beta M >> (1/Q)\log[E(0)/E_s(0)], \tag{5}$$

where $E(0)$ and $E_s(0)$ are the initial pump and signal energies, respectively, and Q is the quality factor for infinitesimal waves. When Eq. (5) holds the rate of amplification will also exceed the loss rate for signal and idler. Equation (5) is easily satisfied in low-temperature superfluid helium.

The experimental cell used in this study has been described previously.[20] A thin film ZnO transducer (190-μm radius) is deposited onto a sapphire rod which is 12 mm long or 1.14 Fresnel lengths. The sound waves launched into the superfluid helium from the sapphire are reasonably planar and have a smooth amplitude distribution. Another sapphire rod with a transducer (100-μm radius) is used to receive the sound waves after some helium propagation distance. The helium path separation and the mechanical tilt between the transmitting and receiving transducers are adjustable as described in Ref. 20. The microwave signal from the 100-μm radius receiving transducer is preamplified by low-noise GaAs field-effect-transistors operating at 4.2 K. At room temperature, the signal is mixed down to an intermediate frequency of 60 MHz, and the envelope detected with a square law crystal detector. A boxcar averager is used to measure the pulse height by adjusting a microwave receive attenuator so as to match the received signal with a fixed reference level.

A sample of the data taken at saturated vapor pressure and 85 mK is shown in Fig. 1. The received intensity is plotted as a function of the transmitted intensity at the frequency 3.25 GHz. A pulsed system was used with 150-ns width pulses at a repetition rate of 20 kHz. No sensitivity to pulse width or repetition rate was observed. The input intensity was roughly calibrated by measuring the microwave input power, measuring the round trip insertion loss of the transmitting transducer, and calculating the effective area of the sound beam in the helium.

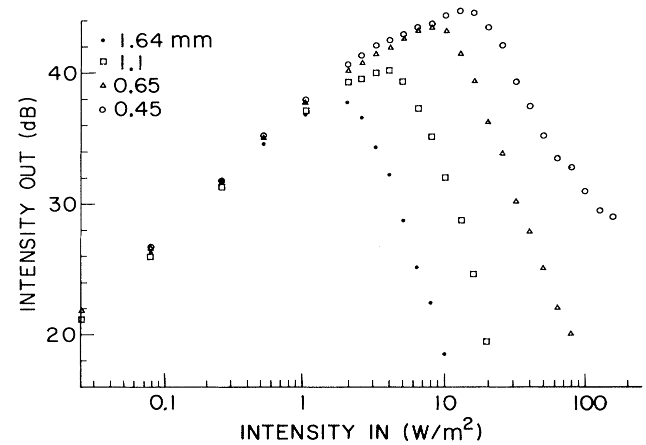

FIG. 1. Output acoustic intensity at 3.25 GHz as a function of input intensity for four different helium path lengths (log-log plot). Data taken at saturated vapor pressure at 85 mK.

The data in Fig. 1 represent measurements for four different helium propagation distances. The distances were measured by the time delay of the sound waves in the helium assuming a sound velocity of 238 m/s. The ZnO transducers and the sound wave propagation in the sapphire rods are linear at these power levels.

For low intensities Fig. 1 demonstrates a linear system in which the output power increases linearly with input power. However, at higher input intensities, corresponding to a Mach number of approximately 10^{-4}, the output intensity falls off sharply. We believe that strong parametric amplification of the spontaneous decay products is taking place at the point where the output intensity rolls over and the input sound wave is being significantly depleted by the buildup of the "noise" field.

When helium is pressurized above 20 bars, it is known that the dispersion becomes first linear and then normal. Normal dispersion is not consistent with Eq. (1) for any angle between the **k** vectors. Indeed, in experiments performed above 20 bars, we have found an output intensity that did not decrease with input intensity as in Fig. 1; rather we have found an output that saturated in a way characteristic of weak shock formation.[21]

In Fig. 2(a) the logarithm of the ouput intensity is plotted as a function of helium path length for three

