Nonperturbative Critical Behavior of Random-Field Systems

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The thermal critical properties of random-field systems in the observable, linear, local-response regime are considered. A consistent nonperturbative approach yields an effective reduced dimension $\bar{d} = [d + 1/\nu(\bar{d})]/2$ for the thermal exponents if $\alpha(\bar{d}) \leq 0$. The consequences for Ising systems are particularly reliable at the upper and lower critical dimensions ($d_{\text{u.c.d.}} = 6$ and $d_{\text{l.c.d.}} = 2$, respectively) as well as at $d = 3$ where $\alpha = 0$ (logarithmic divergence). The results are in agreement with measurements on random antiferromagnets of the specific heat (by linear birefringence) and the correlation length (by neutron scattering).

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The drastic effects of random ordering fields on critical behavior have been debated extensively in recent years. Stability analysis of the ordered ground state initiated by Imry and Ma[1] yields $d_{\text{l.c.d.}} = 2$ as the lower critical dimension (below which no long-range order is present) for systems with discrete symmetry[2] (Ising type) and $d_{\text{l.c.d.}} = 4$ for those with continuous symmetry (Heisenberg type). Following earlier works,[1,3] Aharony, Imry, and Ma[4] and also Young[5] performed renormalization-group calculations near the upper critical dimension $d_{\text{u.c.d.}} = 6$ and derived a dimensional reduction by 2 for the critical exponents of all $O(N)$ models.[6] Random-field (RF) effects in $Z(N)$ models,[7] s-states Potts models,[8] as well as other classical[9] and quantum[10] models have been investigated. The classical Ising model,[11] however, has attracted most recent attention. The contradiction between the ground-state considerations,[1,2] pushed one step short of mathematical rigor,[12] and those[4,5] of the renormalization group, supported by a very elegant supersymmetric formulation,[13,14] still lack a satisfactory explanation. Although my conclusions are applicable to almost all of the systems mentioned above, I focus the present study on the Ising model described by the continuous (bare) Hamiltonian

$$H(\phi) = H_0(\phi) + \int d^d r \, h(\mathbf{r}) \phi(\mathbf{r}), \tag{1}$$

where $H_0(\phi)$ is the usual ϕ^4 Hamiltonian[4] of the pure system and $h(\mathbf{r})$ is a local, uncorrelated, random field for which I assume a Gaussian distribution with zero average and with $[h(\mathbf{r}) h(\mathbf{r}')] = \lambda \delta(\mathbf{r} - \mathbf{r}')$ (square brackets denote configurational averages).

A complex and novel behavior is manifested experimentally. Fishman and Aharony[15] suggested the realization of random-staggered fields by the application of a uniform field to a random antiferromagnet. Random-site antiferromagnets do not exhibit long-range order when cooled in the presence of an external field,[16] although the form factor extracted from neutron scattering shows very large domains (> 700 Å). The approximate value of the correlation length exponent is $\nu \approx 1$ in three dimensions.[17] The long-range order reached in zero-field cooling is not destroyed by the application of the field, while the field-cooled domains remain essentially unchanged if the field is suppressed. This demonstrates the infinitely long-time hysteretic behavior of these systems below T_c.[16,18] The birefringence measurements of the specific heat exhibit a sharp, logarithmic ($\alpha \approx 0$) and symmetric, peak in three dimensions[19] and only a broad maximum is observed in two dimensions.[20] Both are drastically different from the respective random-site antiferromagnetic behavior in zero field.

The importance of the dynamics to the potential realization of the long-range order was mentioned recently.[21,22] The dynamics, however, also has a crucial role in the critical region and will affect the static description as well. The present work addresses the short-time critical regime which is experimentally observable. To outline the new structure of the RF system, I define the scale-dependent averaged susceptibility

$$\chi_l = (l + a)^{-d} \sum_{|\mathbf{r} - \mathbf{r}'| \leq l} [\langle \phi(\mathbf{r}) \phi(\mathbf{r}') \rangle - \langle \phi(\mathbf{r}) \rangle \langle \phi(\mathbf{r}') \rangle] \tag{2}$$

