

$K^0 (\bar{K}^0) \rightarrow 2\gamma$ Decays: Phenomenology and CP Nonconservation

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The mixing and the CP -nonconserving effects are studied for $K^0, \bar{K}^0 \rightarrow 2\gamma$. The Kobayashi-Mashawa scheme can give observable CP -nonconserving effects different from those of the superweak interaction. Experimental ways to detect such effects are pointed out.

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It has been twenty years since the first observation¹ of CP nonconservation, $K_L \rightarrow 2\pi$, and the charge asymmetry of $K_L \rightarrow \pi e \nu$ were reported. So far these are still the only instances of CP nonconservation observed. Further, both phenomena originate from the same CP -nonconserving source, the $K^0 \rightleftharpoons \bar{K}^0$ transition in the mass matrix. CP -nonconserving effects can also arise from weak decay amplitudes. A nonzero ϵ' would be such an effect.²⁻⁴ Unfortunately, $\Delta I = \frac{1}{2}$ dominance gives ϵ' a suppression factor of twenty as compared to ϵ , $|\epsilon'| < |\epsilon|/20$. Recent analyses in the Kobayashi-Maskawa (KM) scheme of CP nonconservation have shown that such CP -nonconserving effects from decay amplitudes can be significant in b decays,⁵ giving partial-decay-rate differences between particles and their antiparticles. It would be interesting to see if there are other such decay-amplitude CP -nonconserving effects in the K decays. The $K \rightarrow 2\gamma$ is such a candidate, especially since the rate for $K_L \rightarrow 2\gamma$ is about one-half the rate of $K_L \rightarrow \pi^0 \pi^0$. Here, we give a detailed analysis of CP -nonconserving effects in

$K^0 \rightarrow 2\gamma$ in the KM scheme. Special attention is paid to how the decay-amplitude effects can be distinguished from the mass-matrix effects. The latter can be calculated precisely from the measured CP -nonconservation effects of $K_L \rightarrow 2\pi$ and $K_L \rightarrow \pi e \nu$. We shall show that, on the basis of current order-of-magnitude estimates, the decay-amplitude CP -nonconserving effects in the KM scheme can be significant, giving a 0.1% to 0.2% effect comparable to that from the pure mass-matrix effects in the partial-decay-rate difference. We then discuss the possible ways to detect such effects. Our calculations and conclusions are substantially different from those of a recent analysis by Decker, Pavlopoulos, and Zoupanos.⁶ These differences will be discussed at the end of the paper.

Formulation.—The two-photon system is a mixture of the CP -even state, $2\gamma_+$, and the CP -odd state, $2\gamma_-$. The decay rates are given by

$$\Gamma_{\pm}(K \rightarrow 2\gamma_{\pm}) = m_K^3 |A_{\pm}|^2 / 64\pi, \quad \Gamma = \Gamma_+ + \Gamma_-. \quad (1)$$

The time evolution of $K^0, \bar{K}^0 \rightarrow 2\gamma$ is

$$\Gamma(\bar{K}^0 \rightarrow 2\gamma)(\tau) = \frac{1}{N} \sum_{j=\pm} |A(K_j \rightarrow 2\gamma_j)|^2 [e^{-\Gamma_j \tau} + |\eta_j|^2 e^{-\Gamma_j \tau} \frac{1}{2} |2\eta_j| e^{-\bar{\Gamma} \tau} \cos(\Delta m \tau - j\phi)], \quad (2a)$$

where $|K_{\pm}\rangle = |K_{S,L}\rangle$, N are normalization factors, and η_{\pm} are the parameters characterizing CP nonconservation (e.g., η_{+-}, η_{00}):

$$\eta_{\pm} = |\eta_{\pm}| e^{i\phi_{\pm}} \equiv A(K_{\mp} \rightarrow 2\gamma_{\pm}) [A(K_{\pm} \rightarrow 2\gamma_{\pm})]^{-1} = [r_{\pm} - (1 - \bar{\epsilon}) / (1 + \bar{\epsilon})] [r_{\pm} + (1 - \bar{\epsilon}) / (1 + \bar{\epsilon})]^{-1}, \quad (2b)$$

$$r_{\pm} = |r_{\pm}| e^{i\theta_{\pm}} = \mp A(K^0 \rightarrow 2\gamma_{\pm}) [A(\bar{K}^0 \rightarrow 2\gamma_{\pm})]^{-1}. \quad (2c)$$

