Electron Acceleration in High-Frequency Longitudinal Waves, Doppler-Shifted Ponderomotive Forces, and Landau Damping

Joseph Kupersztych
Service de Physique des Atomes et des Surfaces, Centre d'Etudes Nucléaires de Saclay, 91191 Gif-sur-Yvette Cedex, France
(Received 23 March 1984)

An electron in the field of a high-frequency longitudinal wave experiences both Doppler-shifted intensity-gradient and phase-gradient ponderomotive forces. It is shown that Landau damping can be considered as a phase-gradient force effect. By taking the wave amplitude space dependence into account, we demonstrate another collisionless effect due to the intensity-gradient force. Either damping or amplification of the wave can occur depending on whether the electron density increases or decreases.

PACS numbers: 52.35.Fp, 03.20.+i, 32.90.+a, 41.70.+t

As is well known, an electron in the field of a high-frequency (HF) transverse or longitudinal electromagnetic wave shows a low-frequency (LF) behavior resulting from the action of the so-called "ponderomotive forces." These LF forces play a fundamental role in the laser interaction with free electrons,[1] atoms,[2] and plasmas.[3] The basic purpose of this paper is to show that in addition to the usual ponderomotive intensity-gradient force $\nabla \mathscr{E}^2$, another ponderomotive force exists, involving a slow time variation of the wave amplitude together with the Doppler shift, which plays an important part inasmuch as it is responsible for the well-known Landau damping[4] of longitudinal waves. Another aim of this paper is to show that a novel collisionless effect involving the spatial variation of the wave amplitude can result in either damping or amplification of the wave when Landau damping is negligible.

We consider a longitudinal wave of the form[5]

$$E(x,t) = e^{\gamma t}\mathscr{E}(x)\cos\omega\tau, \tag{1}$$

where $\mathscr{E}(x)$ is a space-dependent amplitude with a slowly varying time dependence $e^{\gamma t}$ ($\gamma << \omega$), physically describing an adiabatic switching on or turning off of the interaction. $\tau = t - \phi(x)/\omega$, where $\phi(x)$ is the phase.

A convenient way to solve the nonlinear equation of motion for a test electron,

$$m\, d^2x/dt^2 = -eE(x,t), \tag{2}$$

is to introduce a double-time scale procedure. The position x and velocity v are split into a HF part (x_h, v_h) corresponding to the linearized motion and a LF part (x_l, v_l) describing the nonlinear behavior. The latter is defined by $x_l = \langle x\rangle_{\rm av}$ and $v_l = \langle v\rangle_{\rm av}$. The linearized electron response is then described by the HF equation:

$$dv_h/dt = -(e/m)E(x_l,t). \tag{3}$$

If the field amplitude is such that the condition

$$|\nabla \mathscr{E}/\mathscr{E}| << |\omega/v_l| \tag{4}$$

is satisfied, the solution of Eq. (3) may be written

$$v_h = -[e\mathscr{E}(x_l)/m\omega'\alpha']e^{\gamma t}(\sin\omega\tau + \beta'\cos\omega\tau), \tag{5}$$

$$x_h = [e\mathscr{E}(x_l)/m\omega'^2\alpha'^2]e^{\gamma t}(\eta'\cos\omega\tau - 2\beta'\sin\omega\tau), \tag{6}$$

where $\omega' = \omega - k(x)v_l$ is the Doppler-shifted frequency "seen" by the electron in its instantaneous rest frame, $k(x) = \nabla\phi$, $\beta' = \gamma/\omega'$, $\alpha' = 1+\beta'^2$, and $\eta' = 1 - \beta'^2$. Under the assumption that $x_h\nabla << 1$ (or $v_h\nabla << \omega$), the field $E(x,t)$ may be expanded in Taylor series about x_l as $E(x,t) = E(x_l,t) + [(x_h\nabla)E]_{x_l}$. The LF motion equation can then be written in the form

$$m\, dv_l/dt = \mathscr{F}, \tag{7}$$

where $\mathscr{F} = -e\langle[(x_h\nabla)E]_{x_l}\rangle_{\rm av}$. Using Eqs. (1) and (6) we obtain

$$\mathscr{F} = -(e/4m\omega'^2\alpha'^2)e^{2\gamma t}[\eta'\nabla\mathscr{E}^2 - 4\beta'\mathscr{E}^2\nabla\phi]. \tag{8}$$

The first term in square brackets of Eq. (8) is a generalization of the well-known intensity-gradient force, taking into account the Doppler shift and γ^2/ω^2 terms (the usefulness of which will become apparent later on). The second term is connected to LF phase-gradient effects of collisionless origin. To our knowledge, the collisionless Doppler-shifted LF force has not been discussed in the available literature.[6] However, we wish to show that it plays a basic role for electron acceleration in longitudinal waves.