(a is a short-distance cutoff, e.g., the lattice spacing). The relevant scaled parameter is $y = l/\xi$, where ξ is the correlation length. The two extreme regimes are the global response (g.r.) regime with $y \to 1$ in which $\chi(y=1) \sim \xi^{\gamma/\nu}$ and the local response (l.r.) regime with $y \to 0$. As shown below $\chi(y=0)$ scales differently which implies a nontrivial dependence on y of the critical response. The parameter y has a similar role to that of $1 - x$, where x is Parisi's parameter[23] (related by Sompolinsky to relaxational dynamics[24]), in the infinite-range spin-glass mean-field theory. The present effect, however, is due to fluctuations and is confined to the linear-response "point" ($x = 1$). After rescaling, this extra variation with y has to be reflected in the internal structure of the coarse-grained variables. I propose to incorporate it into a self-similar replication procedure which is discussed elsewhere.[25] The renormalization-group flows, in the presence of the relevant length scale $y\xi$, may pursue a course of continuous crossover asymptotically attracted to a "fixed-point universality class," depending on the regime. A detailed description of this potential behavior (which accounts for the apparent scale invariance observed experimentally[17,19]) will also be deferred to a future publication.[25]

Here I choose to present a self-consistent approach to compute the effective dimensional reduction[4] from the behavior of the free energy in the l.r. regime. We first recall the argument of Aharony, Imry, and Ma[4] for the dimensional reduction: In the presence of RF the averaged free energy F_ξ in a volume of linear extent ξ increases as ξ^{ρ} with $\rho > 0$. Its density $f_\xi = \xi^{-d} F_\xi$ decays like $\xi^{-\bar{d}}$, with

$$\bar{d} = d - \rho. \tag{3}$$

Hence, $\bar{d}$ is another critical exponent of the system related (from their definitions) to the exponents α (of the specific heat) and ν (of ξ) by the ("hyper")scaling relation

$$\bar{d}\nu = 2 - \alpha. \tag{4}$$

The exponents $\bar{\alpha}$ and $\bar{\nu}$ of the pure $\bar{d}$-dimensional model ($\bar{d}$ model) obey the same relation (4). A full equivalence between the leading thermal singularities (related to local even operators) of this $\bar{d}$ model and the corresponding RF system follows if $\alpha = \bar{\alpha}$ (or equivalently $\nu = \bar{\nu}$). We assume this relation to hold in order to derive self-consistently the relation between d and $\bar{d}$. Our main interest, however, is to find those special dimensionalities at which the RF system assumes particular values of the exponents.

In order to estimate the leading singular behavior of the averaged free energy, we examine the contribution due to the RF (all others are at most as divergent[4]) which may follow a new behavior, different from the bare one[4] $[F_\xi(\lambda) \sim \lambda\xi^2]$, due to the critical response of the random system. Let us consider the change in the free energy $[\delta F_\xi(\lambda)]$ due to virtual, uncorrelated, variations $h(\mathbf{r}) \to h(\mathbf{r}) + \delta h(\mathbf{r})$ in the bare local RF (which leaves the critical behavior unchanged). The contribution due to fluctuations is given by linear response[26]: $a^d \delta\lambda \chi(0)$ [see Eq. (2)], where $\delta\lambda = [\delta h^2(\mathbf{r})]$. The change in the persistent part will be proportional to $\delta\lambda Q$ with $Q = \int d^d r\, q(\mathbf{r})$ and $q(\mathbf{r}) = [\langle\phi(\mathbf{r})\rangle^2]$. In the regime under consideration the scaling behavior of Q and $\chi(0)$ may be derived from that of the corresponding local operators. Perturbatively, if we follow the conventional approach ($d \to d-2$), it can be shown explicitly[27] that both $q(\mathbf{r})$ and $\tilde{q}(\mathbf{r}) = [\langle\phi^2(\mathbf{r})\rangle - \langle\phi(\mathbf{r})\rangle^2]$ scale as the local energy density $\langle\phi^2(\mathbf{r})\rangle$ in the $(d-2)$ model [contrary to their behavior near the pure fixed-point,[15,26] $h(\mathbf{r}) \to 0$, where they scale as the pure magnetization squared]. It is more than plausible to assume that a similar relation will continue to hold as long as dimensional reduction is maintained in the l.r. regime. Accordingly, both $\tilde{q}(\mathbf{r})$ and $q(\mathbf{r})$ follow (up to unimportant constants) a behavior analogous to that of $\langle\phi^2(\mathbf{r})\rangle \sim AL^{-(1-\bar{\alpha})/\bar{\nu}} + BL^{-1/\bar{\nu}}$ in the $\bar{d}$ model (L is an arbitrary length scale). We define