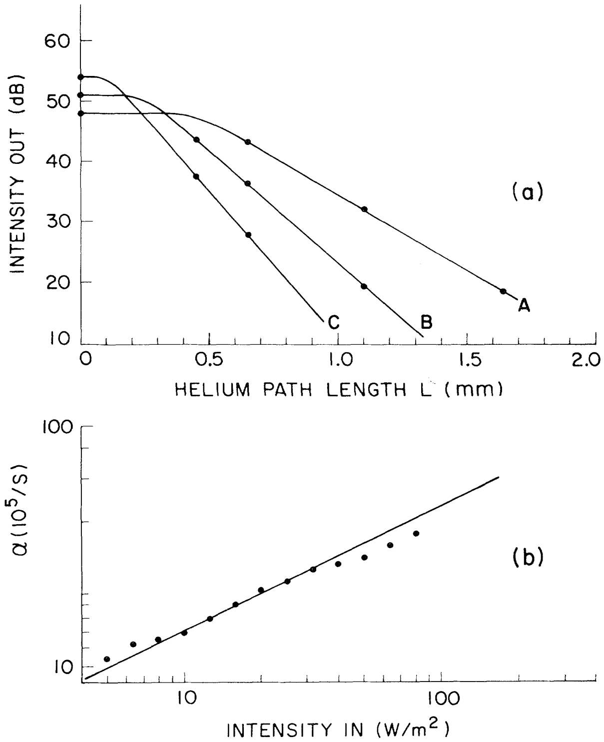

FIG. 2. (a) Output intensity as a function of helium path length for fixed input intensities. Curves A, B, and C correspond to input intensities of 10, 20, and 40 W/cm^2, respectively. Straight lines drawn through the points in each curve represent exponential attenuation. (b) The exponential attenuation coefficient associated with (a) plotted as a function of input intensity. The straight line drawn through the points corresponds to α varying as the square root of input intensity.

different input intensities. The data are taken from Fig. 1. The three data points of curves A and B lie on a straight line of the log plot, indicating that the intensity of the sound wave is falling exponentially with distance after an initial rollover. Higher input intensities result in a large attenuation coefficient as can be seen in Fig. 2(a). In Fig. 2(b), the exponential attenuation coefficient (α) is plotted a a function of input intensity in a log-log plot. Again, the data are taken from Fig. 1. The line drawn through the points corresponds to a slope of $\frac{1}{2}$, i.e., α proportional to the square root of the input power.

In Fig. 2(a), two regions of the pump decay are apparent. In the first region, as the wave is launched into the helium, a small proportion of the enormous number of phonons in the pump spontaneously decay into pairs of phonons with lower frequencies. These low-frequency phonons are then parametrically amplified at the expense of the pump. The distance x_d at which the pump has been depleted to about 50% of its initial energy $E(0)$ can be estimated using the standard solution for parametric amplification.[8,22] This distance is

$$x_d \approx (1/\beta M k_p)\log[E(0)/E_s(0)], \tag{6}$$

where $E_s(0)$ is the initial signal energy, M is the Mach number of the pump wave, $E(0)=M^2\rho c^2 V_p/2$ with V_p the pump pulse volume equal to the pulse length times the area of the sound beam in the helium $\sim 10^{-6}$ cm^3. Setting $E_s(0)=\frac{1}{2}hw_p$ leads to a value for x_d which is in agreement with our experiment. The logarithmic dependence in Eq. (6) causes the rollover distance to be relatively insensitive to the fact that $E_s(0)$ is small compared to $E(0)$. Equation (6) applies only after the first signal phonon has been created by spontaneous decay. The distance for initial spontaneous decay, x_q, is determined by

$$[E(0)/\hbar\omega_p](\alpha_0/c)x_q = 1. \tag{7}$$

For this experiment $x_q \sim 1$ Å but, for example, at 30 MHz and similar pump energy $x_q \sim 1$ cm.

In the standard solution for parametric processes only three modes are considered: the pump, the signal, and the idler. For that case, the pump depletes rapidly to nearly zero amplitude, the phase relationship between the three waves changed by π, and the pump is then regenerated by the large signal and idler. In the present case, however, many pairs of decay products have been produced since $x_q << x_d$. These pairs are not correlated with each other and are independently amplified. As a result, there is no single regeneration time, but instead a mix of incommensurate modulation frequencies that eventually lead to an exponential decay (Fig. 2) which is well described by

$$\alpha = (0.17)(w\beta)[2I(0)/\rho c^3]^{1/2} \tag{8}$$

where $I(0)=M^2\rho c^3/2$ is the pump's initial intensity.

This second region is best understood in terms of a sound wave interacting with a noise background wherein all the noise has been created by the pump and is in resonance with the pump wave. In this sense this region is equivalent to a one-dimensional scattering of sound by noise in a nondispersive medium. The data presented here constitute the first observations relevant to this long-standing unsolved theoretical question.[23] The attenuation does not vary as the intensity of the noise background as predicted by Landau-Rumer theory and observed in the sound attenuation in helium.[17] Instead, the attenuation as shown in Fig. 2(b) is proportional to the square root of the intensity.

It is pertinent to discuss here any other effects associated with high intensity sound propagation in helium which might give rise to the results described in this paper. In particular, we consider cavitation and streaming which exceed the critical velocity for vortex creation. If the sound amplitude in a liquid exceeds a critical value, then cavitation can occur and cause an attenuation of sound. However, evidence from another system indicates that the cavitation amplitude threshold increases linearly with frequency,[24] and extrapolation from helium experiments[25] would lead to a threshold far above the sound amplitudes used in this experiment. On the other hand, the present experimental amplitudes are larger than those used to generate vorticity in helium from streaming.[26,27] However, the acoustic absorption (which gives rise to streaming) is very small in the present experiment compared to the absorption in the experiments cited above, which were performed at temperatures above 1 K. Also, the present experiment used pulsed sound (rather than cw) which greatly lowers the resulting streaming velocity. As already noted, the results given here showed no sensitivity to pulse width or repetition rate, which indicates that streaming is not important. In addition, transient behavior would be expected with vortex effects, since vorticity typically has a very slow decay time (> 1 s).[28] However, no transient behavior was observed in our experiment.