The time-integrated rates are $I(\tau_1) = \int_0^{\tau_1} d\tau \Gamma(\bar{K}^0 \rightarrow 2\gamma)(\tau)$. The interesting measurable CP -nonconserving quantities are the partial-decay-rate difference,

$$\Delta(\tau) = \frac{\Gamma(K^0 \rightarrow 2\gamma)(\tau) - \Gamma(\bar{K}^0 \rightarrow 2\gamma)(\tau)}{\Gamma(K^0 \rightarrow 2\gamma)(\tau) + \Gamma(\bar{K}^0 \rightarrow 2\gamma)(\tau)}, \quad (3)$$

and the integrated partial-decay-rate difference,

$$\Delta_I(\tau_1) = [I(\tau_1) - \bar{I}(\tau_1)] [I(\tau_1) + \bar{I}(\tau_1)]^{-1}. \quad (4)$$

With the measured $A(K_L \rightarrow 2\gamma) = 3.13 \times 10^{-12} \text{ MeV}^{-1}$, the unknown quantities in $K^0 \rightarrow 2\gamma$ are η_{\pm} (or equivalently r_{\pm}) and $R = \Gamma(K_S \rightarrow 2\gamma_+) / \Gamma(K_L \rightarrow 2\gamma_-)$.

Note that $\bar{\epsilon}$ is phase-convention dependent.³ If a common phase convention can be found such that $r_+ = r_- = 1$, then from Eq. (2)

$$\bar{\epsilon} = \eta_+ = \eta_-. \quad (5)$$

If such a convention cannot be found, then there must be CP -nonconserving effects from the decay amplitudes.

There has been a long history⁷⁻¹³ of estimating $A(K^0 \rightarrow 2\gamma_{\pm})$ based upon diagrams of quarks ($u\bar{u}$ for $2\gamma_{-}$), and diagrams with dominant physical intermediate states (2π intermediate states for $2\gamma_{+}$, various pole intermediates for $2\gamma_{-}$). We shall use these methods and results, and incorporate our own, which are based upon the most recent knowledge, to estimate the CP -nonconserving effects in the KM scheme. To present a contrast, we first discuss the superweak model.¹⁴

Superweak model.—In this model the CP nonconservation comes solely from the mass matrix; therefore the conditions of Eq. (5) are satisfied. Therefore the only unknown quantity that needs to be calculated is $\Gamma(K_S \rightarrow 2\gamma_{+})$.

This CP -even transition is dominated by the $\pi^+\pi^-$ intermediate state (the lowest-order one-loop quark diagram is known⁸ not to contribute to $K_S \rightarrow 2\gamma_{+}$). The $K_S \rightarrow 2\gamma_{+}$ via the $\pi^+\pi^-$ intermediate state has been calculated to be⁹⁻¹¹

$$A(K^0 \rightarrow 2\gamma_{+}) = 2\alpha A(K^0 \rightarrow \pi^+\pi^-) \zeta_{2\pi}, \quad (6)$$

where α is the fine-structure constant and $\zeta_{2\pi}$ results from the loop integration of the 2π intermediate state, with the assumption of constant off-shell $A(K^0 \rightarrow 2\pi)$ in the loop integration. In the superweak model $A(K^0 \rightarrow 2\pi) = A(\bar{K}^0 \rightarrow 2\pi)$. We use the experimental values for $A(\bar{K}^0 \rightarrow \pi^+\pi^-) = A(K^0 \rightarrow \pi^+\pi^-) = 2.74 \times 10^{-4} \text{ MeV}^{-1}$, $A(K_L \rightarrow 2\gamma)$, and the calculated⁹⁻¹¹ $\zeta_{2\pi}$ to obtain

$$R = 2.4. \quad (7)$$

The predictions for $\Delta(\tau)$ from this result are given in Fig. 1. The corrections to Eq. (7) from other physical intermediate states are rather small. A possible source of deviation from this value of R is the deviation of the off-shell $A(K^0 \rightarrow \pi^+\pi^-)$ from being constant.

