# Pressure Changes and Magnetostriction in Finite Magnetic Fields in $^3$He and other Normal Fermi Liquids

Kevin S. Bedell
*Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794*
(Received 9 October 1984)

Calculations of the change in pressure and magnetostriction which are exact to second order in the polarization are presented. From these effects it is possible to extract a combination of the linear field dependence of the Landau parameters and effective masses from a thermodynamic measurement in the normal phase of a Fermi liquid.

PACS numbers: 67.50.Dg, 67.80.Jd

Recently much effort has been directed towards achieving highly polarized Fermi liquids by means of rather novel techniques.[1-5] These methods are all designed to achieve high polarizations in relatively small magnetic fields. High polarization is needed, for example, to enhance the rather interesting phenomena predicted for those systems. These include, among others, the reduction in the effective mass[6-8] for fully polarized $^3$He and the suppression of superfluidity first predicted by Bedell and Quader.[6,9] To achieve the equivalent polarizations reached for liquid $^3$He in the rapid melting experiments[2,3] would require dc magnetic field of the order of 40 to 100 T. This is larger than the currently available dc magnets can produce.[10]

In the currently accessible field range 10 to 20 T changes in the usual thermodynamic quantities, e.g., $C_v$, the specific heat, $\kappa$, the compressibility, and $\chi$, the susceptibility, of liquid $^3$He, are expected to be small.[1,11] It follows from thermodynamics that changes in these quantities are all quadratic in $\Delta$, where $\Delta$ is the polarization. Since the largest value for $\Delta$ in a 10-T field is only 4.4%, at the melting pressure of $^3$He, the changes in $C_v$, $\chi$, and $\kappa$ will be difficult to observe.[12] Moreover, direct information about the linear field dependence of the Landau parameters and effective masses cannot be obtained from these measurements.[9,11]

In this Letter I present some new and unexpected results for polarized normal Fermi liquids. The results I have derived show, for the first time, that it is possible to extract a combination of the linear field dependence of the Landau parameters from a thermodynamic experiment in the normal phase of a Fermi liquid. In particular I have calculated the change of the pressure and the molar volume (magnetostriction) exactly to order $\Delta^2$. The coefficient of the $\Delta^2$ term in these phenomena has two distinct sources: The first arises from the Landau parameters of the unpolarized system and the linear change in the Fermi momenta, $k_F^\uparrow$, for up spins, and, $k_F^\downarrow$, for down spins. The other term arises from the linear field dependence of the Landau parameters and effective masses.[9,11] By making use of thermodynamic arguments these effects are also shown to be simply related to the density derivative of the spin-fluctuation temperature, $T_{SF}$, for small polarizations.

To obtain expressions for the pressure reduction and magnetostriction I use the generalization, to finite polarizations, of the Landau theory of a normal Fermi liquid.[13] A considerable body of literature exists on this subject.[6,11,14-17] However, much of the results that I will present do not appear in the literature.[18] In what follows I will outline briefly the derivation for the pressure reduction and magnetostriction.

Here I consider a paramagnetic system with a number density $n=N/V$, where $N$ is the number of particles and $V$ is the volume, in a magnetic field $\mathbf{B}$. The change in the energy density, $\epsilon=E/V$, is given by[6,11,13-18]

$$\delta\epsilon=\sum_{p\sigma}\epsilon^0_{p\sigma}\,\delta n_{p\sigma}+\tfrac{1}{2}\sum_{p\sigma,p'\sigma'}\tilde{f}^{\sigma\sigma'}_{pp'}\,\delta n_{p\sigma}\,\delta n_{p'\sigma'}, \tag{1}$$

where $\delta n_{p\sigma}=n_{p\sigma}-n^0_{p\sigma}$ and $n^0_{p\sigma}$ is the equilibrium distribution function in the presence of the field $\mathbf{B}_0$. The quasiparticle energy is defined such that $\epsilon^0_{k_F^\uparrow}-\epsilon^0_{k_F^\downarrow}=2B_0$ and $\epsilon^0_{k_F^\uparrow}+\epsilon^0_{k_F^\downarrow}=2\mu_0$, where $\mu_0$ and $B_0$ are the equilibrium chemical potential and magnetic field, respectively. The Fermi momenta are defined as $k_F^\sigma=k_F(1+\sigma\Delta)^{1/3}$, where $\sigma=+1\ (-1)$ for up spins (down spins). The polarization $\Delta$ is given by $\Delta=\mathscr{M}/n$, where the magnetization density $\mathscr{M}=M/V$ and $M$ is the magnetization. The quasiparticle interaction $\tilde{f}^{\sigma\sigma'}_{pp'}$ has three distinct components, $\uparrow\uparrow$, $\uparrow\downarrow$, and $\downarrow\downarrow\ (\neq\uparrow\uparrow)$, which characterize the longitudinal fluctuations in the system. They reduce to the usual Landau interactions in zero field.[13] (Note that in the above and in what follows I have set $\hbar$, $k_B$, and the magnetic moment of $^3$He equal to 1.)

