Nonlinear Transmission of Zero Sound in Superfluid ^{3}He-A: A Saturation of the Pair-Breaking Attenuation Mechanism

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We have studied the marked transparency to zero sound observed in ^{3}He-A over a broad range of temperature. This nonlinear effect is induced by sound excitation levels small in comparison with the condensation energy. It is attributed to the creation by Cooper pair breaking of a nonequilibrium distribution of quasiparticles localized over a small region of the Fermi surface and relaxing toward equilibrium at a rate typical of normal processes.

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We have observed a very pronounced acoustic transparency effect in the A phase of superfluid ^{3}He over a broad temperature range at zero and medium (11 bars) pressures which sets in as the sound-pulse power is increased. The effect is particularly strong at low pressure[1] and is absent from the previously reported[2] studies of zero-sound propagation in ^{3}He-A at high pressure. Such nonlinear wave-propagation phenomena occur in a host of different physical situations. They may reflect a coherence property of the medium as in the self-induced transparency effect discovered by McCall and Hahn.[3] Another example of nonlinearities is provided by the saturation of two-level systems in optics[4] or in acoustics.[5] Let us also mention the puzzling nonlinear features of sound propagation observed by Polturak *et al.*[6] close to the real-squashing mode in ^{3}He-B. In contrast with this last case, the nonlinear behavior of the A phase occurs over a wide frequency range and involves energies which are very small compared to intrinsic energies of the superfluid (except those associated with textures). We shall ascribe below the power dependence of the attenuation in ^{3}He-A to the creation by phonon irradiation of a sizable overpopulation of excitations localized over a narrow region of the Fermi surface. This localization is a consequence of the anisotropy of the A-phase order parameter. The resulting nonequilibrium state is analogous to those already met in superconductors[7] and provides an insight on the behavior of the superfluid under strong perturbation at high frequencies.

The experimental observations have been carried out in the same setup as used previously to study the transmission of sound in ^{3}He-A in the linear regime.[8] As the shape of the received signal turns out to be power dependent, the commonly used concepts of attenuation and velocity become inadequate. A full account of the nonlinear regime requires a detailed understanding of the distorted signal shape and an accurate knowledge of the excitation power. These factors have been put under control as follows. As the low-loss transmission lines to the ultracold transducers are carefully matched to 50 Ω, the power reaching the quartz crystal is reasonably well known. The impedance mismatch between the line and the crystal depends on parameters which have been measured with use of transmission-line techniques or else inferred from the crystal geometry and its acoustical response function.[9] Both methods agree to about 20% and give a knowledge of the cw power radiated into the liquid to that accuracy.

In a steady state, the energy flux density of a plane wave traveling in the liquid, $\phi = c\mathscr{E}$, where $\mathscr{E}$ is the energy per unit volume, is defined as[10]

$$\phi = (\delta\mu)^2/m_3 c v_3, \tag{1}$$

where m_3 is the ^{3}He bare mass, v_3 the volume per atom, c the sound velocity, and $\delta\mu$ the chemical-potential fluctuation in the wave. The energy of an acoustic pulse traveling in the fluid can now be evaluated if its envelope is known. Its initial shape results in the convolution by the transducer response function of the tone burst generated by the rf transmitter which has a clean rectangular envelope of duration θ (4 μs). The crystal response is governed by an exponential ringing time constant τ_R (12.8 μs at 0 bar, 6 μs at 11 bars). This analysis yields a total energy per unit area in the sound pulse expressed as $E_t = \phi\{\theta - \tau_R[1 - \exp(-\theta/\tau_R)]\}$. In Fig. 1 is shown

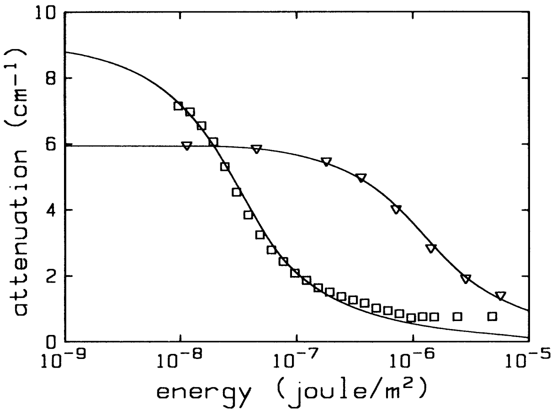

FIG. 1. Apparent attenuation α_a vs total energy per unit area in the sound wave packet at 0 bar, 14.8 MHz, $T/T_c=0.578$ (squares) and 11 bars, 44.7 MHz, $T/T_c=0.407$ (inverted triangles). α_a is defined as $\ln(A_0/A_p)/z$, A_p being the peak amplitude of the signal, A_0 a calibrated reference level, and z the sound flight path (0.4 cm). The solid curves are obtained from theory as explained in the text.

