# Meson Theory of the Dirac Impulse Approximation

J. A. Tjon[(a)]
*Theoretical Division and Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

and

S. J. Wallace
*Department of Physics and Astronomy and Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742*
(Received 15 October 1984)

A new formulation of the Dirac impulse approximation is discussed which connects the $p$-nucleus optical potential to a dynamical description of the relativistic $NN$ amplitude. Use of pseudovector $\pi N$ coupling eliminates rapid energy dependence found in the original impulse approximation. First calculations demonstrate success for 181 MeV with no loss of effectiveness at 500 and 800 MeV.

PACS numbers: 24.10.Ht, 13.75.Cs, 25.10.+s, 25.40.Cm

Recently the highly successful Dirac description of proton-nucleus scattering has drawn much attention.[1-3] In this approach, a basic assumption is made that the physical nucleon-nucleon scattering amplitudes can be extended to describe both positive- and negative-energy matrix elements. In Ref. 1 the five Fermi covariants[4]—scalar ($S$), vector ($V$), tensor ($T$), pseudoscalar ($P$), and axial vector ($A$)—are used to extend the positive-energy $NN$ scattering amplitudes to predict negative-energy matrix elements. The resulting impulse-approximation optical potential then predicts low-momentum-transfer virtual-pair effects which are intrinsic to scattering solutions of the Dirac equation. Excellent parameter-free fits to spin observables follow for $E = 300$ to 800 MeV.[1-3] However, the construction of an optical potential requires knowledge of the fully off-shell relativistic $NN$ amplitude and a five-term representation is not sufficient to give an unambiguous characterization.[5,6] In particular, more information is needed than just the physical scattering data. Therefore, it has become a central issue whether the Dirac successes are compatible with relativistic meson theory which, in principle, gives a complete $NN$ amplitude. We present results which show that the Dirac approach does have a foundation in meson theory.

Calculations based on meson theory accurately reproduce the observed $NN$ phase shifts with suitable choices of meson coupling constants. In particular, meson theory predicts those components which play no role in positive-energy $NN$ scattering, but which directly affect the pair couplings that are essential to the Dirac approach. We have reconstructed all the covariants of a meson-theoretical $NN$ amplitude[7] and reformulated the Dirac impulse approximation to include their effects.

There is an evident flaw in the original Dirac impulse approximation of Ref. 1 due to the prediction of overly strong scalar and vector potentials at low energy. Consequently, the impulse approximation contains overly large pair couplings that are inconsistent with the Dirac phenomenology[8] or with the low-energy, nuclear-matter optical potential of Anastasio *et al.*[9] This flaw can be traced to the way that pionic matrix elements associated with Pauli antisymmetrization are embedded as a purely exchange pseudoscalar contribution to the amplitude.[4] When the exchange pseudoscalar covariant $\tilde{P}$ of Ref. 4 is replaced by an equivalent (on positive-energy states) exchange pseudovector covariant, $\tilde{P}_V$, the resulting new version of the impulse approximation has much reduced energy dependence. However, an arbitrary coefficient times $\tilde{P}_V - \tilde{P}$ can be added to the amplitude without changing positive-energy matrix elements. Therefore, the simple replacement of $\tilde{P}$ by $\tilde{P}_V$ represents an example of the ambiguity in the representation of $NN$ amplitudes when the only constraint is to agree with experiment for the positive-energy states. Nevertheless, we find that it gives comparable scalar and vector potentials to what we obtain using the complete meson-theoretical $NN$ amplitudes discussed below. The original impulse approximation using a pseudoscalar covariant is inadequate for pseudovector $\pi N$ coupling. It omits terms with negative-energy projections which are required to represent the difference between pseudovector and pseudoscalar covariants.

