Vector-Meson Mass Generation by Chiral Anomalies

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We show that the chiral Schwinger model, which is anomalous, yields a consistent and unitary, although not gauge-invariant, theory. The model is exactly solvable and contains a free massive vector boson plus harmonic excitations.

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Quantum field theories with gauge couplings to chiral fermions can suffer from anomalous nonconservation of the gauge current.[1] This leads not only to loss of gauge invariance, but also threatens the existence of a consistent theory. In particular, unitarity and renormalizability are called into question.[2] The potential inconsistency arises because the field equation for the gauge field tensor, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+[A_\mu,A_\nu]$, namely,

$$D^\mu F_{\mu\nu}=J_\nu, \tag{1}$$

implies

$$D^\nu J_\nu = D^\nu D^\mu F_{\mu\nu}=0, \tag{2}$$

while at the same time, thanks to the anomaly,

$$[D^\nu J_\nu]^a = \pm(i\hbar/24\pi^2)\,\mathrm{Tr}[T^a\partial_\mu\epsilon^{\mu\alpha\beta\gamma}(\partial_\alpha A_\beta A_\gamma+\tfrac{1}{2}A_\alpha A_\beta A_\gamma)], \tag{3}$$

where T^a are antiunitary fermion representation matrices and D denotes covariant differentiation. The conventional remedy for this problem is to adjust the fermion content of the theory so that the trace in (3) vanishes, thereby removing the anomaly. For the standard electroweak theory, this criterion yields the so-far successful prediction that the number of quarks equals the number of leptons.[2,3] Another type of remedy that has been used to deal with anomalies involves modifying gauge field dynamics instead of fermion content. In $(2+1)$-dimensional non-Abelian theory with massless fermions, the gauge noninvariance of the fermionic determinant can be compensated for by the addition of a Chern-Simons term in the gauge field action with half-integral coefficient.[4] In four dimensions, a similar compensation is achieved by adding scalar fields to the system, carrying a gauge noninvariant, nonpolynomial Wess-Zumino term.[2,5] In all these modifications, gauge invariance is restored to the theory by changing the original bosonic action, although renormalizability in the presence of the nonpolynomial addition remains unclear.

In this Letter, we adopt an alternative approach. We would like to consider an anomalous chiral gauge model without changing its fermion content or making *ad hoc* changes in gauge field dynamics, and examine whether, by simply giving up gauge invariance, a consistent and unitary theory can be rescued from the model. We have nothing to say about renormalizability; indeed, we work in $(1+1)$ dimensions, where we show that the chiral Schwinger model [a U(1) gauge field coupled to chiral fermions], although anomalous, does yield a consistent unitary theory. We exploit the fact that the fermionic determinant as well as the anomaly have some arbitrariness associated with the regularization of fermionic radiative contributions. This arbitrariness cannot be used, in this theory, to make the current divergence free and restore gauge invariance. But, we show that it can be used to define a unitary theory with a consistent set of propagating solutions to the field equations. The divergence of the current (i.e., of the source for $\partial_\mu F^{\mu\nu}$), although not identically zero, vanishes by virtue of these Heisenberg field equations. There is then no inconsistency between the analog of Eqs. (2) and (3) in this theory. Further, the resulting theory has the spectrum of a free massive vector particle plus massless harmonic excitations. Thus, at least in this simple model, the presence of the anomaly, together with the demand for a consistent unitary theory with particle interpretation, forces the spontaneous breaking of gauge symmetry through the generation of a mass for the gauge boson.

The chiral Schwinger model is described by the action

$$I=\int d^2x\,\{-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}+\hbar\bar\psi[i\partial\!\!\!/+e\sqrt{\pi}A\!\!\!/(1+i\gamma_5)]\psi\}, \tag{4}$$

where $\gamma_5=i\gamma^0\gamma^1$. The fermionic determinant for this two-dimensional problem can be exactly evaluated,[1] giving the gauge field an effective action

$$I_g(A)=\int d^2x\left\{-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\hbar e^2}{2}A_\mu\left[ag^{\mu\nu}-(g^{\mu\alpha}+\epsilon^{\mu\alpha})\frac{\partial_\alpha\partial_\beta}{\Box}(g^{\beta\nu}-\epsilon^{\beta\nu})\right]A_\nu\right\}, \tag{5}$$

$$I_g(A)=\tfrac{1}{2}\int d^2x\, A_\mu M^{\mu\nu}A_\nu. \tag{6}$$

In (5), a is a constant not uniquely determined by the different procedures for calculating the fermionic determinant since it governs a local contribution. We are free to choose it to suit our requirements on the theory. If the model were not anomalous, a would have been fixed by gauge invariance. Here, no choice of a can restore gauge invariance. Other considerations could be used to pick a. For instance, $a=0$ is consistent with the fact that the chiral fermions in Eq. (4) couple only to the combination $(g^{\mu\nu}-\epsilon^{\mu\nu})A_\nu$, since $i\gamma^\mu\gamma_5=\epsilon^{\mu\nu}\gamma_\nu$. The choice of $a=1$ may be preferred on the mathematical grounds that it reduces the anomaly to the pure differential form $\epsilon^{\mu\nu}\partial_\mu A_\nu$, which descends through coboundary operations starting from the four-dimensional Pontryagin density.[6]