the nonlinear behavior of the apparent attenuation coefficient α_a at fixed temperature as E_t is increased. The onset of the nonlinear regime occurs at very low E_t, 3 to 4 orders of magnitude below the level at which heating effects begin to be felt, i.e., minute with respect to the superfluid condensation energy. This onset energy is much smaller at 0 bar than at 11 bars although the values of $\omega/\Delta_0(T)$ are comparable. We have used sequences of two successive pulses to probe the remnant of the transparent state. Even with time intervals as short as 100 μs, no trace of memory could be detected in the conditions of Fig. 1. This double-pulse sequence places an upper limit of ~ 30 μs on the relaxation time of the excited state and serves as a further check that heating effects were not noticeable in these experiments.

In order to give a framework in which this observed nonlinear behavior can be analyzed quantitatively, we have constructed a semiphenomenological model based on the following considerations. The saturation-curve shape shown in Fig. 1 indicates that damping is dominated by a single mechanism. This mechanism lies most likely in the breaking of Cooper pairs as shown in Ref. 8. The physical process responsible for the appearance of transparency resides in its saturation: The absorption of sound by creation of excitations is balanced by the stimulated emission due to the recombination of excitations into pairs. Textural effects have been ruled out because (1) the nonlinear state relaxes in a time comparable to (or smaller than) the quasiparticle collision time, and (2) the magnitude of the attenuation is approximately accounted for by the pair-breaking mechanism.

Next, we take into account the fact that the incoming phonon energy is small with respect to the maximum value of the gap, i.e., $\omega \ll \Delta_0(T)$. This implies that a small portion of phase space, defined by the condition $\omega^2/4=E_k^2=\xi_k^2+\Delta_k^2$ [$\Delta_k=\Delta_0(T)\sin(\hat{\mathbf{k}}\cdot\hat{\mathbf{l}})$, ξ_k^2 is the kinetic energy counted from the Fermi level], is involved in the pair-breaking process. This fact brings about the two following consequences which greatly simplify the theoretical approach. First, the propagative aspect of the sound-wave perturbation is unimportant because the wavelength is large compared to the coherence length. We shall therefore work at zero wave vector. Secondly, neither the shape of the gap nor its overall amplitude $\Delta_0(T)$ are significantly altered by the sound irradiation, even at fairly high sonic levels. This is true because $\Delta_0(T)$ is linked by the gap equation to the distribution of quasiparticles over the entire Fermi surface, the larger portion of which is unaffected by pair breaking. As for the shape of the gap, its distortion would imply—apart from the unlikely occurrence of high-order spherical harmonics—the excitation of pair-vibration eigenmodes. This requires frequencies higher than those used experimentally. Besides, the observed nonlinear phenomena are clearly nonresonant. We shall therefore take the gap structure and its value as constant.

To summarize, we look for the nonlinear response of the quasiparticle system to a homogeneous effective field $\delta\epsilon(t)$ coupled to number-density fluctuations δn via a term $\delta\epsilon(t)\delta n$, the superfluid parameters being held constant. The external field is assumed to be of the form $\delta\epsilon(t)=A\cos\omega t$, where A is a slowly varying function of t on the scale of $1/\omega$.

We then resort to the standard kinetic equation theory[2] and go over to the canonical Bogoliubov quasiparticle representation: The off-diagonal part of the equation describes pair breaking and, as long as the pulse time scale is large compared to $1/\omega$, it can be solved in the linear approximation. When the result is carried back into the diagonal equation, which describes the time evolution of the quasiparticle distribution f_k, we obtain after time averaging

$$\partial_t f_k=\frac{\pi A^2\Delta_k^2}{2E_k^2}(1-2f_k)\delta(\omega-2E_k)-\Gamma_1(f_k-f_k^0). \tag{2}$$

Equation (2) is nothing but the Fermi's "golden rule" in which transition probabilities for both pair breaking and recombination are taken into account, with a relaxation term toward the Fermi equilibrium distribution f_k^0 involving the time $1/\Gamma_1$. Likewise, we introduce a relaxation time $1/\Gamma_2$ for the off-diagonal terms. We assume that Γ_2/Δ_0 is a small quantity and also that Γ_1 and Γ_2 are independent of ω and k. We can then replace $\pi\delta(\omega - 2E_k)$ by $\Gamma_2/[(\omega - 2E)^2 + \Gamma_2^2]$.