A complete and unambiguous expansion of the $NN$ amplitude into sixteen classes has been developed which preserves the original impulse approximation as one class of terms but also provides for new classes of terms which vanish unless one or more of the initial- or final-state nucleons are represented by negative-energy basis states of the free Dirac equation. Reference 3 has identified the main new ingredient of the Dirac approach to be due to negative-energy propagation of just the projectile. Therefore, for simplicity, we have truncated our expansion to the four classes which are necessary to characterize completely matrix elements of particle 1 (projectile) with particle 2 restricted to positive-energy states,

$$\hat{F}=\hat{F}^{(1)}+\Lambda_{1'}^{(-)}\hat{F}^{(2)}+\hat{F}^{(3)}\Lambda_{1}^{(-)}+\Lambda_{1'}^{(-)}\hat{F}^{(4)}\Lambda_{1}^{(-)}, \tag{1}$$

where initial momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ and final momenta $\mathbf{p}_{1'}$ and $\mathbf{p}_{2'}$ are implicit and $\Lambda_1^{(-)}\equiv\Lambda_1^{(-)}(\mathbf{p}_1)$, for example, is a negative-energy projection operator in the Dirac space of particle 1. Projection operators to the left act on final states whereas ones to the right act on initial states. Each class $c=1$ to 4 of (1) is expanded in terms of eight covariants as follows:

$$\hat{F}^{(c)}=\sum_{n=1}F_n^{(c)}\mathscr{K}_n,$$
$$\mathscr{K}_n=\{S,\ V,\ T,\ A,\ P,\ \mathscr{K}_6,\ \mathscr{K}_7,\ \mathscr{K}_8\}, \tag{2}$$

where covariants 6, 7, and 8 are chosen[10] on the basis of the work of Scadron and Jones. Matrix elements of $\hat{F}$ between positive-energy helicity basis states $\phi_n(++,++)$ $(n=1,8)$ are determined by the class-1 terms, i.e., $\hat{F}^{(1)}$, of Eq. (1), and vice versa. This follows because terms involving $\Lambda^-$ projection operators cannot contribute to positive-energy matrix elements, i.e., $\Lambda^-u^+=0$, where $u^+$ is a positive-energy basis state. The new covariants $\mathscr{K}_6$, $\mathscr{K}_7$, and $\mathscr{K}_8$ have zero coefficients for class 1 because of symmetries in $NN$ scattering and thus the class-1 amplitudes are identical to those used in Ref. 1.

For helicity matrix elements such as $\phi_n(-+,++)$ or $\phi_n(++,-+)$, where one and only one negative-energy basis state of particle 1 enters, amplitudes of classes 1 and 2 or 1 and 3 play a role. Since $\hat{F}^{(1)}$ is already uniquely fixed by the $(++,++)$ case, the new matrix elements can be used to determine uniquely $\hat{F}^{(2)}$ and $\hat{F}^{(3)}$, and so on. The input negative-energy helicity matrix elements can be obtained in consistent fashion from solution of the $NN$ integral equations based on meson theory. This is what we have done to determine the new covariant amplitudes $F_n^{(c)}$ of (2) for classes $c=2$, 3, and 4. The original impulse approximation 1 follows from dropping all but the class-1 terms of Eq. (1).

Calculations of the necessary helicity amplitudes using pseudovector pion coupling have been performed with the methods developed by Tjon and van Faassen.[7] We employ a set of three-dimensional coupled integral equations to describe $NN$ and $N\Delta$ channels using the Blankenbecler-Sugar reduction. Reference 7 contains details of the $NN$ dynamics and representative results for the $NN$ phase shifts in the 0- to 1-GeV region. Although the meson-theory results are good for $(++,++)$ physical matrix elements, we have replaced these by amplitudes of Arndt *et al.* based on a recent phase-shift solution.[11] This guarantees that the on-shell information is accurate. Thus, the meson theory is used to calculate classes 2, 3, and 4. To test the model dependence, we have calculated the $NN$ amplitudes in two ways which we find give essentially the same answers for the proton-nucleus scattering observables. One way includes the $N\Delta$ coupled channel while the other does not. The $NN$ phase shifts for these two cases are quite similar for $E_{\text{lab}}=0$ to 300 MeV with suitable choices for the scalar-meson coupling constant. The case with $NN\leftrightarrow N\Delta$ coupling also gives reasonable inelasticities.