For this purpose, it is useful to consider the following three cases: (I) $|\omega'| >> |\gamma|$; (II) $|\omega'| << |\gamma|$, and (III) $|\omega'| \sim |\gamma|$.

If R denotes the ratio between the intensity-gradient force (IGF) and the phase-gradient force (PGF) then for a given value of $L^{-1} = |\nabla\mathscr{E}^2/\mathscr{E}^2|$, $R = |\omega'|/4|\gamma|kL >> 1$ in case (I) and $R = |\gamma|/4|\omega'|kL >> 1$ in case (II), that is, the IGF is much more effective than the PGF in both cases. A peculiar situation occurs in case (III) where $R = |\omega'^2 - \gamma^2|/4\gamma^2 kL$. Clearly, if the electron velocity v_l becomes (locally) of a value such that $|\omega - k(x)v_l|$ is of the same order of magnitude as $|\gamma|$, the PGF will be the predominant force. Hence, through the Doppler shift, electrons of a plasma interacting with a longitudinal wave of the form (1) experience both LF forces, in a proportion which varies with the electron speed. The discussion which follows is intended to show that the physical meaning of the PGF is intimately connected with the well-known collisionless damping effect of longitudinal waves (of space-independent amplitude) discovered by Landau,[4] while the IGF is responsible for a novel noncollisional effect.

For this purpose, we compute the mean work per unit time $\partial W/\partial t$ done by the electrostatic force $F = -eE(x,t)$ on a test electron with velocity v:

$$\partial W/\partial t = \langle Fv\rangle_{\rm av} = \langle F_h v_h\rangle_{\rm av} + \mathscr{F} v_l. \qquad (9)$$

Substituting Eqs. (1), (5), and (8) into Eq. (9), we easily obtain

$$\partial W/\partial t = (e^2/4m)e^{2\gamma t}(2p\mathscr{E}^2 - q v_l \nabla \mathscr{E}^2), \qquad (10)$$

where $p = \gamma(\omega^2 - k^2 v_l^2 + \gamma^2)(\omega'^2 + \gamma^2)^{-2}$ and $q = (\omega'^2 - \gamma^2)(\omega'^2+\gamma^2)^{-2}$.

The total mean work per unit time done by the field on the electron population is then

$$\partial \overline{W}/\partial t = \int_{-\infty}^{+\infty} (\partial W/\partial t) f(x, v_l)\, dv_l,$$

where $f(x, v_l)$ denotes the electron distribution function which we assume to be unperturbed by the field in first approximation. It is worth noting that

$$p = (\partial/\partial v_l)[\gamma v_l (\omega'^2 - \gamma^2)^{-1}]$$
$$= (\pi/k)(\partial/\partial v_l)[v_l \delta(v_l - \omega/k)]$$

and

$$q = -(\partial/\partial v_l)[(v_l - \omega/k)(\omega'^2 - \gamma^2)^{-1}]$$
$$= k^{-1}(\partial/\partial v_l)\mathscr{P}(1/\omega'),$$

where $\mathscr{P}$ denotes the Cauchy principal value. It follows that

$$\partial \overline{W}/\partial t = \partial \overline{W}_{\rm L}/\partial t + \partial \overline{W}_\nabla/\partial t, \qquad (11)$$

where

$$\frac{\partial \overline{W}_{\rm L}}{\partial t} = -\pi\left(\frac{\omega}{2mk^2}\right) e^2 \mathscr{E}^2 e^{2\gamma t}\left(\frac{\partial f}{\partial v_l}\right)_{\omega/k} \qquad (12)$$

is the usual Landau damping term[4] (for a space-independent wave amplitude), and where

$$\frac{\partial \overline{W}_\nabla}{\partial t} = -\left(\frac{e^2}{4mk^2}\right) e^{2\gamma t}\nabla \mathscr{E}^2 \mathscr{T}(v_\phi) \qquad (13)$$

with

$$\mathscr{T}(v_\phi) = \mathscr{P}\int_{-\infty}^{+\infty} \frac{\partial(f v_l)}{\partial v_l}(v_l - v_\phi)^{-1}\, dv_l, \qquad (14)$$

and $v_\phi(x) = \omega/k(x)$ the damping term corresponding to a novel effect: the acceleration (deceleration) of nonresonant electrons by the Doppler-shifted IGF resulting from field inhomogeneity, contributing an extra damping (amplification) to the wave.

