SU(2) β Function with and without Dynamical Fermions

U. Heller
CERN, CH-1211 Geneva 23, Switzerland

and

F. Karsch
University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
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We study the β function of SU(2) lattice gauge theory in the presence and absence of dynamical fermions with the (improved) "ratio method." The pseudofermion algorithm used to simulate the fermion action seems to be capable of giving the right contribution to the β function. For two flavors of fermions we find agreement with asymptotic scaling to within $\sim 15\%$ for $\beta \gtrsim 2.0$.

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Recent, more accurate Monte Carlo simulations of pure lattice gauge theories have shown that there are rather large deviations from asymptotic scaling behavior in the range of couplings accessible with today's computing power. This raised the question of whether we are able to see continuum physics in these simulations. The answer requires the knowledge of the β function away from the asymptotic regime where it is dominated by the two leading (universal) terms in its perturbative expansion. During the last year a large effort has been put into the numerical (nonperturbative) determination of the β function for SU(3) gauge theories and has revealed its rather nontrivial behavior which shows an unexpected dip around $\beta(\equiv 6/g^2) \simeq 6.0$.[1-3] Mainly two different methods have been used for this computation.[4] One is based on a combination of the real-space renormalization group (block-spin transformation) with Monte Carlo simulations, called MCRG,[2,3] whereas the other uses the comparison of ratios of Wilson loops which differ by a factor 2 in size, the "ratio method."[1,5]

QCD includes, besides the pure gauge sector, dynamical fermions represented in the path integral by anticommuting (Grassmann) variables. To investigate the scaling behavior of the full theory one needs to be able to compute the β function with dynamical fermions included. The Grassmanian nature of the fermions prevents a straightforward application of the block-spin transformation scheme to compute the β function. This reflects itself in the nonlocal nature of the fermion determinant obtained after one integrates out the fermion fields. The ratio method on the other hand can easily be done with Wilson loops measured in gauge field configurations created with the effect of dynamical fermions taken into account.

The ratio method.—Ratios of Wilson loops of equal perimeter and number of corners (to cancel the perturbative singularities) such as

$$R(i_1,i_2,j_1,j_2)=\frac{W(i_1,i_2)}{W(j_1,j_2)},\quad i_1+i_2=j_1+j_2, \tag{1a}$$

$$R(i_1,i_2,i_3,i_4,j_1,j_2,j_3,j_4)=\frac{W(i_1,i_2)\,W(i_3,i_4)}{W(j_1,j_2)\,W(j_3,j_4)},$$
$$i_1+i_2+i_3+i_4=j_1+j_2+j_3+j_4, \tag{1b}$$

satisfy the homogenous renormalization-group equation

$$R(\{2i_\alpha\};\beta,L)=R(\{i_\alpha\};\beta',L/2), \tag{2}$$

with $\Delta\beta(\beta)=\beta-\beta'$ $(\beta\equiv 2N/g^2)$ the change in the coupling required to compensate for a change of scale by a factor of 2. The computation of the left-hand side and the right-hand side of Eq. (2) on lattices of size L^4 and $(L/2)^4$ minimizes the finite-size effects since the lattice sizes in physical units (femtometers) are the same when the coupling β' is tuned such that Eq. (2) is satisfied. This determines the shift $\Delta\beta(\beta)$. For a SU(N) gauge theory it is related to the β function $[\beta_{\text{funct}}(g)]$ by

$$\ln 2=-\left(\frac{N}{2}\right)^{1/2}\int_{\beta'}^{\beta}\frac{dx}{x^{3/2}\beta_{\text{funct}}((2N/x)^{1/2})}. \tag{3}$$

Asymptotically for large β, when the β function is dominated by the two universal terms, one finds

$$\Delta\beta=(4Nb_0+8N^2b_1/\beta)\ln 2+\sigma(1/\beta^2), \tag{4}$$

with

$$b_0=\frac{1}{16\pi^2}\left(\frac{11}{3}N-\frac{2}{3}n_f\right),$$

$$b_1=\frac{1}{(16\pi^2)^2}\left[\frac{34}{3}N^2-\left(\frac{10}{3}N+\frac{N^2-1}{N}\right)n_f\right], \tag{5}$$

for n_f flavors of massless fermions.

For ratios of Wilson loops of small size Eq. (2) is affected by lattice artefacts which, however, can be canceled order by order in perturbation theory by the use of appropriate linear combinations of ratios.[1,5,6] For example, tree-level improved ratios are linear combinations of two basic ratios, Eq. (1),

$$R_{12} \equiv R_1 + \alpha R_2, \tag{6}$$

with α determined such as to cancel the lattice artefacts in tree-level perturbation theory.[5,6]

In the following we will present the results for $\Delta\beta$ from tree-level improved ratios both for pure SU(2) lattice gauge theory and for SU(2) with two flavors of dynamical fermions, included with the pseudofermion algorithm. Besides providing us with information about deviations from asymptotic scaling, which for pure SU(2) are expected to be less dramatic than for SU(3), the present simulation should help us clarify whether the pseudofermion algorithm is capable of simulating the correct fermion feedback by giving the right β function.