The proton-nucleus optical potential is calculated with use of classes 1 to 4 of Eq. (1). To obtain a local form for use in coordinate space, it is only necessary to approximate the $\gamma^0E(\mathbf{p})$ and $\gamma^0E(\mathbf{p}-\mathbf{q})$ in $\Lambda^-$ projection operators by $\gamma^0E$. The $\boldsymbol{\gamma}\cdot\mathbf{p}$ and $\boldsymbol{\gamma}\cdot(\mathbf{p}-\mathbf{q})$ parts of these projection operators are handled exactly. Upon Fourier transformation, the optical potential is found to contain six of the eight possible terms[12] as follows:

$$U_{\text{DIRAC}}=S(r)+\gamma^0V(r)+\tfrac{1}{2}\{\boldsymbol{\gamma}\cdot\mathbf{p},C(r)/m\}+i\boldsymbol{\alpha}\cdot\hat{\mathbf{r}}T(r)+[S_{LS}(r)+\gamma^0V_{LS}(r)]\boldsymbol{\sigma}\cdot\mathbf{L}, \tag{3}$$

where each term is calculated by folding various of the $NN$ amplitudes with nuclear densities. The anticommutator term $\{,\}$ guarantees time-reversal invariance. The $\Lambda^-$ projection operators of Eq. (1) are implicit in Eq. (3). The Born approximation based on plane-wave matrix elements of $U_{\text{DIRAC}}$ therefore receives contributions from only the class-1 amplitudes. Complete calculations based on the full set of sixteen classes yield the same structure as Eq. (3). Details will be presented elsewhere.[13] The terms $C$, $S_{LS}$, and $V_{LS}$ which involve nonlocal operators were not present in the original impulse approximation: They arise from the $\Lambda^-$ projectors in class-2, -3, and -4 amplitudes. The important scalar and vector terms also receive substantial contributions from classes 2, 3, and 4 which result in dramatically different potential strengths at low energy and substantial cancellation of the class-1 pionic contribution. Figure 1 shows effective scalar and vector potentials defined by

$$S_{\text{eff}}(r)=\frac{S(r)-C(r)}{1+C(r)/m},$$
$$V_{\text{eff}}(r)=\frac{V(r)+(E/m)C(r)}{1+C(r)/m}$$

which are appropriate to the coordinate-space Dirac equation obtained when the $C/m$ term of Eq. (3) is removed exactly by definition of a new wave function $\psi\rightarrow[1+C(r)/m]^{-1/2}\tilde{\psi}$. The calculations are for

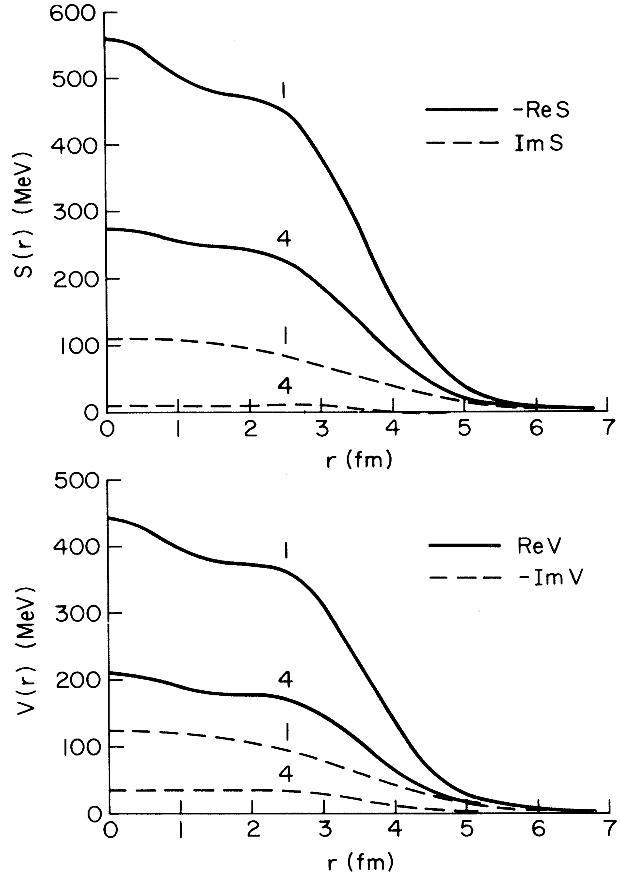


FIG. 1. Scalar and vector potentials for 181-MeV scattering by $^{40}$Ca. Curves labeled 1 include class-1 amplitudes; those labeled 4 include amplitudes of classes 1 through 4.