For a uniform distortion of the Fermi surfaces the change in the distribution functions is given by[18,19]

$$\delta n_{p\sigma}=\delta\epsilon_{k_F^\sigma}\,\delta(\epsilon^0_{k_F^\sigma}-\epsilon^0_{p\sigma})-\tfrac{1}{2}(\delta\epsilon_{k_F^\sigma})^2\delta'(\epsilon^0_{k_F^\sigma}-\epsilon^0_{p\sigma}), \tag{2}$$

where $\delta\epsilon_{k_F^\sigma}=N_\sigma^{-1}(0)\delta n_\sigma$ and $N_\sigma(0)=k_F^\sigma m^*_\sigma/2\pi^2$, with $m^*_\sigma$ the effective mass of spin $\sigma$. Since the second term in Eq. (2) is second order in $\delta n_\sigma$ it will only contribute to the kinetic energy term in Eq (2). Substituting Eq. (2) into Eq. (1), I find[18]

$$\delta\epsilon=\epsilon^0_{k_F^\uparrow}\delta n_\uparrow+\epsilon^0_{k_F^\downarrow}\delta n_\downarrow+\tfrac{1}{2}C^{\uparrow\uparrow}\delta n_\uparrow^2+\tfrac{1}{2}C^{\downarrow\downarrow}\delta n_\downarrow^2+\tilde{f}_0^{\uparrow\downarrow}\delta n_\uparrow\delta n_\downarrow, \tag{3}$$

where $C^{\uparrow\uparrow}=N_\uparrow^{-1}(0)+\tilde{f}_0^{\uparrow\uparrow}$, $C^{\downarrow\downarrow}=N_\downarrow^{-1}(0)+\tilde{f}_0^{\downarrow\downarrow}$, $n_\uparrow=\frac{1}{2}(n+\mathscr{M})$, and $n\downarrow=\frac{1}{2}(n-\mathscr{M})$. The quasiparticle interaction is expanded in the angle between $\mathbf{p}$ and $\mathbf{p}'$, $\tilde{f}^{\sigma\sigma'}_{\mathbf{pp}'}=\sum_l\tilde{f}_l^{\sigma\sigma'}P_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$, and only the $l=0$ moment contributes for uniform distortions. The pressure is given by $P=-(\partial E/\partial V)_{N,M}=-\epsilon+n(\partial\epsilon/\partial n)_{\mathscr{M}}+\mathscr{M}(\partial\epsilon/\partial\mathscr{M})_n$ where $P=P_0+\delta P$, and $P_0$ is the equilibrium pressure. After differentiation of Eq. (3) the change in pressure is given by

$$\delta P=[\tfrac{1}{2}n^\uparrow C^{\uparrow\uparrow}+\tfrac{1}{2}n^\downarrow C^{\downarrow\downarrow}+\tfrac{1}{2}n\tilde{f}_0^{\uparrow\downarrow}]\delta n+[\tfrac{1}{2}n^\uparrow C^{\uparrow\uparrow}-\tfrac{1}{2}n^\downarrow C^{\downarrow\downarrow}-\tfrac{1}{2}\mathscr{M}\tilde{f}_0^{\uparrow\downarrow}]\delta\mathscr{M}. \tag{4}$$

The magnetic field is defined by $B=(\partial E/\partial M)_{N,V}=(\partial\epsilon/\partial\mathscr{M})_n=B_0+\delta B$, where

$$\delta B=\tfrac{1}{4}(C^{\uparrow\uparrow}-C^{\downarrow\downarrow})\delta n+\tfrac{1}{4}(C^{\uparrow\uparrow}+C^{\downarrow\downarrow}-\tilde{f}_0^{\uparrow\downarrow})\delta\mathscr{M}. \tag{5}$$