We shall, however, leave a arbitrary and pursue the consequences. Notice that the nonlocal action (5) can be written in local terms by introducing an auxiliary scalar field $\phi(x)$,

$$\exp[(i/\hbar)I_g(A)]=\int[D_\phi]\exp[(i/\hbar)S(A,\phi)], \tag{7}$$

with

$$S(A,\phi)=\int d^2x\left[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)+\sqrt{\hbar}e(g^{\mu\nu}-\epsilon^{\mu\nu})\partial_\mu\phi A_u+\tfrac{1}{2}a\hbar e^2A_\mu A^\mu\right]. \tag{8}$$

Equation (8) is just the bosonized version of the fermionic action (4), where the arbitrary constant a can be viewed as reflecting bosonization ambiguities. The action (8) yields the following coupled field equations:

$$\Box\phi=-e\sqrt{\hbar}\,\partial_\mu(g^{\mu\nu}-\epsilon^{\mu\nu})A_\nu \tag{9a}$$

and

$$j^\nu=\partial_\mu F^{\mu\nu}=-\sqrt{\hbar}e(g^{\nu\alpha}+\epsilon^{\nu\alpha})\partial_\alpha\phi-a\hbar e^2A^\nu. \tag{9b}$$

On inserting (9a) into (9b), one gets

$$\partial_\mu F^{\mu\nu}+a\hbar e^2A^\nu-\hbar e^2(g^{\nu\alpha}+\epsilon^{\nu\alpha})\frac{\partial_\alpha\partial_\beta}{\Box}(g^{\beta\mu}-\epsilon^{\beta\mu})A_\mu=0. \tag{10}$$

Equation (10) also follows directly from the gauge field effective action (5). After a little algebra, Eq. (9) can be seen to imply

$$(\Box+m^2)\sigma(x)=0, \tag{11}$$

where

$$m^2=\hbar e^2a^2/(a-1),$$

$$\sigma(x)=\phi(x)+h(x),$$

and

$$A_\mu=-(1/\sqrt{\hbar}ea)[\partial_\mu\phi+(1-a)\epsilon_{\mu\nu}\partial^\nu\phi-a\epsilon_{\mu\nu}\partial^\nu h]. \tag{12}$$

In (12), the function h is a harmonic function ($\Box h=0$). Thus, we see that as long as $a>1$, our system consists of (i) a free massive degree of freedom described by the field σ, and (ii) harmonic excitations propagating on the light cone, described by the field h. For later use, note that the combination $F=\epsilon^{\mu\nu}\partial_\mu A_\nu$ obeys the same free massive Klein-Gordon equation (11) as obeyed by σ.

These conclusions can be confirmed by examining the propagator of this theory obtainable directly from the action (5). The propagator $G_{\mu\nu}$ [the inverse of the operator $-(i/\hbar)M^{\mu\nu}$ in (6)], is, in momentum space,

$$G_{\mu\nu}=\frac{i\hbar}{k^2-m^2}\left\{-g_{\mu\nu}+\frac{1}{a-1}\left[k_\mu k_\nu\left(\frac{1}{\hbar e^2}-\frac{2}{k^2}\right)-\epsilon_{\mu\alpha}\frac{k^\alpha k_\nu}{k^2}-\epsilon_{\nu\alpha}\frac{k^a k_\mu}{k^2}\right]\right\}. \tag{13}$$

The residue matrix of $-iG_{\mu\nu}$ at the pole $k^2=m^2$ has two eigenvalues, one of which vanishes and the other is postive for $a>1$. The residue matrix of the pole at $k^2=0$ also has one vanishing eigenvalue and another equal to

$(2/\hbar a^2 e^2)(k_0+k_1)^2$. This confirms that for $a>1$, the system is unitary and contains one massive free degree of freedom, plus the massless excitations which are self-dual in the sense that the associated pole is really in (k_0-k_1). We presume that these are pairs of unconfined fermions.

Note the following: (i) $a=1$ is a singular point where the mass m diverges. The massive vector meson disappears and one is left with just the harmonic excitations. (ii) For $a<1$, the theory produces tachyons and is not unitary. (iii) All values of $a>1$ yield a sensible theory, so that the $(\text{mass})^2$ of the gauge meson, $m^2=\hbar e^2 a^2/(a-1)$, is left undetermined in this model and can lie anywhere between $4\hbar e^2$ and infinity. This is in contrast to the Schwinger model, where gauge invariance fixes the constant a and the gauge boson mass.