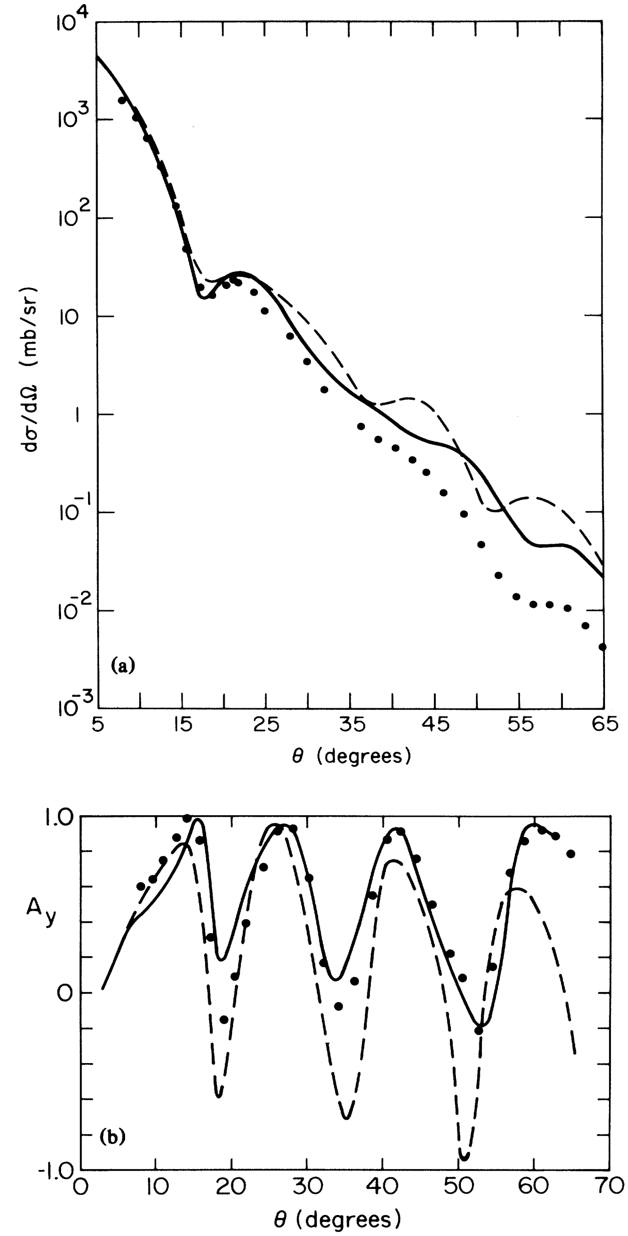


FIG. 2. (a) Differential cross section and (b) analyzing power for 181-MeV scattering by $^{40}$Ca compared with data of Ref. 15. Dashed curve is based on class 1; solid curve includes amplitudes of classes 1 through 4.

181-MeV proton energy and for a $^{40}$Ca target using a nuclear density based upon relativistic Hartree wave functions of Horowitz and Serot.[14] The matrix elements in the nuclear space include positive-energy projectors to be consistent with the truncation of the full *NN* amplitude given by Eq. (1). There are significant but much smaller changes to the potentials at the higher energies. Our calculations at 500 and 800 MeV show improved results for cross sections and comparable results for spin observables to those found in the original impulse approximation. The tensor ($T$), scalar spin-orbit ($S_{LS}$), and vector spin-orbit ($V_{LS}$) potentials are all about 1 MeV or less for 181 MeV. They are of minor importance.

