## Fractals with Local Bridges

In a recent Letter[1] Helman, Coniglio, and Tsallis argued that a linear chain (fracton dimension $d_{fr}=1$) arranged in $d$-dimensional space so as to have a fractal dimension $d_f$ (not too much less than $d$) could be converted into a new fractal with $d_{fr}=d_f$ by the incorporation of a sufficiently large number of massless cross-connecting bridges.

If the bridges are required to be *local* (i.e., of length much smaller than the linear size of the fractal), then the argument of Ref. 1 requires some modification, leading to a significant qualification of the above result. This is important, because the restriction to local bridges is often indicated on physical grounds. The qualification arises because local (rather than long-range) bridges are very sensitive to the detailed structure of the chain arrangement, which is not determined solely by the value of $d_f$, but can also depend on many other parameters. I argue here that at least one of these (the lacunarity) is relevant to determining the new value of $d_{fr}$.

Consider, for example, the case of an ideal random chain ($d_f=1$) with conducting self-intersections (bridges), on a lattice in $d$ dimensions. Banavar, Harris, and Koplik[2] studied, by a variety of methods, an exponent $x(d)$ describing the end-to-end resistance $R(N)$ of such a chain (of $N$ steps). They found $R(N)\sim N^{x(d)}$, where $x(2)\cong 0.46$ and $x(3)\cong 0.73$. Straightforward scaling arguments suggest that $x=\tilde{z}/2$, where $\tilde{z}$ is the exponent describing the resistance $R(r)$ between two points on the fractal at spatial separation $r$ (i.e., $R\sim r^{\tilde{z}}$). Then by the Einstein relation,[3] $d_w=d_f+\tilde{z}$, we find $d_{fr}=1.37$ in $d=2$, and $d_{fr}=1.15$ in $d=3$. Thus, even in $d=2$, $d_{fr}\neq d_f$ when the conducting bridges are included. This result,[4] which is admittedly at first surprising, arises because the fractal (if finite) is not space-filling, even though $d_f=d$. Instead there are large "lacunae"[5] or gaps crossed by thin strands or "bottlenecks," which hamper the free diffusion of a particle and prevent attainment of $d_w=2$.

A second example is provided by the self-avoiding walk (SAW). An explicit calculation to order $\epsilon=4-d$, and more general considerations concerning the distribution of contacts within a long SAW,[6,7] indicate that for this case loops are irrelevant in determining $d_{fr}$. If we introduce a large number of bridges of length $l$, the SAW simply becomes a self-avoiding sausage of a width comparable to $l$. Thus for time scales of interest in probing the larger scale chain structure, $d_w=2d_f$, and $d_{fr}=1$. These remarks should apply to SAW's in $d=2$ and $d=3$.

In summary, the inclusion of local bridges in any fractal will lead to a new value of $d_{fr}$ which depends crucially on the internal correlations of the original structure, and not just on its $d_f$. Thus, even if $d_f$ is close to or equal to $d$, $d_{fr}\neq d_f$ unless further special conditions are met. It seems at least possible that a sufficient condition is that the lacunarity[5] be zero; this would eliminate the possibility of bottlenecks of the type discussed above. It is not obvious whether such a condition is actually met in protein systems[1] of interest, for which the restriction to local bridges is presumably appropriate.

I wish to thank Dr. Itzhak Webman for helpful discussions.

M. E. Cates[(a)]
Corporate Research Science Laboratories
Exxon Research and Engineering Company
Annandale, New Jersey 08801

Received 1 October 1984
PACS numbers: 87.15.By, 63.50.+x, 76.30.−v

[(a)]Permanent address: Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom.

[1]J. S. Helman, A. Coniglio, and C. Tsallis, Phys. Rev. Lett. **53**, 1195 (1984).

[2]J. R. Banavar, A. B. Harris, and J. Koplik, Phys. Rev. Lett. **51**, 1115 (1983).

[3]This was originally derived for percolation clusters by Y. Gefen, A. Aharony, and S. Alexander, Phys. Rev. Lett. **50**, 77 (1983), but is general for $d_{fr}\leq 2$ (e.g., M. E. Cates, unpublished).

[4]It should be noted that the result $d_{fr}=1.37$ in two dimensions is disputed by S. Havlin, G. H. Weiss, D. Ben Avraham, and D. Movshovitz, J. Phys. A **17**, L849 (1984). These authors argue that $d_w=d_{fr}=2$; their reasoning is similar to that of Ref. 1. They find, however, that their exact enumeration data do not support this hypothesis. Unfortunately, they mention no test of their data against the alternative hypothesis that $d_{fr}=1.37$.

[5]B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, San Francisco, 1982).

[6]R. C. Ball and M. E. Cates, J. Phys. A **17**, L531 (1984).

[7]J. des Cloizeaux, J. Phys. (Paris) **41**, 223 (1980).

© 1985 The American Physical Society