Kobayashi-Maskawa model.—The KM scheme gives CP -nonconserving effects both in the $K^0 \rightleftharpoons \bar{K}^0$ mass matrix and in the decay amplitudes; thus $A(K^0 \rightarrow \pi^+\pi^-)/A(\bar{K}^0 \rightarrow \pi^+\pi^-)$ in Eq. (6) is no longer real.

However, because of this $\pi^+\pi^-$ dominance in $K_S \rightarrow 2\gamma_{+}$, it is easily shown by substitution of Eq. (6) into Eq. (2) that the KM case for $K_S \rightarrow 2\gamma_{+}$ is the same as for the superweak case, i.e., $\eta_{+} = \eta_{+-}$, or equivalently $|r_{+}| = 1$, $\phi_{+} = 2t_0$, where³ $t_0 = \text{Im}a_0/\text{Re}a_0$, and a_0 is the $I=0$ part of $K \rightarrow 2\pi$. This conclusion is quite general, more so than the derivation for the value of R in Eq. (7), since R depends on the evaluation of $\zeta_{2\pi}$.

To calculate η_{-} , we need to calculate $A(K^0 \rightarrow 2\gamma_{-})$.

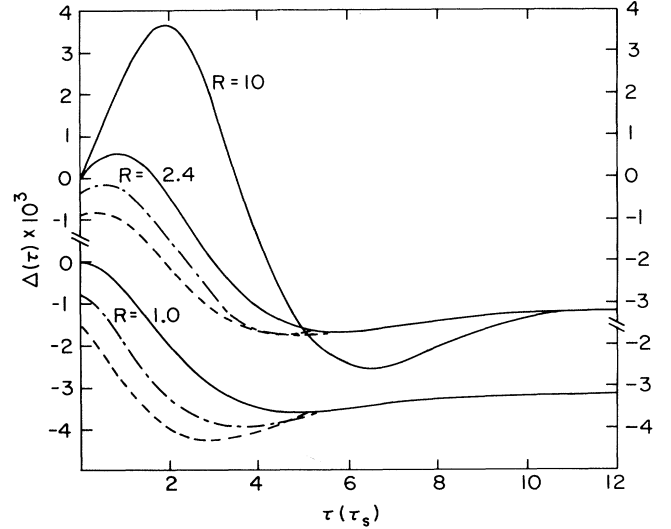


FIG. 1. The time evolution for the partial-rate difference $\Delta(\tau)$: the solid (broken) lines are from the superweak (KM) model; for parameters see the text.

First we discuss the quark contributions from the $u\bar{u}$, $c\bar{c}$, and $t\bar{t}$:

$$A_Q(K^0 \rightarrow 2\gamma_{-}) = \frac{G_F \alpha}{\sqrt{2}\pi} \sum_{i=u,c,t} V_{is}^* V_{id} A_i. \quad (8)$$

For $\bar{K}^0 \rightarrow 2\gamma_{-}$, $V_{is}^* V_{id}$ is replaced by $V_{is} V_{id}^*$ in this equation. It is known⁸ that $|A_u| \gg |A_c|, |A_t|$. Both A_c and A_t are real, while A_u has an absorptive part $\text{Im}A_u$. As we shall show later, together with the imaginary part from V_{ij} , $\text{Im}A_u$ can contribute to CP -nonconserving effects. Here, we generalize the calculation of A_Q in Ref. 12 to include the KM phase δ . Using their results for A_i , we find that the quark contribution is rather small,

$$|A_Q(K_L \rightarrow 2\gamma)/A_{\text{exp}}(K_L \rightarrow 2\gamma)| \approx 10\% - 27\%$$

for $m_u = 330 \text{ MeV}$ and 0, respectively. The low-mass intermediate-state contributions are usually handled by the pole-dominance model.