Before going into details on this effect, let us derive the total energy transfer rate. The energy density of the wave is

$$S = \left(\frac{\langle E^2\rangle_{\rm av}}{8\pi}\right)\left(\frac{\partial}{\partial\omega}\right)\left[\omega\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)\right]_{\omega=\omega_{pe}},$$

where $\omega_{pe}^2 = 4\pi n_e e^2/m$ and where $n_e(x) = \int_{-\infty}^{+\infty} f\, dv$ is the electron density. It is related to $\partial\overline{W}/\partial t$ through the energy conservation equation $\partial\overline{W}/\partial t = -\partial S/\partial t$. Using Eqs. (1) and (11) we immediately find

$$\gamma = \gamma_{\rm L} + \gamma_\nabla,$$

where

$$\gamma_{\rm L} = (2\pi^2 e^2 \omega/mk^2)(\partial f/\partial v_l)_{\omega/k}, \qquad (15)$$

which is recognized as being the well-known Landau damping rate,[4] and

$$\gamma_\nabla = (2\pi^2 e^2/mk^2)(\nabla\mathscr{E}^2/\mathscr{E}^2)\mathscr{T}(v_\phi), \qquad (16)$$

the damping rate corresponding to the above-mentioned new effect.

The present derivation of Landau damping considered as an LF collisionless PGF effect has shed some more light onto this basic effect. As $\beta'(\omega') = -\beta'(-\omega')$, Eq. (8) shows that those electrons whose velocity v_l is greater than the local phase velocity v_ϕ of the wave are accelerated by the PGF and, conversely, decelerated if $v_l < v_\phi$. The classical term $(\partial f/\partial v_l)_{\omega/k}$ expresses the corresponding balance between the number of accelerated and decelerated particles. The fact that Landau damping is a resonant effect at $v_l = v_\phi$ is obvious from the singularity $\beta'(\omega'=0)$.

The above-mentioned effect resulting from field inhomogeneity has quite a simple physical representation. The IGF is "odd" with respect to the relevant quantity $\omega'^2 - \gamma^2$. This means that, if for example $\nabla\mathscr{E}^2 > 0$, those electrons whose speed is such that $|\omega'| > |\gamma|$ are accelerated by the IGF, whereas they are decelerated when $|\omega'| < |\gamma|$. Obviously, if $\nabla\mathscr{E}^2 < 0$ the opposite is true. The resulting momentum balance is expressed by the term $\partial(fv)/\partial v$ in $\mathscr{T}(v_\phi)$. As there is no singularity at $|\omega'| = |\gamma|$ in the IGF, we are dealing here with a nonresonant effect, a fact suggested by the principal value in $\mathscr{T}$.

At a first glance, the additional effect appears as a correction to Landau damping, due to field inhomogeneity. In fact, we are dealing with another noncollisional effect independent of Landau's effect. In order to emphasize its physical importance, we assume the unperturbed electron velocity distribution to be a shifted Maxwellian: $f_M(x,v_l) = n_e(x) a \exp[-b(v_l - u)^2]$, with $a = (b/\pi)^{1/2}$ and $b = 1/2v_t^2$, where v_t denotes the thermal velocity; u denotes any drift velocity.

In this case we easily obtain from Eq. (14)

$$\mathscr{T}_M(\sigma,\rho) = (n_e/\sqrt{2}v_t)[(1-2\sigma^2-2\sigma\rho)X - 2(\sigma+\rho)],$$

where $\sigma = (v_\phi - u)/\sqrt{2}v_t$, $\rho = u/\sqrt{2}v_t$, and where $X(\sigma)$ denotes the real part of the Hilbert transform of the Gaussian[7] of real argument. Equations (15) and (16) become

$$\gamma_L = -\sqrt{\pi}(\omega_p^2/\omega)(\sigma+\rho)^2\sigma\exp(-\sigma^2), \quad \gamma_\nabla = \epsilon(\omega_p^2/4\omega)(\sigma+\rho)[(1-2\sigma^2-2\sigma\rho)X - 2(\sigma+\rho)],$$

where ϵ is an algebraic parameter defined by $\nabla\mathscr{E}^2/\mathscr{E}^2 = \epsilon k$. $\epsilon \ll 1$ as a result of (4).