Consider first the change in pressure at fixed density, i.e., $\delta n=0$. Since the number of particles are fixed this is equivalent to keeping the volume fixed. Since here I am interested in terms that are second order in $\Delta$, only the linear terms in the coefficient of the $\delta\mathscr{M}(=n\,\delta\Delta)$ term in Eq. (4) are needed. As shown by Bedell and Quader[9] the Landau parameters $\tilde{f}_0^{\sigma\sigma'}$ and effective masses $m_\sigma^*$ are to linear order in $\Delta$ given by $\tilde{f}_0^{\sigma\sigma}=f_0^{\uparrow\uparrow}(1-b_0\sigma\Delta)$, $m_\sigma^*=m^*(1-a\sigma\Delta)$, and $\tilde{f}_0^{\uparrow\downarrow}=f_0^{\uparrow\downarrow}$, with $f_0^{\sigma\sigma'}$ and $m^*$ the zero-field Landau parameters and effective mass. Substituting these into Eq. (4) and integrating over $\Delta$, I find

$$P(\Delta)-P(0)=\tfrac{1}{3}n\epsilon_F\Gamma\Delta^2, \tag{6}$$

where $\Gamma=[1+F_0^s+a-\frac{1}{3}-\frac{1}{2}b_0F_0^{\uparrow\uparrow}]$ and $P(0)$ is the pressure in zero field. Here $F_0^s$ and $F_0^{\uparrow\uparrow}$ and $\epsilon_F=k_F^2/2m^*$ are the Landau parameters and Fermi energy, respectively, in zero field. The polarization is given by the low-field result

$$\Delta=(\mathscr{M}/n)=\tfrac{3}{2}(B/T_{SF}), \tag{7}$$

where $T_{SF}=(1+F_0^a)T_F$. Higher-order corrections to $\Delta$ can be ignored since they would lead to higher-order corrections to Eq. (6).

Magnetostriction can be defined as the amount by which the density must change (for fixed $N$ this corresponds to a change in $V$) with polarization such that the pressure remains constant, i.e., $\delta P=0$. From Eq. (4) with $\delta P=0$, I find, after integrating, that

$$\frac{n(\Delta)}{n}\simeq1-\frac{\Gamma}{1+F_0^s}\frac{\Delta^2}{2}, \tag{8}$$

where $F_0^s$ and $n$ are the zero-field Landau parameter and density, respectively. This definition of magnetostriction is equivalent to the one employed by Castaing and Nozieres,[1] who use the Maxwell relation

$$\left(\frac{\partial v}{\partial B}\right)_P=-\left(\frac{\partial\Delta}{\partial P}\right)_B=-\frac{1}{n}\left(\frac{\partial\mathscr{M}}{\partial P}\right)_B+\frac{\mathscr{M}}{n^2}\left(\frac{\partial n}{\partial P}\right)_B, \tag{9}$$

where $v=1/n$ is the molar volume. The two partial derivatives in Eq. (9), $(\partial\mathscr{M}/\partial P)_B$ and $(\partial n/\partial P)_B$, can be determined from Eqs. (4) and (5). Keeping terms to leading order in $\Delta$ will after integration give Eq. (8). The size of this effect, e.g., in $^3$He, is not as large as the pressure reduction. The simple reason is that here we are trying to change the density, whereas, the pressure reduction is accomplished at fixed density. To change the density we must overcome the large incompressibility of the liquid, and thus the screening factor $1+F_0^s$ appears in the denominator.

The result that I find for magnetostriction is related to that found by Castaing and Nozieres.[1] The result found by Castaing and Nozieres[1] can be written as

$$\frac{n(\Delta)}{n}=1-\frac{n}{(1+F_0^s)\epsilon_F}\left(\frac{\partial T_{SF}}{\partial n}\right)_B\frac{\Delta^2}{2}. \tag{10}$$

This they obtained[1] by using Eq. (7) to evaluate the partial derivative $(\partial\Delta/\partial P)_B$. This simple relationship, Eq. (7), between $\Delta$ and $B$ comes from integration of Eq. (5) at constant density. However, what I used was the integration of Eq. (5) at constant field. To see this I use Eq. (4) to evaluate $(\partial\mathscr{M}/\partial P)_B$, where

$$\left(\frac{\partial\mathscr{M}}{\partial P}\right)_B\simeq\frac{3/2}{(1+F_0^s)T_F}\left(\frac{\partial\mathscr{M}}{\partial n}\right)_B \tag{11}$$

for small $\mathscr{M}$ $(=n\Delta)$. The partial derivative $(\partial\mathscr{M}/\partial n)_B$ is then obtained from Eq. (5) at constant $B$,

$$\left(\frac{\partial\mathscr{M}}{\partial n}\right)_B\simeq\frac{(a-\frac{1}{3}-\frac{1}{2}b_0F_0^{\uparrow\uparrow})}{1+F_0^a}\frac{\mathscr{M}}{n}. \tag{12}$$