In the pole-dominance model the $K^0 \rightarrow 2\gamma_{-}$ amplitude is dominated by the π^0 , η , and η' pole contributions.^{7,8,12,13} For $\langle \pi^0 | H_W | K^0 \rangle$, we use the soft-pion reduction¹⁵ of the $K \rightarrow 2\pi$ amplitude, and obtain

$$|\langle \pi^0 | H_W | K_L^0 \rangle|^2 = 2.6 \times 10^{-12} \text{ MeV}^2,$$

where $f_{\pi} = 0.95 m_{\pi}$. For η_8 , we use the SU(3) relation $\langle \eta_8 | H_W | K^0 \rangle = (1/\sqrt{3}) \langle \pi^0 | H_W | K^0 \rangle$. For the singlet η_0 , $\langle \eta_0 | H_W | K^0 \rangle$ needs to be calculated. Following Ref. 15 and Donoghue and Holstein¹⁶ we define a complex parameter ρ ,

$$\langle \eta_0 | H_W | K^0 \rangle = -2(\frac{2}{3})^{1/2} \rho \langle \pi^0 | H_W | K^0 \rangle.$$

We then relate the physical states¹⁷ η , η' to η_0 , η_8 by $\eta = \eta_8 \cos\theta - \eta_0 \sin\theta$ and $\eta' = \eta_8 \sin\theta + \eta_0 \cos\theta$. Putting

all these relations together, we obtain

$$A_P(K^0 \rightarrow 2\gamma_-) = \langle \pi^0 | H | K^0 \rangle A(\pi^0 \rightarrow 2\gamma) \left\{ \frac{1}{m_K^2 - m_\pi^2} + \frac{1}{\sqrt{3}} \frac{C_\eta}{m_K^2 - m_\eta^2} \frac{A(\eta \rightarrow 2\gamma)}{A(\pi^0 \rightarrow 2\gamma)} \right. \\ \left. + \frac{1}{\sqrt{3}} \frac{C_{\eta'}}{m_K^2 - m_{\eta'}^2} \frac{A(\eta' \rightarrow 2\gamma)}{A(\pi^0 \rightarrow 2\gamma)} \right\}, \quad (9)$$

where $C_\eta = \cos\theta + 2\sqrt{2}\rho \sin\theta$, $C_{\eta'} = \sin\theta - 2\sqrt{2}\rho \cos\theta$. In our convention the sign of the ratio $A(\eta' \rightarrow 2\gamma)/A(\pi^0 \rightarrow 2\gamma)$ is the same sign as for $A(\eta \rightarrow 2\gamma)/A(\pi^0 \rightarrow 2\gamma)$, in order to satisfy the U -spin relation,¹² $A(\eta \rightarrow 2\gamma)\cos\theta + A(\eta' \rightarrow 2\gamma)\sin\theta = (1/\sqrt{3})A(\pi^0 \rightarrow 2\gamma)$. With use of the experimental values,¹⁷

$$|A(\eta \rightarrow 2\gamma)[A(\pi^0 \rightarrow 2\gamma)]^{-1}| = 0.783 \pm 0.061, \quad |A(\eta' \rightarrow 2\gamma)[A(\pi^0 \rightarrow 2\gamma)]^{-1}| = 1.38 \pm 0.2, \\ A(\pi^0 \rightarrow 2\gamma) = 2.53 \times 10^{-5} \text{ MeV}^{-1},$$

we find that the $K_L \rightarrow 2\gamma$ amplitude is totally accounted for by the π^0 , η , and η' pole contributions provided that¹⁸ $\rho = 0.94$, or $\rho = 0.06$, allowing a small (10^{-3}) imaginary part. By use of the vacuum insertion calculation, it was shown in Refs. 13 and 16 that $\text{Re}\rho = (3f-1)/2$, $\text{Im}\rho = -3(1-f)t_0/2$. The parameter f is the fraction of the $\Delta I = \frac{1}{2}$ transition amplitude that comes from the "penguin" diagram. So for the two solutions of $\rho \approx 1$ and $\rho \approx 0$, $f = 1$ or $\frac{1}{3}$, respectively. When $f = 1$, the "penguin" is the only contribution for the $\Delta I = \frac{1}{2}$ transition, and $\text{Im}\rho = 0$; then $\arg\langle \eta_0 | H_W | K^0 \rangle = \arg\langle \pi^0 | H_W | K^0 \rangle$. When $f \neq 1$, there are other contributions, which are real, other than the "penguin" in the $\Delta I = \frac{1}{2}$ transition; then $\arg\langle \eta_0 | H_W | K^0 \rangle \neq \arg\langle \pi^0 | H_W | K^0 \rangle$.