$$d_q = \min[(1-\bar{\alpha})/\bar{\nu},\, 1/\bar{\nu}], \tag{5}$$

and replace the spatial dimension by $\bar{d}$ in relating the critical behavior of the local and macroscopic quantities (this is implied by the parallel interchange $d \to \bar{d}$ in the hyperscaling relations), to find the required leading singularity,

$$\chi(0) \sim Q \sim \xi^{\bar{d} - d_q}, \tag{6}$$

from which follows $\delta F_\xi(\lambda) \sim \delta\lambda\xi^{\bar{d}-d_q}$. Thus, the RF contribution is strongly affected by the new critical behavior. This, in return, will influence the shift in dimension. We set $\rho = \bar{d} - d_q$ in (3) and, with (4) and (5), we find that

$$\bar{d}(d) = \begin{cases} d - 1/\bar{\nu}, & \bar{\alpha} \geqslant 0, \quad \text{(7a)} \\ (d + 1/\bar{\nu})/2, & \bar{\alpha} \leqslant 0, \quad \text{(7b)} \end{cases}$$

The conclusions for these particular cases in which the dimensional reduction is the most reliable are the following: (a) For d (and $\bar{d}$) large enough, $\nu = \bar{\nu} = \frac{1}{2}$ and $\bar{d} = d - 2$ follows. This holds for any $d \geqslant d_{\text{u.c.d.}} = \bar{d}_{\text{u.c.d.}} + 2$. (b) At $d_{\text{l.c.d.}}$, $\nu = \bar{\nu} = \infty$ and $\alpha = \bar{\alpha} = -\infty$ which yields $d_{\text{l.c.d.}} = 2\bar{d}_{\text{l.c.d.}}$, reproducing $d_{\text{l.c.d.}} = 2$ (4) for Ising (Heisenberg) models. (c) The dimensions d_0 and $\bar{d}_0(d_0)$ at which $\bar{\alpha} = \alpha = 0$ and $\bar{d}_0\bar{\nu} = 2$ are related by $2d_0 = 3\bar{d}_0$. This is the case, e.g., for O(N) models at $d_{\text{u.c.d.}} = 6$. For the Ising systems it also holds at $d = 3$ [with $\bar{d}_0(3) = 2$]. In this case we therefore anticipate $\nu = 1$ and a logarithmic divergence of the specific heat, in agreement with experimental measurements[17,19] (site or bond randomness will not modify these conclusions[15]). Another check will be provided by the form of the structure factor measured above T_c.[17] Away from T_c it is well fitted[17] by the expected form

$$S(k) = \frac{C(\xi)}{\xi^{-2} + k^2} + \frac{C'(\xi)}{(\xi^{-2} + k^2)^2}. \tag{8}$$

Some information concerning the magnetic exponents η and γ may be extracted from the behavior of $S(k)$ at $k = 0$. According to the conventional theory, the dependence of the coefficients on ξ is of the form $C(\xi) = c\xi^{-\eta}$ and $C'(\xi) = c'\xi^{-\eta}$. Another theory[28] predicts a different behavior for the second coefficient: $C'(\xi) = c'\xi^{-2\eta}$. A preliminary fitting (first paper of Ref. 17) of the experimental data is not inconsistent with the latter form with the $\bar{d} = 2$ value of $\eta = \frac{1}{4}$.

However, the former form, as well as other possibilities, cannot be ruled out on the basis of the existing data. Clearly, more precise measurements as well as further theoretical analysis, which account for the new effects discussed above, are necessary before conclusions can be drawn.

In their paper Aharony, Imry, and Ma[4] also raised the possibility of a reduction in dimension $d \rightarrow d' = d - 2 + \eta(d')$ from the argument that it is the excess in the total RF (on a length scale of ξ) which should couple to the averaged magnetization to yield the dominant contribution to $F_\xi(\lambda)$. They derived the same result by integrating the momentum-dependent susceptibility $\chi(k)$ over all small momenta with $k \leqslant 1/\xi$. Thus, this reduction (derived recently from an approximate equivalent annealed model[28]) may hold in the g.r. regime. The present result (7), on the other hand, follows from the integration of $\chi(k)$ over all allowed momenta, $k \lesssim 1/a$, and is dominated by the behavior on shorter ($k > 1/\xi$) wavelengths.

To conclude, the RF system is far more complex than its pure counterpart. I have presented a nonperturbative approach which for the first time provides an explanation for the observed critical behavior and which, in certain other respects, supports previous theoretical results. The critical fluctuations may exhibit similar effects in other systems such as short-range spin-glasses and amorphous magnetic materials with random anisotropies.

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