Figure 2 compares proton scattering data of Arnold *et al.*[15] with results based on solution of the Dirac equation using the potentials described above. The meson-theory extension significantly improves the impulse approximation for this case. We believe that the new formulation is essential to further progress for two reasons. First, the extension of the *NN* amplitudes to Dirac operators based on just the five nonvanishing class-1 amplitudes would be correct only if all the other classes were essentially zero. This is far from true at low energy for the preferred nuclear physics model employing pseudovector $\pi N$ coupling. Secondly, the very rapid energy dependence and the overly large $S$ and $V$ components of the class-1 amplitudes, both of which can be traced to pionic contributions, must be suppressed to obtain reasonable low-energy optical potentials.

In conclusion, there are three main points. (1) A new approach to the Dirac optical potential has been developed which provides a meson-theoretical basis for virtual-pair couplings. (2) Starting from conventional *NN* integral equations and meson theory, a complete set of Lorentz-invariant amplitudes has been calculated for the first time. This eliminates ambiguities due to the choice of covariants which have heretofore been assumed in the Dirac impulse approximation. (3) Proton-nucleus calculations show that the new approach has much reduced energy dependence of scalar and vector potentials and it provides a more accurate description of experimental data at 181 MeV.

The authors acknowledge support by the U. S. Department of Energy under Contract No. DE-AS05-86ER-05126, by the Institute of Physical Sciences and Technology, University of Maryland, by the University of Maryland Center for Theoretical Physics, and by Los Alamos National Laboratory. In addition, we are grateful for the computer support by the University of Maryland Computer Science Center, Los Alamos National Laboratory, and the University of Maryland Experimental Nuclear Physics Computer Facility.

[a]Permanent address: Institute for Theoretical Physics, Princetonplein-5, P.O. Box 80.006, 3508 TA Utrecht, The Netherlands.

[1]J. A. McNeil, J. R. Shepard, and S. J. Wallace, Phys. Rev. Lett. **50**, 1439 (1983); J. R. Shepard, J. A. McNeil, and S. J. Wallace, Phys. Rev. Lett. **50**, 1443 (1983); J. A. McNeil, L. Ray, and S.J. Wallace, Phys. Rev. C **27**, 2123 (1983).

[2]B. C. Clark, S. Hama, R. L. Mercer, L. Ray, and B. D. Serot, Phys. Rev. Lett. **50**, 1644 (1983); B. C. Clark *et al.*, Phys. Rev. Lett. **51**, 1809(C) (1983); B. C. Clark, S. Hama, R. L. Mercer, L. Ray. G. W. Hoffmann, and B. D. Serot, Phys. Rev. C **28**, 1421 (1983).

[3]M. V. Hynes, A. Picklesimer, P. C. Tandy, and R. M. Thaler, Phys. Rev. Lett. **52**, 978 (1984).

[4]M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. **120**, 2250 (1960).

[5]D. L. Adams and M. Bleszynski, Phys. Lett. **136B**, 10 (1983).

[6]M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. (N.Y.) **2**, 226 (1957).

[7]J. A. Tjon and E. van Faassen, Phys. Lett. **120B**, 39 (1983); E. E. van Faassen and J. A. Tjon, Phys. Rev. C **28**, 2354 (1983), and **30**, 285 (1984).

[8]B. C. Clark, S. Hama, and R. L. Mercer, in *The Interaction Between Medium Energy Nucleons in Nuclei— 1982,* edited by H. O. Meyer, AIP Conference Proceedings No. 97 (American Institute of Physics, New York, 1983), p. 260.

[9]M. R. Anastasio, L. S. Celenza, W. S. Pong, and C. M. Shakin, Phys. Rep. **100**, 327 (1983), and references therein.

[10]M. D. Scadron and H. F. Jones, Phys. Rev. **173**, 1734 (1963).

[11]R. A. Arndt, L. D. Roper, R. A. Bryon, R. B. Clark, B. J. VerWest, and P. Signell, Phys. Rev. D **28**, 97 (1983); R. A. Arndt, private communication.

[12]L. S. Celenza and C. M. Shakin, Phys. Rev. C **28**, 1256 (1983).

[13]J. A. Tjon and S. J. Wallace, unpublished.

[14]C. J. Horowitz and B. D. Serot, Nucl. Phys. **A368**, 503 (1981).

[15]L. G. Arnold, B. C. Clark, R. L. Mercer, and P. Schwandt, Phys. Rev. C **23**, 1969 (1981).