We shall see later that this is responsible for the difference of η_+ between the KM model and the superweak result. There are indications¹⁹ that the "penguin" is not the sole source of the $\Delta I = \frac{1}{2}$ transition and $f \leq \frac{1}{3}$ is actually consistent with the present bound on ϵ'/ϵ . Next, by virtue of $\arg\langle \pi^0 | H_W | K^0 \rangle = \arg\langle \pi^0 \pi^0 | H_W | K^0 \rangle = t_0$ obtained from the current algebra and the partially conserved axial-vector-current relation we determine from Eq. (9) that $\arg A_P(K^0 \rightarrow 2\gamma_-) = t_0\{1 - 3.4(1-f)\} = t_0'$. Then we write

$$A(K^0 \rightarrow 2\gamma_-) = A_Q + A_P = A(K^0 \rightarrow 2\gamma_-)(1 + it_0') - i(G\alpha/\sqrt{2}\pi)f_K V_{us}^* V_{ud} \text{Im}A_u \\ = A(K^0 \rightarrow 2\gamma_-)(1 + it_0' - i0.3 \text{Im}A_u).$$

Putting this result into Eq. (2) gives $|r_-| = 1 - 0.6t_0' \text{Im}A_u$, and $\theta_- = 2t_0'$. We see here that for $|r_-|$ being from unity, imaginary parts from both weak and strong interactions are needed. The partial-rate difference at $\tau = 0$,

$$\Delta(\tau = 0) = \frac{|r_+|^2 - 1 + |r_-|^2(|r_-|^2 - 1)}{2(1 + |r_-|^2)},$$

only results²⁰ from $|r_i| \neq 1$.

To estimate t_0 , we use the current experimental bounds (with one standard deviation) on ϵ'/ϵ ,

$$t_0 = -\sqrt{2}20e^{-i\pi/4}\epsilon' \approx -7.4 \times 10^{-4}.$$

Using this number and our calculated $R = 2.4$ we find for $\rho = 0.94$, $|r_-| = 1 - t_0$, and $\theta_- = 2t_0$, that $\eta_- = 2.65 \times 10^{-3} e^{i41^\circ}$, which is very close to the superweak result. For $\rho = 0.06$, $|r_-| = 1 + 1.7t_0$, $\theta_- = -2.8t_0$, or equivalently, $|\eta_-| = 3.65 \times 10^{-3}$ and $\phi_- = 75^\circ$, we show its corresponding $\Delta(\tau)$ (the dash-dotted curve) in Fig. 1. They differ from the superweak results by $\sim 0.1\%$. For comparison we also show the corresponding case for $R = 1$. In addition, we also present a case (the dashed curve) for

$|r_-| = 1 + 1.7t_0$, $\theta_- = 4.8t_0$, or, equivalently, $|\eta_-| = 4.43 \times 10^{-3}$, $\phi_- = 78^\circ$, for both $R = 2.4$ and 1. As R increases, the deviation from the superweak results decreases; it becomes unnoticeable at $R = 10$. One important feature is that the KM scheme differs from the superweak scheme mainly during small $\tau \leq 5\tau_S$. Thus if time-integrated observations are made, the integration range ought not to exceed $5\tau_S$.

These results are fundamentally different from those of Ref. 6. From the brief descriptions in their paper, we think that the differences are due to their different quark intermediate-state calculation (none of Refs. 9-13 are cited in their paper) and to their apparent ignorance of the pole contributions. Further, all the points made below, except for point 2, are different from those of Ref. 6, or are absent.

Concluding remarks and experimental outlook.—(1) The ratio R is calculated to be $R = A(K_S \rightarrow 2\gamma)A(K_L \rightarrow 2\gamma) \approx 2.4$, based on the very general observation that $K_S \rightarrow 2\gamma$ is dominated by the 2π intermediate state. The only assumption here is that the amplitude $A(K_S \rightarrow 2\pi)$ used in the loop integration of

$K_S \rightarrow 2\pi \rightarrow 2\gamma$ is constant.