An easy numerical computation shows that the optimum value of γ_∇ arises when $\sigma \sim 0$, that is, when $\omega \sim ku$ (top of the shifted distribution function) and $v_t \ll u$ (such a physical situation is exemplified by an electron beam). In this case the Landau damping rate γ_L is negligibly small in comparison with γ_∇ which takes the simple value

$$\gamma_\nabla(0) = -\epsilon u/4k\lambda_D^2,$$

where $\lambda_D = v_t/\omega_{pe}$ is the Debye length. It is to be noted that, in contrast with γ_L, γ_∇ can represent either damping or amplification depending on whether ϵ is positive or negative. Furthermore, since in the present case the LF electron current density is approximately $j_e = n_e u$, Maxwell's equation $\partial E/\partial t = -4\pi j$ yields $\nabla\mathscr{E}^2 \sim \nabla n$. Therefore, damping or amplification of longitudinal waves occurs respectively in those regions where the electron density increases or decreases.

To conclude, we have shown that Landau damping can be considered as a ponderomotive phase-gradient effect and that a further noncollisional (damping or amplification) effect due to field inhomogeneity exists. This last effect can play an important role in physical situations where Landau damping is ineffective.

Stimulating discussions with Dr. C. Cohen-Tannoudji, Dr. G. Mainfray, Dr. C. Manus, and Dr. P. Monchicourt are gratefully acknowledged.

[1]T. W. B. Kibble, Phys. Rev. Lett. **16**, 1054 (1966), and Phys. Rev. **150**, 1060 (1966). A classic review paper on the subject is J. H. Eberly, in *Progress in Optics,* edited by E. Wolf (North-Holland, Amsterdam, 1968), Vol. 7.

[2]E. A. Martin and L. Mandel, Appl. Opt. **15**, 2378 (1976); F. Fabre, G. Petite, P. Agostini, and M. Clement, J. Phys. B **15**, 1353 (1982); V. S. Letokhov and V. G. Minogin, Phys. Rep. **73**, 1 (1981); C. Cohen-Tannoudji, Cours au Collège de France, 1983–1984 (unpublished).

[3]L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, New York, 1960); A. Bers, in *Plasma Physics,* edited by C. De Witt and J. Peyraud (Gordon and Breach, New York, 1975); J. A. Stamper, Phys. Fluids **18**, 735 (1975); V. B. Krapchev, Phys. Rev. Lett. **42**, 497 (1979); J. R. Cary and A. N. Kaufman, Phys. Fluids **24**, 1238 (1981); M. M. Skoric and D. ter Haar, Phys. Fluids **27**, 2375 (1984).

[4]L. D. Landau, Zh. Eksp. Teor. Fiz. **16**, 574 (1946) [*Collected Papers of Landau* (Pergamon, Oxford, 1965), p. 445]. Enlightening physical discussions on the Landau effect can be found in J. Dawson, Phys. Fluids **4**, 869 (1961); T. O'Neil, Phys. Fluids **8**, 2255 (1965); E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981), p. 124.

[5]Such longitudinal waves exist in weakly inhomogeneous plasmas; see, e.g., H. L. Berk, M. N. Rosenbluth, and R. N. Sultan, Phys. Fluids **9**, 1606 (1966).

[6]The influence of a slowly varying time dependence of the wave amplitude was discussed by H. Washimi and V. I. Karpman, Zh. Eksp. Teor. Fiz. **71**, 1010 (1976) [Sov. Phys. JETP **44**, 528 (1976)]; M. Kono, M. M. Skoric, and D. ter Haar, Phys. Rev. Lett. **45**, 1629 (1980), and references therein; D. D. Tskhakaya, J. Plasma Phys. **25**, 233 (1981).

[7]B. D. Fried and S. D. Conte, *The Plasma Dispersion Function* (Academic, New York, 1961).