The results are obtained from Monte Carlo data on 8^4 and 4^4 lattices at different values of β. These lattice sizes were chosen such as to make the inclusion of fermions feasible. To increase the number of ratios we used Wilson loops up to size 3×3 on the 4^4 lattice and 6×6 on the 8^4 lattice. This is a valid procedure since the (strong) finite-size effects of these loops are the same on both lattices and cancel in the matching procedure.

Our results for $\Delta\beta(\beta)$ obtained from basic and tree-level improved ratios for the pure SU(2) theory are given in Table I. The tree-level results are shown in Fig. 1. Included in the figure are $\Delta\beta$'s as extracted from string-tension measurements[8,9] and measurements of the deconfinement transition temperature.[10,11] At $\beta = 2.6$ the prediction from a tree-level matching of a 16^4 lattice[12] to an 8^4 lattice is also shown. It agrees within errors with the matching from 8^4 to 4^4, showing that the finite-size effects in the matching procedure cancel. The structure of $\Delta\beta$ as a function of β is very similar to the one found in SU(3), though the dip seen around $\beta \simeq 2.5$ is somewhat less pronounced. This is consistent with the interpretation that the dip is connected to a critical point in the fundamental-adjoint plane which is further away and thus less influential for SU(2) than for SU(3).[13] Our results indicate that the deviations from asymptotic scaling are less than $\sim 20\%$ for $\beta \gtrsim 2.2$. A qualita-

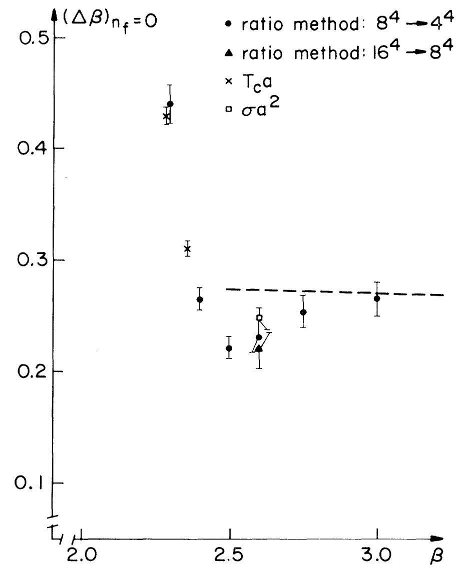

FIG. 1. Collection of the available information on $\Delta\beta$ for the pure SU(2) gauge theory. Shown is $\Delta\beta$ vs β obtained from the ratio method for tree-level improved ratios on a 8^4 lattice (circles) and $16^4 \to 8^4$ lattice (triangle). Also shown are data taken from measurements of the deconfinement temperature (crosses) (from Refs. 10 and 11) and the string tension (square) (from Refs. 8 and 9). The dashed line indicates the expectation based on the asymptotic scaling relation, Eq. (4): $\Delta\beta = 0.2575 + 0.0403/\beta$.

TABLE I. Summary of results obtained for $\Delta\beta$ for the pure SU(2) gauge theory ($n_f = 0$) at various values of β.

β	Basic ratios	Tree-level ratios
2.3[a]	0.489 ±0.053	0.440 ±0.018
2.4[a]	0.273 ±0.030	0.265 ±0.010
2.5	0.251 ±0.36	0.221 ±0.010
2.6	0.270 ±0.055	0.232 ±0.013
2.75	0.285 ±0.058	0.253 ±0.014
3.0	0.333 ±0.082	0.266 ±0.016

[a]Multihit method of Ref. 7 has been used to measure the Wilson-loop expectation values.

TABLE II. Same as Table I but for the SU(2) gauge theory with two flavors of dynamical fermions ($n_f = 2$). As the multihit method of Ref. 7 is not applicable in the presence of dynamical fermions it becomes much harder to measure large Wilson loops at small values of β. At $\beta = 2.1$ only three basic ratios could be measured. For these we give the mean value and in parentheses the smallest and largest values measured.

β	Basic ratios	Tree-level ratios
2.1	0.48 (0.52,0.41)	...
2.2	0.276 ±0.027	0.242 ±0.012
2.3	0.221 ±0.032	0.193 ±0.009
2.4	0.249 ±0.024	0.225 ±0.010