Substituting Eqs. (11) and (12) along with $(\partial n/\partial P)_B$ into Eq. (9) yields, after integration, Eq. (8). That these two expressions for magnetostriction, Eqs. (8) and (10), are equivalent follows from the fact that $\delta B$, Eq. (5), is a total differential. From this it follows that $(\partial^2B/\partial n\,\partial\mathscr{M})=(\partial^2B/\partial\mathscr{M}\,\partial n)$ and for small polarizations this leads to the relation $\Gamma=(n/\epsilon_F)(\partial T_{SF}/\partial n)$.

The quantity $(n/\epsilon_F)(\partial T_{SF}/\partial n)$ can be obtained from experiment by use of the values of the Landau parameters obtained from Greywall's effective masses.[20] From this I can obtain the combination $a-\frac{1}{2}b_0F_0^{\uparrow\uparrow}$ which ranges from $-0.43$ at saturated vapor pressure (SVP) to $-0.52$ at the melting pressure where $n=2.38\times10^{-2}$ Å$^{-3}$. A direct measurement of $\Gamma$ can be obtained by placing the Fermi liquid in a rigid cell and measuring the pressure change when the magnetic field is applied. In $^3$He at the melting pressure this would result in a decrease of 1.5 mbar in a 10-T field. Although this would be an interesting and easy experiment to perform it will not, for small magnetic fields, give us any more information than we would get from the magnetostriction effect. This can then be used as a check on the various microscopic theories of polarized Fermi liquids.

In the theory recently proposed by Vollhardt[21] this combination, $a-\frac{1}{2}b_0F_0^{\uparrow\uparrow}$, would be identically 0 since there are no linear field terms in the theory. The model of Bedell and Quader[9] also does rather poorly on this combination. For example, at SVP their parameters[9] give $-1.3$ and at melting this would be $-13.9$. This at first appears rather surprising since the theory of Bedell and Quader[9] provides an excellent account of the density and field dependence of the linear field splitting in the superfluid phase of $^3$He.[22,23] It is important to determine why this theory works[9] well for the linear field splitting but rather poorly on the combination $a-\frac{1}{2}b_0F_0^{\uparrow\uparrow}$. This can be understood by obtaining estimates of $b_0$ and $a$.

Separate estimates of $b_0$ and $a$ can be obtained by making use of the forward-scattering sum rules[11,16] for scattering between $\uparrow\uparrow$ and $\downarrow\downarrow$ particles, keeping only the $l=0$ and 1 moments of the interactions, and Eq. (12). At SVP this gives $b_0\simeq0.17$ and $a\simeq0.3$ and at the melting pressure $b_0\simeq0.02$ and $a\simeq0.5$. The values of $a$ obtained this way are close to those of Bedell and Quader[9]; however, $b_0$ is off by an order of magnitude at high pressure. This sum-rule argument provides only a rough check on the parameters but from this it is clear that $b_0$ is overestimated by Bedell and Quader.[9] However, because of the sum-rule constraints $b_0$ does not couple very strongly into the linear field splitting nor into the calculation of the coefficient $a$. Thus, improvements in the calculation of Bedell and Quader,[9] such as including the momentum dependence of the quasiparticle interaction, will have a large effect on $b_0$ but only a small effect on $a$.

In this Letter I have shown that it is possible to obtain direct information about the linear field dependence of a particular combination of the Landau parameters and effective masses from a thermodynamic measurement in the normal phase of a Fermi liquid. This applies rather generally to any normal Fermi liquid. In particular in $^3$He it has been shown to provide an additional check on the models for the polarized phase of $^3$He.

I would like to thank the Kammerlingh Onnes Laboratorium for their hospitality during my stay where some of this work was done. I also want to thank C. N. Archie, G. Frossati, and K. F. Quader for many valuable discussions. This work was supported in part by the U. S. Department of Energy under Contract No. DE-AC02-76ER13001.

[1]B. Castaing and P. Nozières, J. Phys. (Paris) **40**, 257 (1979).

[2]G. Schumacher, D. Thoulouze, B. Castaing, Y. Chabre, P. Segransan, and J. Joffin, J. Phys. (Paris), Lett. **40**, 143 (1979); M. Chapelier, G. Frossati, and F. B. Rasmussen, Phys. Rev. Lett. **42**, 904 (1979).