Within this framework, the mean power absorbed per unit volume $\mathscr{P}(t) = \langle \partial t(\delta\epsilon)\delta n \rangle$ is expressed by

$$\mathscr{P}(t) \propto \frac{\omega\Gamma_2 A^2}{4} \sum_{\hat{k}} \left(\frac{\Delta_k}{E_k}\right)^2 \frac{1 - 2f_k}{(\omega - 2E_k)^2 + \Gamma_2^2}. \quad (3)$$

The energy density in the zero-sound wave $\mathscr{E}$ defined by Eq. (1) propagates along z according to the equation

$$\partial_t \mathscr{E} + c\partial_z \mathscr{E} = -\mathscr{P}, \quad (4)$$

in which $\mathscr{E}$ is now the slowly varying envelope of the traveling wave packet. To close this set of equations, we need first to reproduce the known linear regime for which $f_k \sim f_k^0$ and $\mathscr{P} = 2\alpha_0 c\mathscr{E}$, where the linear-regime attenuation coefficient α_0 is obtained directly by extrapolation of the low-power data. We need also to relate the effective field A to the isotropic perturbation $\delta\mu$ caused by the sound wave whose effect on the superfluid is screened by molecular fields. In the low-temperature limit, we expect the usual renormalization of $\delta\mu$ such that $A = \delta\mu/(1 + F_0^s)$. This Fermi-liquid correction is also required to obtain the correct attenuation in the linear regime. As F_0^s increases strongly with pressure, it explains the experimental fact that the attenuation is less easily saturated at high pressure.

Equations (2), (3), and (4) describe the departure from equilibrium of the distribution function f_k and the nonlinear behavior of zero sound at high-power level. Their general features are expected to hold beyond weak-coupling theory. For each k vector, they bear a close resemblance with the population-evolution equations for two-level systems.[4,5] Γ_1 governs the diffusion of quasiparticles and Γ_2 their recombination into pairs. Γ_2 enters the equations as a normalizing factor of A^2 and fixes the energy scale for the onset of nonlinearities. Although analytical solutions of this set of nonlinear integrodifferential equations can be obtained in a few limiting cases, we had to perform a numerical integration to calculate the signal shapes in realistic situations. After adjustment of the outcome of the simulation to the actual signals, we find $1/\Gamma_1 = 15$ and 12 μs and $1/\Gamma_2 = 30$ and 3.3 μs at 0 and 11 bars, respectively, for the conditions of Fig. 1. Γ_1 and Γ_2, determined to about 50% only, have a magnitude characteristic of quasiparticle processes as was to be expected on general grounds. From the magnitude of Γ_2, we infer that only a relatively small portion of phase space is deeply perturbed in the process and that the energy $\mathscr{E}$ required to fully saturate the pair-breaking mechanism at the frequency ω remains small with respect to the superfluid condensation energy. This result agrees with the experimental observations and with the initial assumptions of our model. It confirms the internal consistency of our approach. Without additional information, we can now evaluate α_a as a function of temperature. We use the parameter values determined in the physical situation corresponding to Fig. 1 as a starting point and assume for Γ_1 and Γ_2 a $(T/T_c)^4$ dependence characteristic of quasiparticle relaxation rates in ^{3}He-A.[11] The outcome of this evaluation, shown in Fig. 2 for 11 bars, reproduces the observed variation of α_a with temperature at various power levels in a quite satisfactory manner.

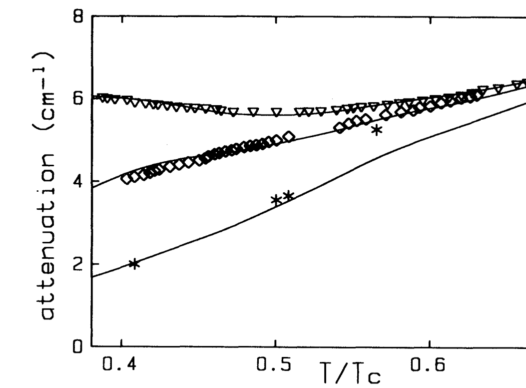

FIG. 2. Measured and calculated apparent attenuation vs temperature at 11 bars for three excitation levels: 44×10^{-3} (inverted triangles), 7.0×10^{-2} (lozenges), and 2.8×10^{-1}, (asterisks) in nJ/cm^2.

This agreement brings additional support to the physical picture that we put forward which lies midway between a two-level-system description[3-5] and the case of nonequilibrium superconductivity[7]: The two-level systems are evoked out of the particle-hole continuum by phonon irradiation and live for a time governed by quasiparticle collisions. At power levels still higher than those reported here, the agreement between the model predictions and experiment breaks down. The received signal is seen to refocus sharply and significant phase shifts occur. The model leaves room for the appearance of coherence phenomena but not for changes in the phase velocity which may be interpreted as a sign of local heating effects. The experimental situation becomes too complex to allow us to draw any definite conclusion.

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