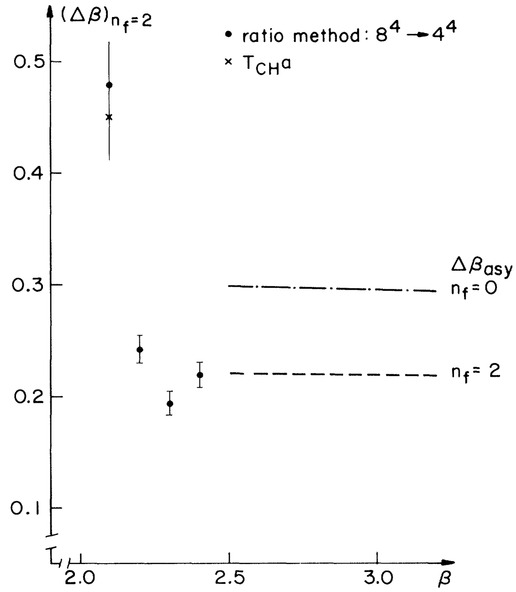

FIG. 2. $\Delta\beta$ vs β for the SU(2) gauge theory with two flavors of dynamical fermions. Shown are the results obtained from the ratio method for tree-level improved ratios (circles) and results from the chiral phase transition (cross) (Ref. 15). The dash-dotted line shows the asymptotic scaling prediction in the absence of dynamical quarks, while the dashed line is the expectation for two flavors: $\Delta\beta = 0.2107 + 0.0258/\beta$. At $\beta = 2.1$ the error bar indicates only the spread of the three basic ratios measured.

tively similar behavior of $\Delta\beta$ has been found by Mackenzie using the block-spin method.[14]

The inclusion of dynamical fermions in numerical simulations requires working at finite (bare) quark mass. Under a scale change by a factor of 2 this mass (in lattice units) changes by a factor of 2 up to logarithmic corrections due to anomalous dimensions. We neglected the logarithmic corrections and worked on a 8^4 lattice with fermions of mass $ma = 0.1$ and on the 4^4 lattice with $ma = 0.2$. Since in weak coupling fermions affect Wilson loops only at the one-loop level (order $1/\beta^2$) the tree-level mixing coefficients are the same as in the pure gauge sector.[6] The results for $\Delta\beta$ from matching of basic and tree-level improved ratios for SU(2) with two fermion flavors are summarized in Table II and the tree-level results are shown in Fig. 2, where we included also one point extracted from the measurement of the chiral-symmetry restoration temperature.[15] For $\beta \gtrsim 2.2$ $\Delta\beta$ agrees within errors with the asymptotic behavior, the deviations being less than $\sim 15\%$. Since $\Delta\beta(\beta)$ reflects the scaling behavior between $\beta - \Delta\beta(\beta)$ and β this implies that physical observables should show (nearly) asymptotic scaling behavior above $\beta \simeq 2.0$.

Comparing $\Delta\beta(\beta)$ with and without dynamical fermions we see that the onset of asymptotic scaling occurs at a smaller value of β (shifted by ~ 0.2) in the former case. The fact that $\Delta\beta$ for two fermion flavors is compatible with the asymptotic prediction indicates that the pseudofermion algorithm is capable of correctly simulating the effects of dynamical fermions. Unfortunately the presence of the dip in the pure gauge sector around $\beta \simeq 2.5$ weakens this conclusion somewhat since it makes the difference in $\Delta\beta$ between the pure gauge sector and the theory with fermions less pronounced in this coupling-constant regime. To strengthen the argument it would therefore be interesting to determine $\Delta\beta$ for larger values of β or in the presence of more flavors of fermions which should increase the difference in $\Delta\beta$.

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[1]A. Hasenfratz, P. Hasenfratz, U. Heller, and F. Karsch, Phys. Lett. **143B**, 193 (1984).

[2]R. Gupta, G. Guralnik, A. Patel, T. Warnock, and C. Zemach, Phys. Rev. Lett. **53**, 1727 (1984).

[3]K. C. Bowler, A. Hasenfratz, P. Hasenfratz, U. Heller, F. Karsch, R. D. Kenway, H. Meyer-Ortmanns, I. Montvay, G. S. Pawley, and D. J. Wallace, University of Edinburgh Report No. 85/225 (to be published).

[4]For a pedagogical introduction to the renormalization-group approach to lattice gauge theories see P. Hasenfratz, CERN Report No. Ref.-Th. 2999/84 (to be published).

[5]A. Hasenfratz, P. Hasenfratz, U. Heller, and F. Karsch, Phys. Lett. **140B**, 76 (1984).

[6]U. Heller and F. Karsch, Nucl. Phys. **B251** [FS13], 254 (1985).

[7]G. Parisi, R. Petronzio, and F. Rapuano, Phys. Lett. **128B**, 418 (1983).

[8]F. Gutbrod and I. Montvay, Phys. Lett. **136B**, 411 (1984).

[9]F. Karsch and C. B. Lang, Phys. Lett. **138B**, 176 (1984).

[10]J. Engels, F. Karsch, I. Montvay, and H. Satz, Nucl. Phys. **B205** [FS5], 545 (1982).

[11]C. Curci and R. Tripiccione, Phys. Lett. **151B**, 145 (1985).

[12]We thank F. Gutbrod and I. Montvay for providing us with unpublished Wilson-loop data of Ref. 8.

[13]Yu. Makeenko and M. I. Polikarpov, Phys. Lett. **135B**, 133 (1984).

[14]P. Mackenzie, in *Gauge Theory on a Lattice: 1984,* edited by C. Zachos *et al.*, Proceedings of the ANL Workshop (Argonne National Laboratory, Argonne, 1984).

[15]U. Heller and F. Karsch, CERN Report No. Ref.-Th. 4078/84 (to be published).