[3]G. Bonfait, L. Puech, A. S. Greenberg, G. Eska, B. Castaing, and D. Thoulouze, Phys. Rev. Lett. **53**, 1092 (1984).

[4]M. Himbert, V. Lefevre-Seguin, P. J. Nacher, J. Dupont-Roc, M. Leduc, and F. Laloë, J. Phys. (Paris), Lett. **44**, 523 (1983); C. Lhuillier, in *Proceedings of the Third International Conference on Recent Progress in Many Body Theories,* edited by H. Kümmel and M. L. Ristig, Lecture Notes in Physics, Vol. 198 (Springer, New York, 1984).

[5]M. Chapellier, J. Phys. (Paris), Lett. **43**, 609 (1982); S. A. Langer, K. De Conde, and D. L. Stein, J. Low Temp. Phys. **57**, 249 (1984).

[6]K. S. Bedell and K. F. Quader, Phys. Lett. **96A**, 91 (1983).

[7]H. R. Glyde and S. I. Hernadi, Phys. Rev. B **29**, 3873 (1984).

[8]E. Krotscheck, J. W. Clark, and A. D. Jackson, Phys. Rev. B **28**, 5088 (1983).

[9]K. S. Bedell and K. F. Quader, Phys. Rev. B **30**, 2894 (1984).

[10]L. G. Rubin and P. A. Wolff, Phys. Today **37**, No. 8, 24 (1984).

[11]K. B. Bedell, in *Proceedings of the Third International Conference on Recent Progress in Many Body Theories,* edited by H. Kümmel and M. L. Ristig, Lecture Notes in Physics, Vol. 198 (Springer, New York, 1984).

[12]B. Sen and C. N. Archie, Phys. Rev. B **29**, 1490 (1984). No measurable change in $C_v$ was observed up to 8 T in this experiment.

[13]L. D. Landau, Zh. Eskp. Teor. Fiz. **30**, 1058 (1956) [Sov. Phys. JETP **3**, 920 (1957)].

[14]A. A. Abrikosov and I. E. Dzyaloshinski, Zh. Eksp. Teor. Fiz. **35**, 771 (1958) [Sov. Phys. JETP **8**, 535 (1959)]; P. S. Kondratenko, Zh. Eksp. Teor. Fiz. **47**, 1536 (1964) [Sov. Phys. JETP **20**, 1032 (1965)], and **46**, 1438 (1964) [**19**, 972 (1964)]; J. Czerwonko, Acta Phys. Pol. **36**, 763 (1969).

[15]E. P. Bashkin and A. E. Meyerovich, Usp. Fiz. Nauk **130**, 279 (1980) [Sov. Phys. Usp. **23**, 156 (1980)], and references therein; A. E. Meyerovich, J. Low Temp. Phys. **53**, 485 (1983).

[16]P. C. E. Stamp, Ph.D. thesis, Sussex, England, 1983 (unpublished).

[17]K. F. Quader and K. S. Bedell, J. Low Temp. Phys., to be published.

[18]K. S. Bedell and B. Friman, unpublished.
[19]G. Baym and C. J. Pethick, in *The Physics of Liquid and Solid Helium,* edited by K. H. Bennemann and J. B. Ketterson, (Wiley, New York, 1978), Vol. II.
[20]D. S. Greywall, Phys. Rev. B **27**, 2747 (1983).
[21]D. Vollhardt, Rev. Mod. Phys. **56**, 99 (1984).
[22]W. J. Gully, D. D. Osheroff, D. T. Lawson, R. C. Richardson, and D. M. Lee, Phys. Rev. A **8**, 1633 (1973); W. P. Halperin, C. N. Archie, F. B. Rasmussen, T. A. Alvesalo, and R. C. Richardson, Phys. Rev. B **13**, 2124 (1976); L. R. Corruccini and D. D. Osheroff, Phys. Rev. Lett. **45**, 2029 (1980); R. Ruel and H. Kojima, Phys. Rev. B **28**, 6582 (1983).
[23]U. E. Israelson, B. C. Crooker, H. M. Bozler, and C. M. Gould, Phys. Rev. Lett. **53**, 1943 (1984); D. C. Sagan, P. G. N. De Vegvar, E. Polturak, L. Friedman, S.-S. Yan, E. L. Ziercher, and D. M. Lee, Phys. Rev. Lett. **53**, 1939 (1984).