Comment on "Solitons in Superfluid ^{3}He-A: Bound States on Domain Walls"

In a very recent paper,[1] Ho, Fulco, Schrieffer, and Wilczek (HFSW) have studied the effect of solitons on the quasiparticle spectrum in ^{3}He-A. They find that bound states exist with the possibility that some states have zero energy. These results are due to the existence of two nodes for the gap on the Fermi surface. The effect of these nodes has already been considered previously[2-4] because of its importance for the statics and the dynamics of ^{3}He-A, and conclusions similar to those of HFSW have already been reached.[3] Here we want to comment on the physical significance of the HFSW results.

The existence of bound states is not related to the presence of topological solitons in the sample. Bound states are linked to local properties of the sample and not to global ones. They are due to textural inhomogeneities in the anisotropy axis $\hat{\mathbf{l}}$. As such they exist in any sample because the boundary condition of $\hat{\mathbf{l}}$ perpendicular to the surface makes it impossible to have a completely homogeneous sample. It is fairly easy to understand why any bending in the $\hat{\mathbf{l}}$ field produces bound states. Quasiparticles with fixed $\hat{\mathbf{p}}$ travel along straight lines parallel to $\hat{\mathbf{p}}$ in the quasiclassical approximation. A quasiparticle with $\hat{\mathbf{p}} \simeq \hat{\mathbf{l}}(M)$ can have a very low energy since the gap is almost zero for this value of $\hat{\mathbf{p}}$. But such a low-energy quasiparticle cannot go far from point M if the $\hat{\mathbf{l}}$ field lines bend away, because at a nearby point M' we will have $\hat{\mathbf{l}}(M') \neq \hat{\mathbf{p}}$ and the gap at M' will be too high for quasiparticle $\hat{\mathbf{p}}$ to reach M' in the classical approximation. Therefore this quasiparticle is trapped around M which can be anywhere in the sample and this trapping causes bound states.

Bending is the dominant[5] source of bound states and it appears impossible to avoid it in a real sample. Lack of bending, $\mathbf{a} \equiv (\hat{\mathbf{l}} \cdot \nabla)\hat{\mathbf{l}} = 0$, means that the field lines are straight lines which is usually incompatible with boundary conditions. In this respect HFSW's case a is rather peculiar since there is no bending. In case b bound states are due to bending. The analysis of this case can be simplified if we note that, rather than a sharp wall, we have in any actual sample a texture with a typical length scale $1/a$ long compared to the coherence length $\xi = v_F/\Delta$. For a given $\hat{\mathbf{p}}$, the order parameter can be linearized[2] around the zero z_p of its real part. This leads to the following low-energy spectrum[2,3]:

$$E_0 = \Delta \hat{\mathbf{p}} \cdot \hat{\mathbf{a}} \times \hat{\mathbf{l}}, \quad E_n = (E_0^2 + 2n\Delta a v_F)^{1/2}, \quad n \geqslant 1. \qquad (1)$$

For E_0, this result agrees with HFSW's case b. The corresponding spinor wave functions $(\psi_1^n e^{i\mathbf{p}\cdot\mathbf{r}}, \psi_2^n e^{i\mathbf{p}\cdot\mathbf{r}})$ are given by

$$\psi_1^n(\mathbf{r}) = (e^{i\pi/4}/2)[(1+E_0/E_n)^{1/2} f_n(\rho) - i(1-E_0/E_n)^{1/2} f_{n-1}(\rho)] \qquad (2)$$

and $\psi_2^n = (\psi_1^n)^*$, where $f_n(\rho)$ is the normalized eigenstate of the harmonic oscillator and $\rho = (a/\xi)^{1/2}(z - z_p)/\hat{\mathbf{p}}_z$. The low-energy states are localized around z_p in a region of size $(\xi/a)^{1/2}$ large compared to ξ but small with respect to $1/a$ which justifies the linearization of the order parameter. It should be noted that, although $|\mathbf{p}| = p_F$ in the above wave functions, the whole low-energy spectrum is affected by the trapping of the excitations and corresponds to bound states, which are not confined to a two-dimensional manifold in momentum space. The set of quantum numbers $\mathbf{p}$ of the free quasiparticle is replaced by the set $(\hat{\mathbf{p}}, n)$ with the corresponding modification Eq. (1) of the spectrum.

Because the bound states are localized in narrow regions, the above spectrum goes basically unmodified for a general texture. A main consequence[2] is the existence of a nonzero excitation density of states at zero energy $N(0) = N_0 a v_F/2\Delta$ due to the zeroth level. This results in a finite normal density at $T=0$ and a corresponding $\rho_s \neq \rho$. We note that the existence of zero-energy excitations with nonzero density of states can be shown[6] to be independent of the gap linearization. Finally the lack of inversion symmetry and the corresponding quasiparticle current are already present in the $\mathbf{C}\cdot\mathrm{curl}\hat{\mathbf{l}}$ term of the standard gradient expansion. This is quite analogous to the situation created by a superfluid velocity $\mathbf{v}_s$.

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[3]R. Combescot and T. Dombre, in *Quantum Fluids and Solids—1983*, edited by E. D. Adams and G. G. Ihas, AIP Conference Proceedings No. 103 (American Institute of Physics, New York, 1983), p. 261; T. Dombre and R. Combescot, Phys. Rev. B **30**, 3765 (1984).

[4]P. Muzikar and D. Rainer, Phys. Rev. B **27**, 4243 (1983).

[5]But not the only one. The general case is considered in Refs. 2 and 3.

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