In conclusion, the rapid and highly nonlinear depletion of coherent, monochromatic sound has been observed in low-temperature superfluid helium. The depletion is due to parametric self-enhancement of the spontaneous decay of sound. The rate of exponential amplification of the signal (spontaneous decay products) is proportional to the amplitude of the sound wave [see Eq. (6)]. This coherent effect is to be contrasted with incoherent feedback instabilities where gain is proportional to intensity and which on occasions have been improperly labeled as parametric processes.[29] Our work shows that high-frequency sound wave propagation in low-temperature helium is a highly nonlinear system capable of entering regimes not available in nonlinear optics. In future research, we plan to study the behavior of highly nonlinear parametric systems, analyze the role of dimensionality in the scattering of sound by noise, and probe the quantum noise limit of parametric amplification.

We would like to acknowledge many helpful discussions with C. F. Quate, D. Rugar, R. L. Byer, S. E. Harris, B. Hadimioglu, R. Youngquist, A. Larraza, and J. Millovich. This work was supported by the Office of Naval Research and one of us (J.S.F.) acknowledges support as a Marvin Chodorow Fellow.

[1]G. L. Slonimskii, Zh. Eksp. Teor. Fiz. **7**, 1457 (1937).

[2]S. T. Belyaev, Zh. Eksp. Teor. Fiz. **34**, 433 (1958) [Sov. Phys. JETP **7**, 299 (1958)].

[3]J. Jäckle and K. W. Kehr, Phys. Rev. Lett. **24**, 1101 (1970).

[4]T. J. Sluckin and R. M. Bowley, J. Phys. C **7**, 1779 (1974).

[5]H. J. Maris, Phys. Rev. A **9**, 1412 (1974).

[6]M. Cabot and S. Putterman, Phys. Lett. **83A**, 91 (1981).

[7]R. Orbach and L. A. Vredevoe, Physics (N.Y.) **1**, 91 (1964).

[8]R. A. Baumgartner and R. L. Byer, IEEE J. Quantum Electron. **15**, 432 (1979).

[9]L. D. Landau and G. Rumer, Phys. Z. Sowjetunion **11**, 18 (1937).

[10]R. Baumgartner, M. Engelhardt, K. Renk, and R. Orbach, Physica (Utrecht) **107B**, 109 (1981).

[11]R. L. Byer and S. E. Harris, Phys. Rev. **168**, 1064 (1968).

[12]D. A. Kleinman, Phys. Rev. **174**, 1027 (1968).

[13]S. E. Harris, Proc. IEEE **57**, 2096 (1969).

[14]T. G. Giallorenzi and C. L. Tang, Phys. Rev. **166**, 225 (1968).

[15]T. A. Rabson, H. J. Ruiz, P. L. Shah, and F. K. Tittel, Appl. Phys. Lett. **21**, 129 (1972).

[16]S. A. Akhmanov, V. V. Fadeev, R. V. Khokhlov, and O. N. Chunaev, Pis'ma Zh. Eksp. Teor. Fiz. **6**, 575 (1967) [JETP Lett. **6**, 85 (1967)].

[17]H. J. Maris, Rev. Mod. Phys. **49**, 341 (1977).

[18]N. S. Shiren, Proc. IEEE **53**, 1540 (1965).

[19]O. V. Rudenko and S. I. Solvyan, *Theoretical Foundations of Nonlinear Acoustics* (Consultants Bureau, New York, 1977), p. 86.

[20]D. Rugar and J. S. Foster, Phys. Rev. B **30**, 2595 (1984).

[21]See, for example, D. A. Webster and D. T. Blackstock, J. Acoust. Soc. Am. **62**, 518 (1977).

[22]J. A. Armstrong, N. Bloembergen, J. Ducuing, and P. S. Pershan, Phys. Rev. **127**, 1918 (1962).

[23]A. C. Newell and P. J. Aucoin, J. Fluid Mech. **49**, 593 (1971).

[24]E. Webster, Ultrasonics **1**, 39 (1963).

[25]R. D. Finch, R. Kagiwada, M. Barmatz, and I. Rudnick, Phys. Rev. **134**, A1425 (1964).

[26]R. F. Carey, J. A. Rooney, and C. W. Smith, J. Acoust. Soc. Am. **66**, 1801 (1979).

[27]J. A. Rooney, C. W. Smith, and R. F. Carey, J. Acoust. Soc. Am. **72**, 245 (1982).

[28]F. P. Milliken and K. W. Schwarz, Phys. Rev. Lett. **48**, 1204 (1982).

[29]R. Orbach, IEEE Trans. Sonics Ultrason. **14**, 140 (1967).