It is also instructive to consider the energy-momentum tensor for this system. Recall that for a free massless Dirac field, the energy-momentum tensor $\theta^{\mu\nu}_{\text{free}}$ can be written in terms of currents[7]:

$$\theta^{\mu\nu}_{\text{free}}=\pi(j^\mu j^\nu-\tfrac{1}{2}g^{\mu\nu}j^\alpha j_\alpha). \tag{14}$$

The same form occurs in the matter portion of the energy-momentum tensor for our system and the current j^ν of Eq. (9b) appears. The symmetric energy-momentum tensor obtained from the Lagrangian in Eq. (8) is

$$\theta^{\mu\nu}=\left[\partial^\mu\phi\,\partial^\nu\phi+\sqrt{\hbar}e\,(A^\mu\partial^\nu\phi+A^\nu\partial^\mu\phi)+\hbar ae^2A^\mu A^\nu-\tfrac{1}{2}g^{\mu\nu}(\partial_\alpha\phi\,\partial^\alpha\phi+\hbar ae^2A_\alpha A^\alpha+2\sqrt{\hbar}eA^\alpha\,\partial_\alpha\phi)\right]$$
$$+[-F^{\mu\alpha}F^\nu_\alpha+\tfrac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}]. \tag{15}$$

Upon using the field equations (9) and their solution (12), the above can be written as

$$\theta^{\mu\nu}=m^{-2}[j^\mu j^\nu-\tfrac{1}{2}g^{\mu\nu}j^\alpha j_\alpha]+[-F^{\mu\alpha}F^\nu_\alpha+\tfrac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}]+[\partial^\mu h\,\partial^\nu h-\tfrac{1}{2}g^{\mu\nu}\partial^\alpha h\,\partial_\alpha h]. \tag{16}$$

The second term is the gauge field contribution, while the third term represents the harmonic excitations. The first term, formed from currents, and the last arise from the matter portion of (15). The first two terms can be combined by use of $j^\nu=\partial_\mu F^{\mu\nu}=\epsilon^{\nu\mu}\partial_\mu F$ to obtain finally an expression in which the single massive excitation F is seen explicitly:

$$\theta^{\mu\nu}=m^{-2}[\partial^\mu F\,\partial^\nu F-g^{\mu\nu}(\tfrac{1}{2}\partial^\alpha F\,\partial_\alpha F-\tfrac{1}{2}m^2F^2)]+[\partial^\mu h\,\partial^\nu h-\tfrac{1}{2}g^{\mu\nu}\partial^\alpha h\,\partial_\alpha h]. \tag{17}$$

In four dimensions, as well as for non-Abelian theories in two dimensions, the field equations are nonlinear and we have not analyzed them. But note that gauge invariance again cannot be maintained with chiral fermions. Hence, gauge noninvariant mass contact terms cannot be excluded and may generate a mass for the vector meson, although the renormalizability of such a theory remains unclear. If our mechanism can operate in physically relevant models, it raises the intriguing possibility that the vector mesons mediating weak interactions become massive because the underlying gauge symmetry is "anomalously" broken by chiral fermions, which are deconfined.

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[1]For a review, see R. Jackiw, in *Relativity, Groups and Topology II,* edited by B. DeWitt and R. Stora (North-Holland, Amsterdam, 1984).

[2]D. Gross and R. Jackiw, Phys. Rev. D **6**, 477 (1972).

[3]C. Bouchiat, J. Iliopolous, and Ph. Meyer, Phys. Lett. **38B**, 519 (1972).

[4]N. Redlich, Phys. Rev. Lett. **52**, 18 (1984), and Phys. Rev. D **29**, 2366 (1984); L. Alvarez-Gaumé and E. Witten, Nucl. Phys. **B234**, 269 (1984).

[5]It has been shown that in a gauge theory with Higgs fields and paired chiral fermions, a Wess-Zumino term is induced when the mass of one of the fermions tends to infinity; see E. D'Hoker and E. Farhi, Nucl. Phys. **B248**, 59, 77 (1984).

[6]A. Niemi and G. Semenoff, Phys. Rev. Lett. **51**, 2077 (1983); L. D. Faddeev, Phys. Lett. **145B**, 81 (1984); B. Zumino, Y. S. Wu, and A. Zee, Nucl. Phys. **B239**, 427 (1984).

[7]H. Sugawara, Phys. Rev. **170**, 1659 (1968); C. Sommerfield, Phys. Rev. **176**, 2019 (1968); S. Coleman, D. Gross, and R. Jackiw, Phys. Rev. **180**, 1359 (1969).