(2) Based upon the same principles as for the calculation of R , one finds that the CP -nonconserving effect in $K_S \rightarrow 2\gamma_+$ is purely from the mass-matrix effect, just as in the superweak model, i.e., $|\eta_+| = 2.3 \times 10^{-3} \approx \eta_{+-}$, and $\phi_+ = 45^\circ \approx \phi_{+-}$.

(3) The method of estimating η_- for $K_L \rightarrow 2\gamma$ is quite complicated and depends upon how the poles π^0 , η_8 , η_0 contribute and the intricate relationship between the pole and the quark contributions. We think that the estimate is at best an order-of-magnitude estimation. In our calculations, we obtain two solutions, depending upon whether the $A(K^0 \rightarrow \eta_0)$ amplitude has the same phase as $A(K^0 \rightarrow (\pi\pi)_{I=0})$ or not. From partial conservation of axial-vector-current and $SU(3)$, $A(K^0 \rightarrow \pi^0)$ and $A(K^0 \rightarrow \eta_8)$ have the same phases as $A(K \rightarrow (2\pi)_{I=0})$. If $\arg A(K \rightarrow \eta_0) = \arg A(K \rightarrow (\pi\pi)_{I=0})$, the CP nonconservation in $K_L \rightarrow 2\gamma_-$ is the same as in the superweak case. If $\arg A(K \rightarrow \eta_0) \neq \arg A(K \rightarrow \pi)$, the CP nonconservation in $K_L \rightarrow 2\gamma_-$ could be quite different from the superweak case by about 0.1% to 0.2%, namely, $|\eta_-| = (3.7 \text{ to } 4.4) \times 10^{-3}$, $\phi_- = 75^\circ \text{ to } 78^\circ$, for our calculated value $R = 2.4$. Such a difference persists for small values of R , but it decreases as R increases. The two different solutions will also have very different implications²¹ for the η_0 contribution to $\text{Im}M_{12}$ of the $K^0 \rightleftharpoons \bar{K}^0$ transition. Therefore, experiments on the CP -nonconservation effects in $K_L \rightarrow 2\gamma_-$ are extremely interesting and will contribute to our understanding, which is now very crude, of the hadronic dynamics in weak decays.

(4) To see such decay-amplitude CP -nonconserving effects, experiments capable of measuring the $(-)$ $K^0 \rightarrow 2\gamma$ difference $\Delta(\tau)$ or $\Delta_I(\tau_1)$ are called for. The current low-energy antiproton ring experiments at CERN²² are ideal for this study. One important point to note in experiments which are capable of measuring only the time-integrated partial decay rates $\Delta_I(\tau_1)$ is that the range of time integration should not be too large, since the decay-amplitude effects are prominent only for the brief interval ($\tau \leq 5\tau_S$) following the moment when K^0, \bar{K}^0 are tagged. This is obvious from Fig. 1.

It is important to note in the figure that the behavior of $\Delta(\tau)$ [even more so for $\Delta_I(\tau_1)$] with such decay-amplitude CP -nonconserving effects for a given value of R (say, 2.4) can be mimicked by a similar $\Delta(\tau)$ without decay-amplitude CP -nonconserving effects, but for a different R (say 1.0). Therefore, it is desirable first to make an accurate measurement of R by measuring $I(\tau)$ and $\bar{I}(\tau)$. Then with such measured R , the decay-amplitude CP -nonconserving effects can be determined by experiments measuring $\Delta(\tau)$ or $\Delta_I(\tau_1)$. Then the time evolution experiments are called for.

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²⁰One can show that $\Delta(\tau=0) = 0$ for $|r_+| = |r_-| = 1$, independent of the values of ϕ_+ , ϕ_- directly from Eq. (3), by observing that $\text{Re}\eta_i = (1 + |\eta_i|^2)\text{Re}\bar{\epsilon} \times (1 + |\bar{\epsilon}|^2)^{-1}$.

²¹Detailed discussions on this point will be given in another publication.

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